ELECTRIC MOTORS

CONTINUOUS CURRENT MOTORS AND INDUCTION MOTORS

Their Theory and Construction

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BY

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With 480 Illustrations

WHITTAKER AND CO.

2 WHITE HART STREET, PATERNOSTER SQUARE, LONDON

AND 66 FIFTH AVENUE, NEW YORK

1904

PREFACE

THIS treatise is characterised by considerable departures from conventional methods, and it is believed that more new light is thereby thrown on the subject than would otherwise have been practicable.

The work is based on a series of articles just completed in *Traction and Transmission*, and what usefulness it may possess is due in large measure to the courtesy of many engineers and manufacturers who have so liberally supplied the author with examples of their modern designs.

In a growing subject, in which one adopts improvements to the utmost extent that other considerations permit, thorough consistency is difficult to attain.

To his friend, Mr Theodore Stevens, E.M., A.M.Inst.C.E., A.M.I.E.E., who has carefully arranged the publication in its present form, the author wishes to express his sincere thanks.

The author trusts that the present volume may in some small degree usefully supplement the excellent treatises of Kapp, Thompson, Arnold, Hawkins and Wallis, and others, which have been so helpful to him in his own practice.

A table of usefully arranged data on copper conductors will be found at the end of the volume.

H. M. H.

LONDON, *June* 1904.

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PART I.—CONTINUOUS CURRENT MOTORS

CHAPTER I

INTRODUCTORY

§ 1. Relative Merits of Alternating and Continuous Current Motors.—On the continent of Europe and in America a considerable percentage of the electric motors employed for other than tramway purposes are of the induction type. In England, however, the continuous current motor has not as yet met with any very serious competition from motors of the alternating current class. This is partly attributable to the very early and extensive introduction of the single-phase alternating current system in England, that system having made here earlier and more rapid progress than in other lands. Hence, at the time of the first induction motors, and the almost simultaneous demonstration of their adaptability to single-phase working, it was inevitable that the extension of the transmission of the electric energy to power purposes, as distinguished from its use for lighting, should in England have tended to await the further development of the single-phase motor in cases where continuous current was not available. But although the last ten years have witnessed a certain amount of improvement in the polyphase motor, single-phase motors still remain far behind. These and other causes, such as the patent situation, and the absence of large water powers in England, have resulted in retarding the introduction of the polyphase induction motor to anything near the extent which has occurred in many other countries.

In view, however, of the present state of development of con-

tinuous current and alternating current motors, this may be of considerable advantage, for there are now good grounds for the opinion that by those systems of distributing power, in which the final transformation into mechanical energy is by means of the continuous current motor, more satisfactory results are, on the whole, to be obtained, not only for traction purposes, but also for motors in factories, mills, and mines, than by the use of alternating current motors, whether single or polyphase. The present tendency seems to be to reconsider the merits of the continuous current motor. At the time of the first introduction of the polyphase motor much was hoped from it on the score of the possibility of avoiding all moving contacts, and on account of the economy of transmitting at high voltage, transforming to a lower voltage by means of apparatus without moving parts, and employing the lower voltage in a motor in which the absence of all moving contacts in the conducting circuits was pointed to as a striking improvement over motors requiring commutators and brushes.

§ 2. **Present Stage of Development.**—But since that time the continuous current motor has undergone great improvement, doubtless in part owing to its threatened supersession by the alternating current motor; and the best motors of to-day are characterised by a complete absence of sparking at the commutator, while the collection of the current with fixed brush position at all loads is in all respects as satisfactory as can be obtained at the slip-rings of induction motors. For the induction motor's development has not generally been on the lines of the earlier machines, with no moving contacts. This type had serious faults, which have considerably limited its use. Prominent among these faults is the low value of the starting torque per ampere input. Far the greater number of large induction motors now in use have slip-rings and starting resistances. Many of those of the type without slip-rings employ a compensator in the primary circuit to reduce the amperes input at starting, and this requires a rearrangement of connections after the motor has started up. It is well known that there is a field excellently served by the induction motor, but, in the light of the above considerations, it is less evident than is sometimes assumed to be the case that its use, in general, leads to greater simplicity and usefulness than does the use of the continuous current motor. A further well-known objection relates to the low power factor of the induction motor, and in the latest device for overcoming this fault we have an induction motor with a multi-segment com-

mutator. It may be well to further mention that one of the most promising lines for developing single-phase motors involves the employment of a commutator. Although in the ordinary polyphase motor the power factor in all but the smallest sizes may exceed 0·90 at full load, it is very low when the motor is running light and with small loads, and this seriously affects the "all-day" efficiency of a plant where any considerable proportion of the load consists in induction motors. The important bearing of this consideration will be the more evident when taken in connection with the subject of "all-day" economy in the opening section on continuous current motors.

The transformation from the high voltage of the transmission line down to the low voltage of distribution to the motors, without resorting to apparatus with moving parts, has been looked upon as leading to an important group of advantages gained by employing alternating current motors. This was much more the case in the earlier days, when plants were on a far smaller scale, and when it was customary to instal numerous small transformers scattered all over the area supplied. But since the extended introduction of the practice of transforming in bulk at a few large sub-stations, rotating apparatus is again coming into fairly general use, with a correspondingly great increase in the flexibility of the transformation. Where this is carried out by motor generators, although the cost may be a little higher, the regulation may be much more satisfactorily provided for than by rotary converters and auxiliary static transformers.

§ 3. **Advantages of Motor Generators.**—Motor generators permit of better automatic control of the secondary voltage than can be obtained with static transformers. One may accomplish, by employing motor generators, transformations not only of the voltage and current, but also of frequency and phase, as well as transformations from alternating to continuous current and the reverse. Not only are such arrangements superior to static transformers on the score of ability to automatically control the voltage regulation, but the question of ventilation of the apparatus, as related to economical design for a given permitted temperature, is much simplified.

But little success has as yet attended the employment of the induction motor for traction; one of the leading objections relates to the question of obtaining variable speed. The same difficulty also limits the field of application of the induction motor for stationary work, since all methods as yet devised for obtaining

wide variations in the speed of the induction motor require serious sacrifices of its other good properties. In the section treating of induction motors the means for varying the speed will be described.

These two types of motors—the continuous current motor and the alternating current induction motor—have now been in competition for some ten years, and after considering the results achieved by them, one inclines to the opinion that the continuous current motor will not only maintain its present position in England, but that, in America and on the continent of Europe, there will be an increasing tendency to employ it in mills, mines, factories, and shipyards, even in the many cases where the electric energy will, at the source, be generated in the form of alternating currents.

No direct mention has yet been made of the synchronous type of alternating current motor. This is widely employed in large sizes, and it may be expected to be yet more used in its own somewhat restricted field. At present its chief usefulness is in transforming stations, generally as the motor member of motor generator sets.

CHAPTER II

CONTINUOUS CURRENT MOTORS

§ 1. **The Two Chief Classes of Continuous Current Motors.**—These are termed respectively shunt motors and series motors. The former find their widest use as constant speed stationary motors, the latter for tramways, and for cranes, hoists, and similar work, where wide ranges of torque and speed are required. No precise subdivision can be made, as there is the intermediate class of compound-wound motors.

Shunt motors are most useful in workshops, mills, and factories, for driving machine tools, looms, lines of shafting, etc. In very many instances of their application there is an analogy between the conditions attending their use and those attending the use of alternating current static transformers. This is referred to at the very start, because, being such a familiar subject and one of admitted importance, as relating to alternating current transformers, and being also of considerable, though less recognised, importance for motors, it should call for consideration now, and be taken into account so far as affects the design of the motor. As will be seen from the following paragraphs, it affects the lines of the design to a great extent.

§ 2. **Low-Load Losses.**—It was long ago pointed out that, inasmuch as alternating current transformers are in circuit for many hours, and are fully loaded for but a small part of the time, economical operation is only possible to the extent to which their "no-load" losses are low. The appreciation of this fact gradually led to great modifications in the design of the transformer, and those to-day on the market have "no-load" losses of one-third and less of the corresponding losses in their predecessors of twelve years ago.

In the alternating current transformer the "no-load" loss is merely that occurring in the laminated iron core due to the periodic

reversals of the magnetic flux in the core. This loss is, at constant voltage, a constant at all loads. The remaining loss is the I^2R loss in the copper windings, which increases as the square of the load.

To design a transformer for minimum losses at no load, one must let the I^2R loss be as high as is consistent with suitably low temperature when running at full load and with suitable regulation in the secondary circuit. In large transformers heating is generally the limit. In small transformers the required regulation is more often the limit—the "percentage drop in voltage at full load."

In attempting to secure lower core losses in transformers one encounters these limitations of heating and regulation, and, in fact, good regulation is still more difficult on a motor load because of the "wattless" component of the current. The most successful direction in which to decrease the core loss for a given required regulation was in increasing the number of turns in the windings (also suitably intermixing the primary and secondary coils, and to a greater extent the more inductive the nature of the load) and decreasing the cross section and weight of iron in the magnetic circuit.

In a shunt motor the "no-load" losses are made up, not only of the core loss, but of the mechanical friction losses as well, and of the loss in the shunt winding. These losses are present at about constant value at all loads. The I^2R losses in the armature winding and in the brush contact resistance increase as the square of the load, and hence, in the interest of low losses at no-load and at light loads, one increases the turns on the armature, increasing thus the I^2R loss and decreasing the above enumerated constant losses. In the shunt motor it is of much less importance than in the transformer to keep the IR drop low, and in such motors we may, in the light of available data, design a thoroughly satisfactory motor as regards freedom from sparking and heating, by employing relatively many turns in the armature winding, and a small cross section of the magnetic circuit. If a constant speed is desired, it may be obtained by giving to the brushes, as their permanent working position, such a number of segments of backward lead that the armature reaction slightly weakens the field as the load increases, which will result in off-setting the resistance drop in the armature winding. As a matter of fact, however, the conditions of operation of the shunt motor are rarely such that a few per cent. greater drop in speed at full load is of any consequence.

But in the alternating current transformer a regulation of 2 per cent. throughout the range of load is generally required, and the drop on inductive load must also be low.

§ 3. **Speed Regulation and Brush Position.**—To better understand the dependence of the speed regulation upon the brush position, the curves of Fig. 1 should be examined. These curves show how the speed would vary as the load increases, with the brushes set respectively with backward lead, in the neutral position, and with forward lead. One sees that in this feature the shunt motor is, as regards speed regulation, capable of being made more perfect than a static transformer in respect to voltage

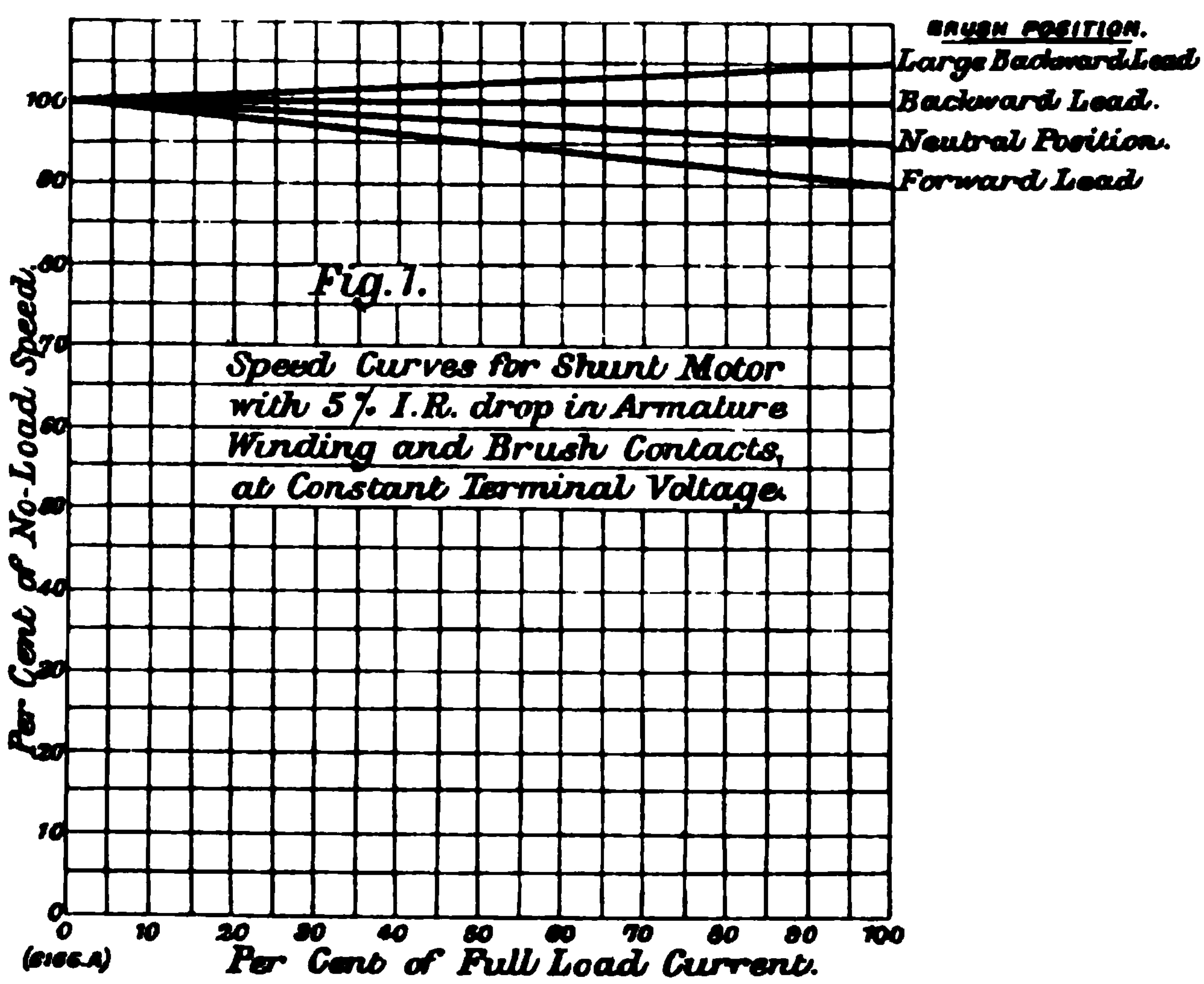


FIG. 1.—Relation of Speed Regulation to Position of Brushes.

regulation, and this point has a rather important bearing in the choice of transforming apparatus for sub-stations. Better automatic control of the voltage may be obtained by suitably designed motor generator sets than by stationary transformers.

§ 4. **Internal Voltage.**—When the voltage available at the terminals of the motor is of a given constant value, say 500 volts, then the internal voltage is, at no load, also practically 500 volts, because the resistance drop is the negligible value corresponding to the current input to the unloaded motor, multiplied by the sum of the resistance of brush contacts and armature winding. For this condition of the motor running with no load, let

E = Electromotive force, "internal voltage,"

T = Armature turns in series between positive and negative brushes,

N = Periodicity of reversals of the magnetic flux in the armature core, expressed in complete cycles per second,

M = Magnetic flux linked with the armature turns, T,

then

$$E = 4TNM \times 10^{-8}$$

For motors it is more convenient to substitute for the periodicity, N, in complete cycles per second, the speed S, in revolutions per minute, and the number of poles, P, of the motor—

$$N = \frac{S}{60} \times \frac{P}{2}$$

and the general formula becomes

$$E = 4 \times T \times \frac{S}{60} \times \frac{P}{2} \times M \times 10^{-8}$$
$$= 3{\cdot}33\ TSPM \times 10^{-6}$$

Now for a shunt motor the excitation will be supplied at constant terminal voltage, hence, except in so far as armature reaction (which, of course, also exerts a magneto-motive force as truly as the field excitation) intervenes, the magnetic flux, M, is also of constant value, and the speed, S, of the motor will be, at no load, when there is but negligible armature current, and hence, also, negligible armature magneto-motive force, in revolutions per minute, of the value

$$S = \frac{E \times 10^6}{3{\cdot}33\ TPM}$$

But when the motor is loaded to such a value that it consumes a current, I, in amperes, then if R be the resistance in ohms, of brushes, brush contacts, and armature winding, the internal voltage will be less than the terminal voltage. If the terminal voltage is 500, then the internal voltage, which is the value which should be entered in the formula, will be $500 - IR$, or, in general, denoting by V the voltage at the terminals of the motor, then, $E = V - IR$, and the formula for the speed becomes

$$S = \frac{(V - IR)\ 10^6}{3{\cdot}33\ TPM}$$

or, more precisely, this formula approximates to the truth to the extent that the magneto-motive force of the armature does not

come into consideration. Until recently the tendency has been to make the armature of but moderate strength as expressed in armature ampere turns per pole piece, and of low resistance. For such motors the above formula would be fairly true; in fact, the backward lead generally given to the brushes enables the slight armature magneto-motive force to exert a small component in opposition to the field magneto-motive force, so that the fall in speed, for constant terminal voltage, V, would be slightly less as the load increases than would be given by the formula.

§ 5. **All-day Economy.**—But motors designed on lines to give minimum "no-load" losses, and hence high "all-day" economy, are characterised by strong armatures (expressed in armature ampere turns per pole piece), and high armature resistance from positive to negative brushes; hence by considerable voltage drop (IR drop) in the armature, and, but for a suitable use of armature magneto-motive force, or of a negative compound winding, the speed would decrease considerably with the load. Such decrease in speed is generally not objectionable; but the remedy, where desired, is at hand, for a moderate backward lead gives, in the case of a strong armature, a fairly considerable magneto-motive force in opposition to that of the field magnet windings. From this it results that in the formula—

$$S=\frac{(V-I\,R)\cdot 10^6}{3{\cdot}33\ TPM}$$

we may have M, in the denominator, automatically decrease in value at nearly the same rate that (V−IR) in the numerator decreases in value, and thus the speed, S, remains nearly constant as the load—and I—vary.

§ 6. **Armature Interference.**—In *Engineering* for September 16th, 1898, page 349, was given a series of curves representing the results of tests on armature interference. These are reproduced in Fig. 2.

These tests were made upon a four-pole machine with a four-circuit drum winding, with seventy-nine coils of six turns each, in seventy-nine slots in the periphery. There were, therefore, 119 turns per pole piece on the armature. The armature current being 71·5 amperes, there were 18 amperes per turn, or $18\times119=2140$ ampere turns per pole piece on the armature. The areas of the curves, which are proportional to the flux entering the armature, are as follows:—

				PER CENT.
A	49 square centimetres		=	100
B	49	„ „	=	100
C	36	„ „	=	74
D	27	„ „	=	55
E	20	„ „	=	41

For curves A and B, the demagnetising component was zero, because the brushes were at the geometrical neutral position, but

Fig. 2.—Diagram of Tests of Armature Interference.

while in A there was no current in the armature, and hence no distortion, the full load armature current present in the case of curve B exerted maximum distortion. But it was pointed out in that article that the results of the tests showed no decrease in the area either of curve B or of any of the curves due to distortion:

the decrease, where existing, could always be accounted for by armature reaction.

In a paper by Hawkins and Wightman, in vol. xxix. (1900) of the *Journal of the Institution of Electrical Engineers*, the authors give the following clear explanation of the absence of any increase in impressed magneto-motive force required to overcome distortion: "It is true that a symmetrical distribution of the field requires the mimimum excitation or magneto-motive force; but when there is distortion, the additional magneto-motive force that is required by the longer and more saturated paths of the lines is just what is supplied by the cross ampere turns of the armature, and corresponds to the self-induction of the armature winding. The only effect which the saturation of the iron of the armature or magnets, or of the teeth in a toothed core, has, is to cause any

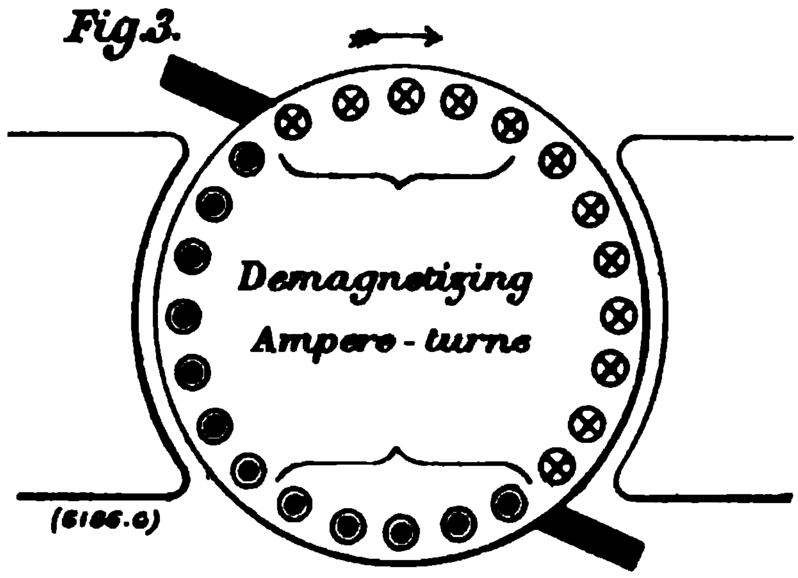


FIG. 3.—Demagnetising Ampere Turns.

two corresponding portions of the cross section to have different permeabilities, so that the magneto-motive force of the cross ampere turns, instead of being divided equally over the two air gaps, or portions of the cross circuits, is divided proportionally to their different reluctances. Thus the saturation of any part of the magnetic circuit simply reduces the distortion, but never reduces the total flux."

Hence it appears that for the approximate estimations one may disregard the distorting component of the armature ampere turns, and consider merely the directly demagnetising component. This will be equal to the ampere turns on the armature included between the double angle of backward lead, as shown in brackets in the diagram Fig. 3.

Suppose, for example, that a motor has, from the field spool windings, an excitation which may be represented by 100, and that at full load the total armature strength in armature ampere

turns per pole piece may also be represented by 100. Then, with a 5 per cent. backward lead of the brushes, the strength of the directly demagnetising component will be $2 \times \cdot 05 \times 100 = 10$, and the resultant magneto-motive force will have been reduced from no load to full load in the proportion of 100 : 90, and the magnetic circuit may be assumed to be worked at such a point of the saturation curve that the flux corresponding to these two values of the magnetisation will be respectively 100 and 93. Thus M in the denominator of the formula

$$S = \frac{(V - IR)\,10^6}{3{\cdot}33\ TPM}$$

for the speed of the motor has been decreased 7 per cent., and the internal drop (IR) may be 7 per cent., or, for a 500-volt motor, $\cdot 07 \times 500 = 35$ volts. If this were a 20-brake horse-power (20 B.H.P.) motor, the current input, I, at full load, would be about 34 amperes, hence the resistance, R, of armature winding, plus brush contacts, must be $\frac{34}{35} = 0{\cdot}97$ ohm. If the resistance is less than 0·97 ohm, then, with 5 per cent. backward lead of the brushes, the speed should increase slightly with the load. With a resistance greater than 0·97 ohm there would be a corresponding decrease in speed.

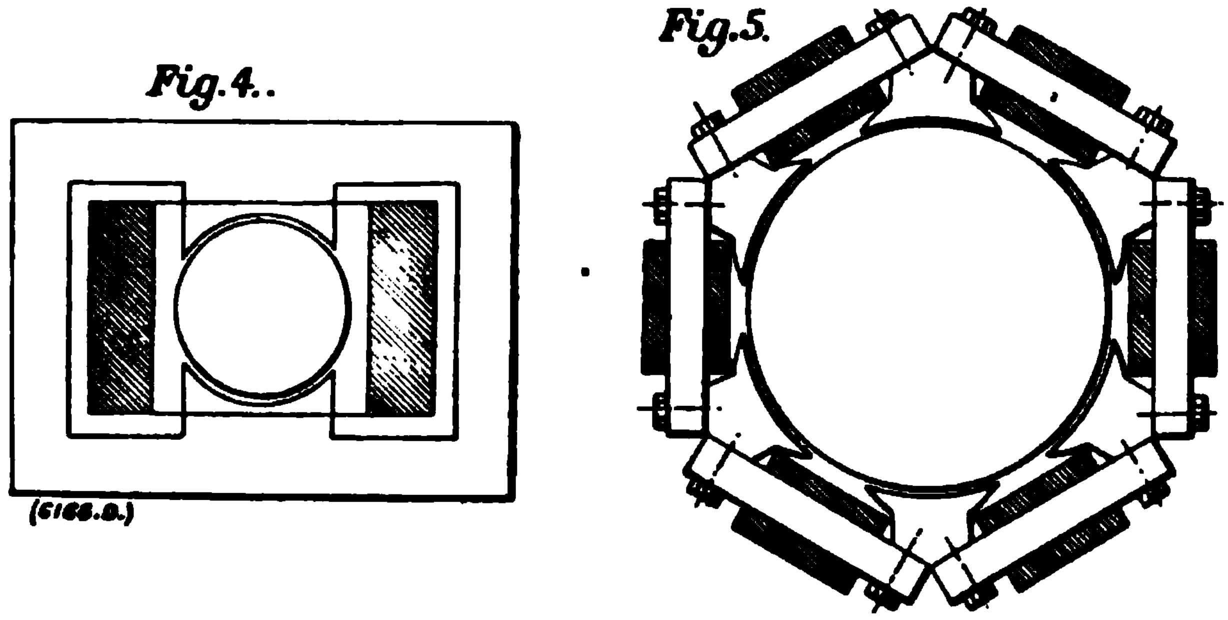
It should be mentioned that this illustration has been worked out without any reference to magnetic leakage, and since the modern motor, in the interests of compactness, must often be built in forms permitting of considerable magnetic leakage, and, in fact, with a percentage varying with the load, no such precise predetermination of the result is practicable. But by designing the motor with a wide neutral zone and low-reactance voltage per segment at full load, a considerable range of choice as to the permanent fixed brush position is practicable after the motor has been constructed.

§ 7. **Magnetic Leakage.**—In order to illustrate the extremes of type, there are given in Figs. 4 and 5 two well-known but by no means modern forms. Fig. 4 (one field coil only) would, so far as relates to the avoidance of increased magnetic leakage with increasing load, be admirably adapted to permitting a fairly precise estimate by means of the method above explained; the latter (Fig. 5) would offer great difficulties.

These general lines of design for obtaining improved economy at light load are, of course, in the interest of the consumer.

They offer, however, another and a highly practical advantage of a nature more directly helpful to the manufacturer. This relates to the increased practicability of employing the same magnetic circuit and windings, in fact practically the same motor, for both open and enclosed types. With the extended use of electric motors, there have arisen large classes of work requiring the employment of totally enclosed motors, owing to their exposure to weather or to other conditions tending to deterioration, such as the accumulation of dust or exposure to flying particles. Indeed it is becoming increasingly evident that, other things being equal, such a motor is the most approved type for all purposes, the chief drawbacks relating at present to the greatly increased cost, the decreased economy of such designs, and even on shorter runs the

Fig. 4. *Fig. 5.*



FIGS. 4 and 5.—Diagrams of Two Types of Motors.

inferior heating guarantees which the manufacturers are prepared to offer.

§ 8. **Open and Enclosed Rating of Motors.**—In some of the earlier instances where such motors were required, the frames of standard open-type motors were so re-modelled as to permit of retaining the old electrical and magnetic design, merely introducing such mechanical alterations in the frame as sufficed to completely enclose the working parts. It was then customary to give such a motor a somewhat reduced rating, so that, while the totally enclosed type had a rating of, say, 100, the corresponding open motor, from which it was derived, may have been rated at 150. Even with this ratio of decrease in the rating, the motor, when totally enclosed, often ran undesirably warm. This result, in the motors of that period, came about as follows:—

Suppose in an open type motor with a rating of 150 the total losses, at full load, amounted to 15, the input thus being 165.

This input of 165, at full load, may have been sub-divided as follows:—

Losses remaining constant at all loads	10·0
Losses increasing as the square of the load	5·0
Mechanical output of motor	150·0
Total input	165·0

Then on rating it down to give, as a totally enclosed motor, an output of but 100, the distribution of the losses became as follows:—

Losses remaining constant at all loads	10·0
Losses increasing with the square of the load	2·2
Mechanical output of motor	100·0
Total input	112·2

The internal losses at full load in this latter case are still 12·2, *i.e.*, 81 per cent. as great as when open, whereas the cooling facilities are reduced in a far greater proportion. Since the temperature rise, when suitably rated as an open motor, was as high as could desirably be permitted, the motor, when totally enclosed, necessarily reached, even with the much lower rating, an excessively high temperature. In fact, even at no load the losses in the totally enclosed motor were already 10·0, *i.e.*, an internal loss equal to 67 per cent. of that occurring at full load as an open motor. Thus, even when running without any load, it would, when totally enclosed, have a greater temperature increase than when running as an open type motor at full load.

But suppose that, as an open type motor, it had been designed with the following distribution of energy:—

Losses remaining constant at all loads	5·0
Losses increasing as the square of the load	10·0
Mechanical output from motor	150·0
Total input	165·0

When, now, it is rated as a totally enclosed motor at 100, as before, the amount and distribution of the energy is as follows:—

Losses remaining constant at all loads	5·0
Losses increasing as the square of the load	4·5
Mechanical output from motor	100·0
Total input	109·5

The internal losses at full load are thus 9·5, as against 15·0 as an

open motor, and as a totally enclosed motor the temperature increase would be only about eight-tenths as great as with the former design.

These values have been chosen largely at random to give simple figures for the purpose of illustrating the important point involved.

The lesson to be learned is that when employing the same electro-magnetic design for an open and a totally enclosed motor, the latter may be rated at a greater percentage of the rating as an open type motor, the greater the ratio of the "variable" to the "constant" losses in the open type motor.

That manufacturers do not even yet clearly comprehend this

Per Cent Commercial Efficiency

fs

Fig. 6.—Efficiency of Different Motors.

point, or at any rate act upon it, is very evident by a perusal of the latest catalogue of any of the best-known firms, and is still more evident by tests of modern motors.

§ 9. **Comparative Designs 100 H.P. Motors.**—We will now make an examination of the components of the "variable" and the "constant" losses in constant speed-shunt motors.

The "variable" losses are those due to the current consumed by the motor in flowing through the resistances of the brush contacts and of the armature winding—in other words, the I^2R losses.

The "constant" losses consist of core loss, friction losses, and shunt-winding losses.

The matter will be most readily made clear by giving an

example of two calculations of the efficiency of two 100 horse-power, constant-speed, shunt-wound motors, the design for the first being for high "variable" and low "constant" losses, and that for the second the reverse. The calculations are carried through in parallel columns to facilitate comparison.

TABLE I.—100 BRAKE HORSE-POWER, CONSTANT-SPEED, 500-VOLT SHUNT MOTORS.

	DESIGN NO. 1. WATTS.	DESIGN NO. 2. WATTS.
Armature I^2R loss ...	6,000	2,700
I^2R loss at brush contact surfaces ...	400	300
Total of "variable" losses ...	6,400	3,000
Core loss ...	1,000	2,300
I^2R loss in field spools ...	300	1,000
I^2R loss in shunt rheostat ...	50	150
Brush friction loss ...	350	400
Bearing friction loss ...	1,000	1,000
Total of "constant" losses ...	2,700	4,850
Full load, total of *all* losses...	9,100	7,850
Three-quarter load, total of *all* losses ...	6,300	6,540
Half „ „ „ ...	4,300	5,600
Quarter „ „ „ ...	3,100	5,000
No-load, „ „ ...	2,700	4,850
Watts output at full load ...	74,600	74,600
Input at full-load output ...	83,700	82,450
„ „ three-quarters of full-load output ...	62,300	62,540
„ „ half ...	41,700	43,000
„ „ quarter ...	21,800	23,700
„ „ no load ...	2,700	4,850
	AMPERES.	AMPERES.
Current input at full load ...	168·0	165·0
„ „ three-quarter load ...	124·5	125·0
„ „ half load ...	83·5	86·0
„ „ quarter load ...	43·5	47·3
„ „ no load ...	5·4	9·7
	PER CENT.	PER CENT.
Commercial efficiency at full load ...	89·1	90·5
„ „ „ three-quarter load ...	89·9	89·6
„ „ „ half load ...	89·6	86·9
„ „ „ quarter load ...	85·7	79·0

For the purposes of the illustration, approximate losses have

been assumed, but sufficiently in conformity with values such as one might actually employ in designs on these lines.

These results are plotted in the curves of Figs. 6 to 11*a*.

§ 10. **Point of Maximum Efficiency.**—In Fig. 11*a* (page 19) are plotted the values of the ratios of the "constant" to the "variable" losses in the two designs. A motor has maximum

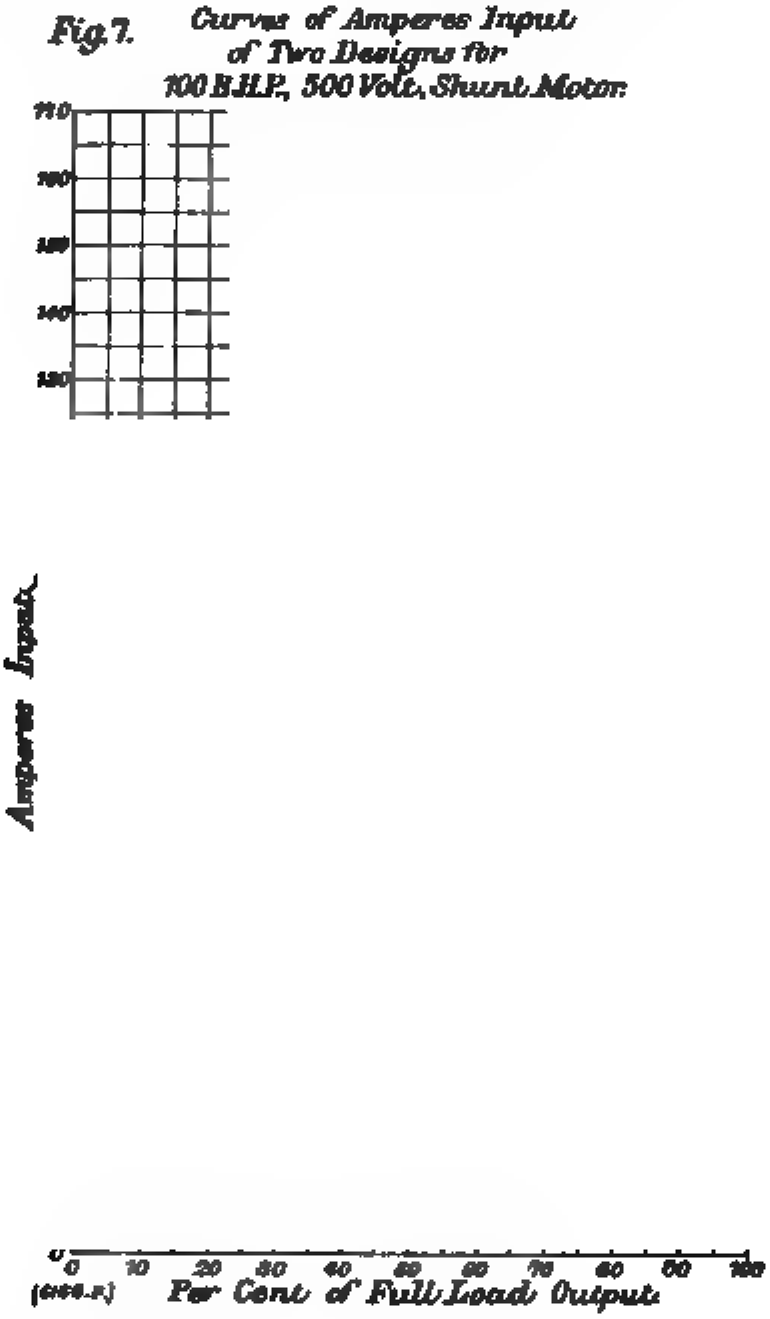


FIG. 7.—Ampere Input of Two Motors.

efficiency at the output at which this ratio has the value 1·00, *i.e.*, when the sum of the "variable" losses equals the sum of the constant losses. In design No. 1 this occurs at 65 per cent. of the rated output; in design No. 2 at 27 per cent. overload.

A motor on the lines of Design No. 1 is therefore seen to be characterised by high efficiency at all loads, though the full-load efficiency (see Fig. 6) is slightly sacrificed to the advantage of the

B

efficiency at light loads and of the "all-day" efficiency. Such proportions would be the most desirable for almost all purposes, though for cases where motors, when in circuit, are constantly carrying loads in the neighbourhood of their full-load capacity, the motor could with advantage be proportioned for heavier losses at light loads and smaller losses at full load. The gain would, however, be but slight, and such cases rarely arise. On the contrary the great majority of motors, and especially small motors,

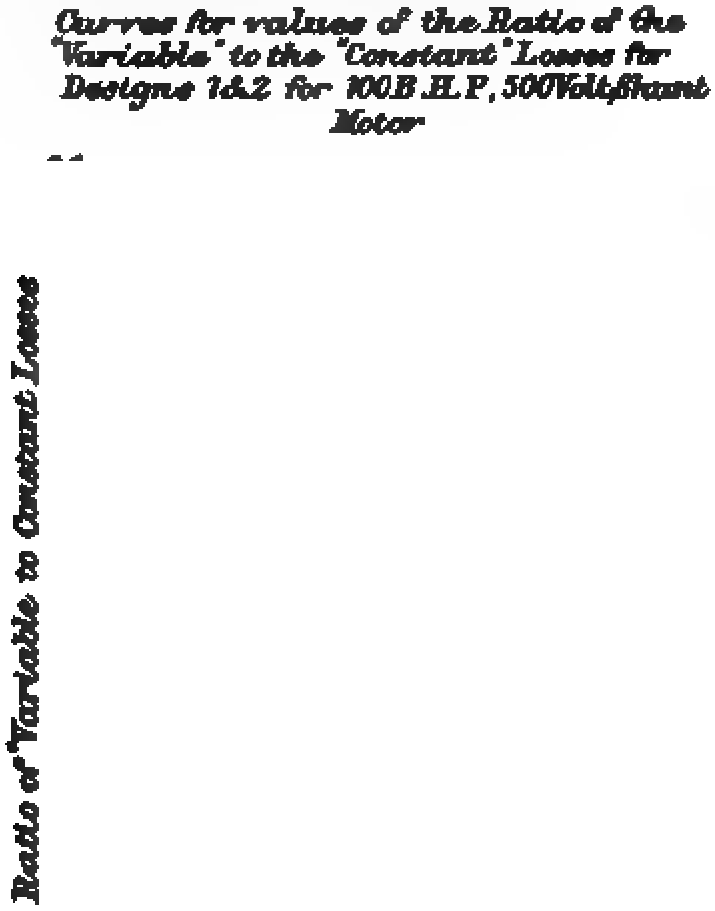


FIG. 11*a*.—Ratio of Variable to Constant Losses.

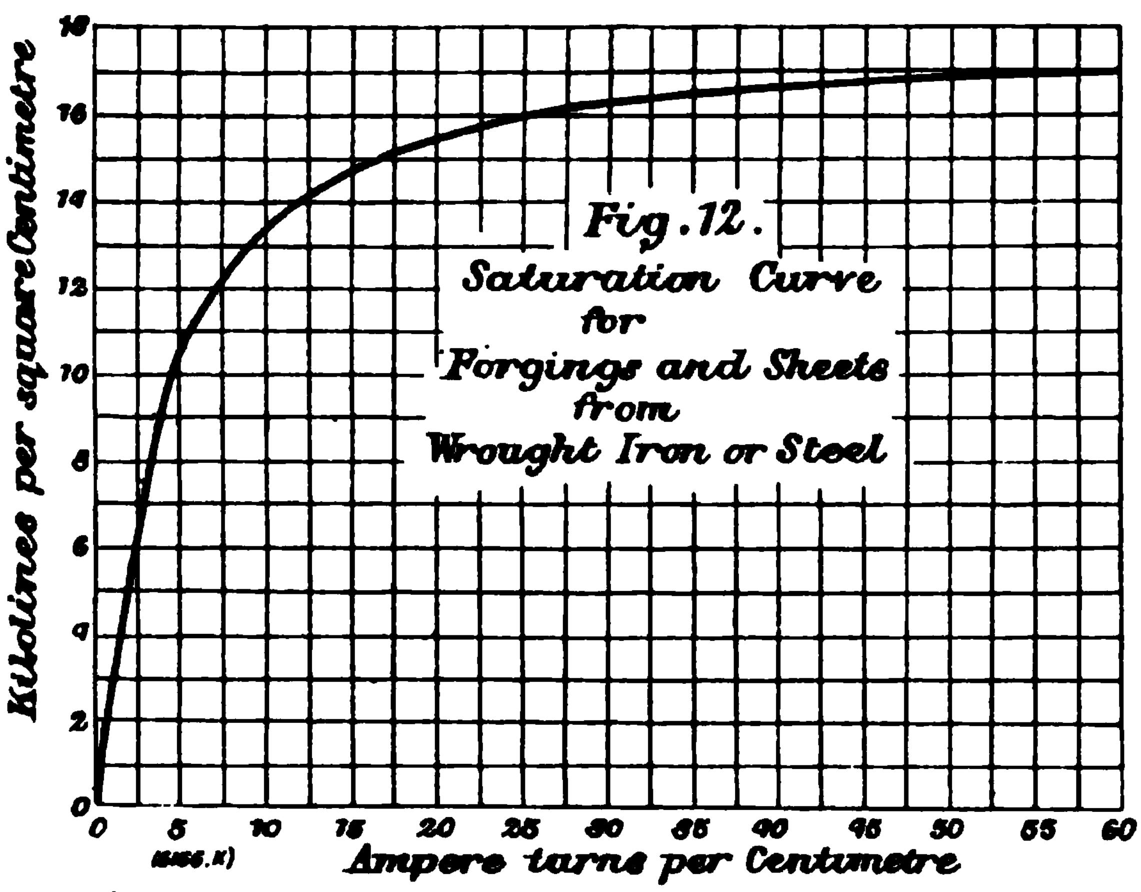
are generally subjected to extremely varying loads, and taking the average load of a large number of motors in a given plant, it will often be found to be considerably less than 25 per cent. of the rated capacity of all the motors in circuit at a given time. It may be said that the saving at average day loads would range from some 4 per cent. to 10 per cent. in the larger motors—say 60 to 120 horse-power normal rated capacity—up to from 8 per cent. to 15 per cent. in the smaller motors, the basis of comparison being motors proportioned respectively on Designs Nos. 1 and 2.

CHAPTER III

DATA FOR MOTOR DESIGNING

§ 1. **Separating Losses.**—The conditions controlling the estimating of values for the losses as set forth in Designs 1 and 2 should next be considered, *i.e.*, the matter of the estimation of each individual loss. Leading up to this we must, however, first briefly touch upon the underlying data for design.

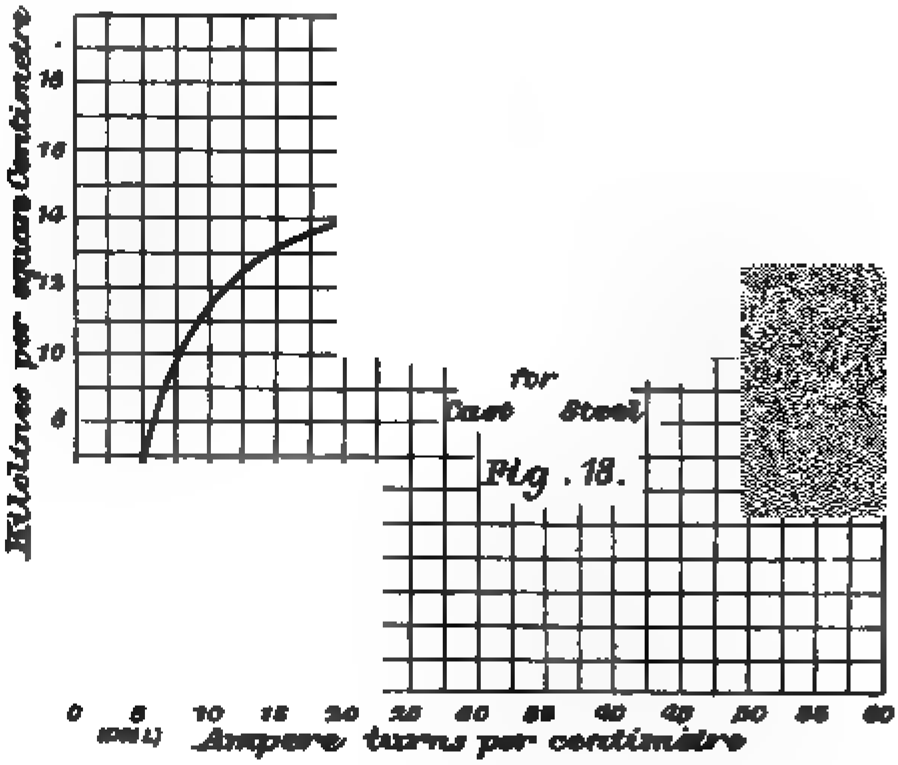
§ 2. **Saturation Curves.**—In Figs. 12, 13, and 14 are given standard curves for the magnetic properties of the grades of



wrought iron, cast iron, laminations, and cast steel which are likely to be obtained. Much better results are generally quoted, and are often obtained, not only in small samples, but in the actual materials delivered; nevertheless, the use of higher values in estimations for designs will involve the danger of under-estimating the required magneto-motive force.

These curves are plotted in terms of the required magneto-motive force at various densities, the magneto-motive force being

expressed in ampere turns per centimetre of length of the magnetic circuit, and the densities in kilolines per square centimetre. The



curve of Fig. 15 gives the same values for air, the magneto-motive forces being expressed in ampere turns per millimetre radial depth

Kilolines Per Sq. Centimeter

Ampere-Turns Per Centimeter

of the air gap, and the densities in kilolines per square centimetre of the pole face.

§ 3. **Magnetic Reluctance of Armature Teeth.**—Still two more curves are necessary, namely, those relating to the magneto-motive force required for overcoming the magnetic reluctance of

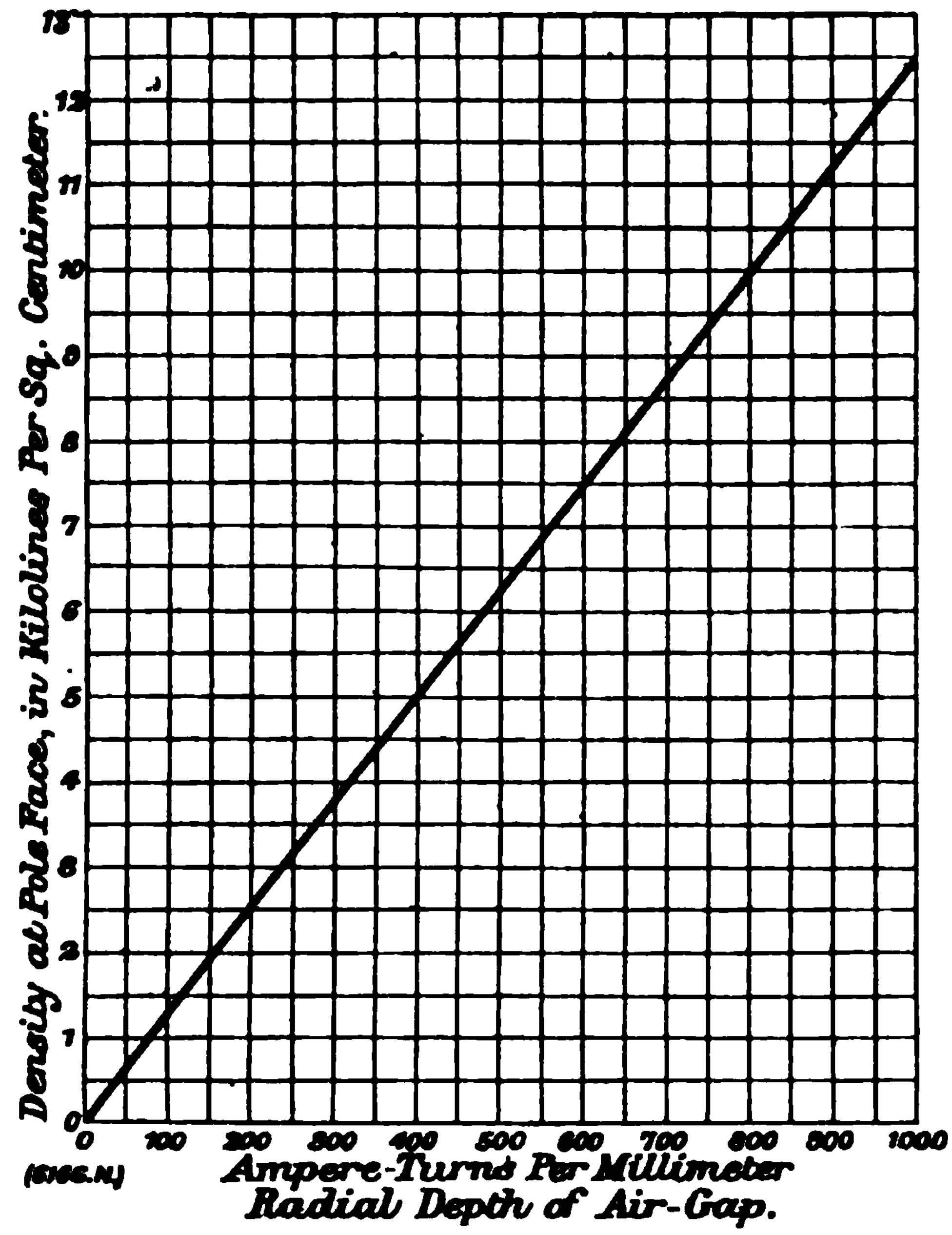


FIG. 15.—Air Gap Reluctance.

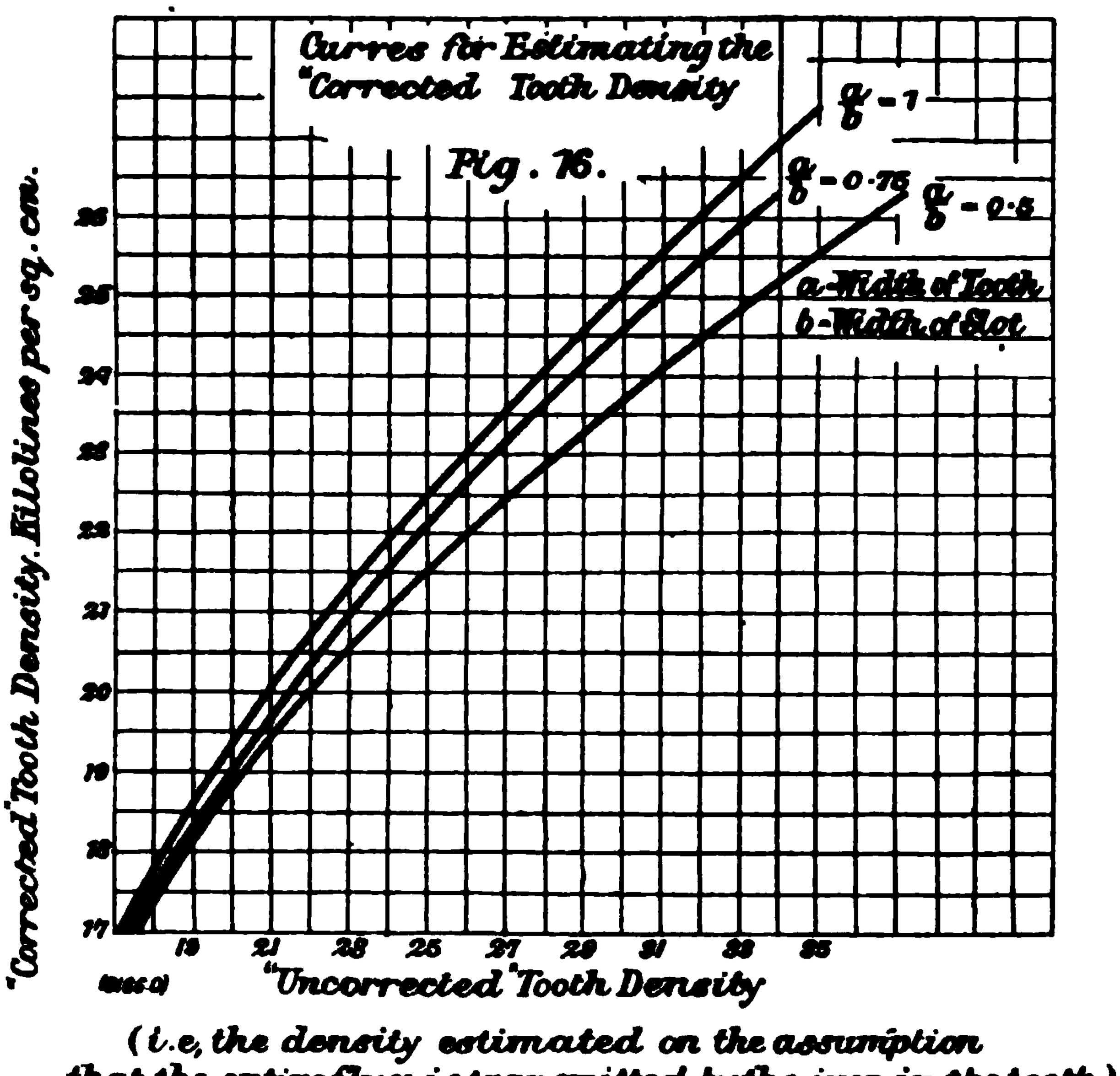


FIG. 16.—Magnetic Reluctance of Armature Teeth.

the armature teeth, for these projections—so-called teeth—are often run up to much higher magnetic saturation than occurs elsewhere in the magnetic circuit, in many cases so high, in fact, that the permeability of the iron is reduced to values not many times greater than that of air, copper, and other non-magnetic substances (whose permeability equals unity), hence a by no means negligible percentage of the total flux passes, in accordance with the law of divided circuits, through the copper, air, and insulation in the slots and ventilating ducts, giving, consequently, a "corrected" density in the projections, much lower than the "apparent" density, and, owing to saturation, a corresponding magneto-motive force in a still far greater proportion less than

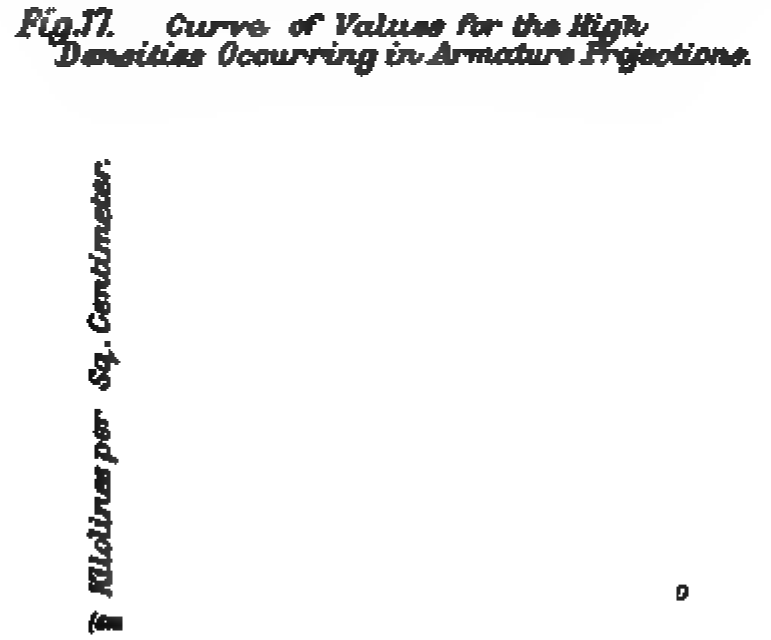


FIG. 17.—Magnetic Reluctance of Armature Teeth.

would have been obtained on the assumption that the entire magnetic flux is transmitted through the laminated iron of the projections. The curves of Figs. 16 and 17 serve for ascertaining the magneto-motive force required for the teeth.

§ 4. **Relative Merits of Different Magnetic Materials.**—Cast iron, cast steel, solid wrought iron, and laminated wrought iron and steel are selected by various makers for magnet cores and yoke. The question of the relative merits of these different materials is a many-sided subject, depending not only upon the relative costs and quality of these materials in various countries, but also upon the relation of the cost of the magnetic materials to the cost of copper, and there must come into the question the cost and quality of labour in the country in question, the facilities at the disposal of the manufacturer, the nature of the work which

the motor is to perform, and the location and space in which it is to be installed.

In the following calculation, which is given for explanatory purposes, cast-steel magnet cores and a cast-iron yoke are taken as the basis, this disposition of material being one fairly extensively adopted, though where the magnet cores may be arranged separable both from the yoke and the pole shoes, a very excellent plan is to employ forgings for the magnet cores (forgings of wrought iron or of steel generally having a more uniform quality than steel castings), and a poor grade of cast iron for the pole shoes. The use of this poor cast iron for the pole shoes is for the purpose both of checking magnetic distortion in virtue of the low magnetic permeability of the material and of reducing eddy currents in the pole face in consequence of the low electrical conductivity of poor grades of cast iron.

Another construction employed for reducing the eddy current losses in the pole face is to make the pole shoe of laminated iron This eddy current loss in the pole shoes is greater the greater the width of the armature slots and the shorter the radial depth of the air gap.

§ 5. **Specific Resistance of Irons and Steel.**—An idea of the relative electrical conductivities of cast iron and other magnetic materials may be obtained from the following table:—

TABLE II.—SPECIFIC RESISTANCE AT 0° CENT. IN MICROHMS PER CENTIMETRE CUBE.

Cast iron ...	100
Cast steel ...	20
Wrought iron and very mild steel ...	10
Nearly pure iron ...	9

§ 6. **Calculation of Field Ampere Turns required per Pole.**—Having derived the flux required at no load and full load from the formulæ already given, we proceed as follows to obtain the magneto-motive force per field spool. As example, take a 4-pole, 50 brake horse-power, 500-volt, 800 revolutions per minute shunt motor.

Flux entering the armature per pole, full load, 3·58 megalines; corresponding internal voltage, 488·5; corresponding terminal voltage, 500; leakage factor, 1·125; flux generated per pole, full load, 4·025 megalines.

Armature.—Depth of iron punchings below teeth, 10·3 centimetres; length of effective magnetic iron (total armature length

minus ventilating ducts and insulation between punchings), 14·8 centimetres; cross section of the core, 305 square centimetres, *i.e.*, $10·3 \times 14·8 \times 2$; density, 11,750 C.G.S. lines; ampere turns per centimetre length of magnetic path, 7 (from Fig. 12); length of path, 13 centimetres, *i.e.*, the average circumference of iron punchings below teeth $\div 2 \times$ number of poles; ampere turns, 90.

Teeth.—Number of teeth per pole, 16; number of teeth directly below a mean pole arc, 10; percentage increase allowed for spread, 20 per cent.; total number of flux-carrying teeth per pole, 12; width of tooth at bottom of slot, 1·11 centimetres; total cross section per pole at the bottom of teeth, 197 square centimetres, *i.e.*, $12 \times 14·8 \times 1·11$; apparent density, 18,200 C.G.S. lines; mean width of tooth, 12·15 millimetres; width of slot, 10·6 millimetres; mean width of tooth $\div$ width of slot, 1·2; corrected density from curve similar to Fig. 16, 18,000 C.G.S. lines; ampere turns per centimetre length of magnetic path, 100 (from Fig. 17); length of tooth, 3·8 centimetres; ampere turns, 380.

Air Gap.—Width of pole, 18·4 centimetres; mean pole arc, 25·6 centimetres; cross section at pole face, 470 square centimetres; density at pole face, 7600 C.G.S. lines; length of air gap, iron to iron, 0·45 centimetres; ampere turns, 2740 (from Fig. 15), *i.e.*, $\frac{10}{4\pi} \times$ density $\times$ length of air gap.

Magnet Core.—Cast-steel poles of circular cross section. Magnet core cross section, 265 square centimetres; density, 15,200 C.G.S. lines; ampere turns per centimetre length of magnetic path, 32 (from Fig. 13); length of path, 17 cent.; ampere turns, 540.

Yoke.—Material cast steel. Cross section, 315 square centimetres, *i.e.*, width of yoke $\times$ radial depth $\times 2$; density, 12,800 C.G.S. lines; ampere turns per centimetre length of magnetic path, 13 (from Fig. 13); length of path, 42 centimetres, *i.e.*, mean circumference of yoke $\div 2 \times$ number of poles; ampere turns, 550.

Terminal voltage	500
Internal voltage	488·5

AMPERE TURNS PER POLE—FULL LOAD.

Armature core	90
Armature teeth	380
Air gap	2740
Magnet core	540
Yoke	550
Total ampere turns ...	4300

§ 7. **Weight of Copper per Field Spool.**—From the value of the required ampere turns per field spool derived by the above method, one is in a position to deduce the relation between the watts lost per field spool and the weight of copper to be employed per field spool. To explain the method used in this part of the calculation, let us take the case of another motor with magnet cores of a circular cross section of the dimensions shown in Fig. 18, *i.e.*, 20 centimetres diameter.

Assume that for a certain case a preliminary estimate has shown that, at normal speed and voltage, there will be required a magneto-motive force of 4000 ampere turns per field spool. The motor is for 500 volts and 6-polar; hence, allowing 15 per cent. for voltage drop in the regulating rheostat in the field circuit, there remain $\frac{500 \times 0{\cdot}85}{6} = 71$ volts per spool.

Suppose, for a trial calculation, we take the spool of the length shown in Fig. 18, *i.e.*, 20 centimetres, and make the trial assumption that its depth of winding will be 4 centimetres. Such a winding would have a "space factor" of about 0·5, that is to say, of the cross section of the completed spool some 50 per cent. will be copper, the remainder being the space occupied by insulation and the lost space.

Mean length of turn	$24\pi = 75{\cdot}5$ centimetres.
Cross section of completed spool winding	$4 \times 20 = 80$ square centimetres.
Cross section of copper in spool	$0{\cdot}5 \times 80 = 40$ square centimetres.
Volume of copper per spool ...	$75{\cdot}5 \times 40 = 3{,}020$ cubic centimetres.
Weight of one cubic centimetre of copper	0·0089 kilogramme.
Weight of copper per spool ...	$3{,}020 \times 0{\cdot}0089 = 26{\cdot}8$ kilogrammes.

The following formula gives the relation between the magneto-motive force in ampere turns per field spool, the mean length of turn in metres, the watts per field spool at 60 degs. Cent., and the kilogramme weight of copper per field spool:—

$$\text{Kilogrammes} = \frac{0{\cdot}000{,}176\ a^2\ b^2}{\text{Watts at } 60^\circ \text{ Cent.}}$$

In which

a = ampere turns per field spool,

b = mean length of one turn in meters.

For the case now under consideration:—

$$a^2\ b^2 = (4000 \times {\cdot}755)^2 = 9{,}100{,}000.$$

Hence watts per field spool at 60 degs. Cent. :—

$$= \frac{0 \cdot 000,176 \times 9,100,000}{26 \cdot 8}$$

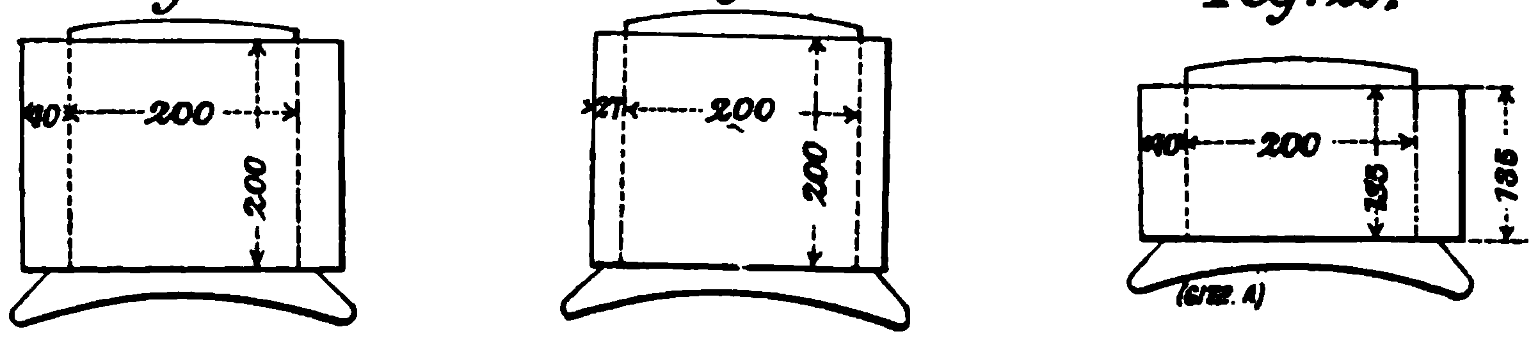
$$= 60 \text{ watts.}$$

There are two considerations controlling the choice of the watts per field spool. First, the magnitude of the loss to be permitted as affecting the efficiency and the no-load loss (the "constant" loss); and, secondly, the permissible temperature increase, this being dependent upon the watts per square decimetre of radiating surface and the facilities for ventilation.

Although not necessarily the best way, since the heat may in special cases be dissipated largely by internal ducts, and also to a great extent by conduction through the iron of the magnet core and yoke, nevertheless it is quite customary and very convenient to judge of the temperature rise partly by the aid of the magnitude of the watts per square decimetre of external cylindrical radiating surface of the field spool.

§ 8. **Typical Field Magnet Cores.**—In the case in hand the external cylindrical radiating surface $= 2 \cdot 8 \times \pi \times 2 = 17 \cdot 6$ square decimetres, and hence there are $\frac{60}{17 \cdot 6} = 3 \cdot 4$ watts per square decimetre. This is an unnecessarily low value, even for a totally enclosed motor, from the temperature standpoint. Hence, if the

Fig. 18. *Fig. 19.* *Fig. 20.*



FIGS. 18, 19, 20.—Typical Magnet Cores.

efficiency considerations would permit of allowing a higher loss per spool, one would reduce the amount of copper per spool—in the case of a medium speed, totally enclosed motor—to, say, 18 kilogrammes per spool, and, instead of the proportions of Fig. 18, we should employ those of Fig. 19, or of Fig. 20, or some intermediate form, various considerations determining the preferable form. Thus, in Fig. 19, we should have substantially the same radiating surface; but we should be employing the same weight of magnet core. In Fig. 20 we should be saving in material of magnet core and in space (*i.e.*, overall dimensions), but should be

reducing the radiating surface considerably—a dangerous step to take, especially in totally enclosed motors, without having ample experimental data at hand to justify such a reduction. The impracticability will now be recognised of following any general rules in these matters, as the requirements of each case determine the suitable proportions. In the calculations for the above cases there have, however, been given sufficient curves, data, and formulæ to serve as a guiding example in determining the spool winding and the spool losses in other shunt motors.

§ 9. **Armature I^2R Loss.**—The estimation of the armature I^2R loss is simply a matter of estimating the resistance from the mean length of turn, the cross-section of the conductor, and the specific resistance of copper. The specific resistance of commercial copper at 60 degs. Cent. may be taken as 0·0000020 ohm per cubic centimetre (*i.e.*, between the opposite faces of a centimetre cube). The resistance through the winding, from negative to positive brushes, is, at 60 degs. Cent.,

$$\frac{0{\cdot}0000020 \times \text{length through winding, from neg. to pos. brushes}}{\text{Cross section of all parallel paths,}}$$

the length and cross section being respectively in centimetres and square centimetres.

§ 10. **Calculation on 15 B.H.P. Shunt Motor.**—The following is an example of such a calculation on a 4-pole, 15 brake horsepower, 220-volt, 700 revolutions per minute shunt motor.

Terminal voltage, full load, 220; terminal voltage, no load, 220; number of face conductors, 516; number of conductors per slot, 12; number of slots, 43; total amperes to commutator, 57; total number of parallel paths through the armature winding, 2; number of conductors in series between brushes, 258; mean length of a single turn, 1·04 metres; total number of turns, 258; number of turns in series between brushes, 129; total length through winding, from negative to positive brushes, 134 metres; cross section of one conductor $0{\cdot}28 \times 0{\cdot}26 = 0{\cdot}073$ square centimetres; resistance through the winding, from negative to positive brushes, at 60 degs. Cent. $= \frac{0{\cdot}0000020 \times 13{,}400}{2 \times 0{\cdot}073} = 0{\cdot}184$ ohm; IR drop in armature winding at full load and 60 degs. Cent., 10·5 volts; IR drop in the brush contact surface, 1·5 volts, calculated, as shown subsequently, under commutator losses; total internal IR loss, 12 volts; total induced voltage, full load, 208.

§ 11. **Current Density in Armature.**—The permissible

current density in the armature conductors is, from the heating standpoint, very high, for by the use of numerous and wide ventilating ducts through the core, current densities up to 500 amperes per square centimetre of conductor may often be used in moderate, and high speed, open type motors. The heating is, in fact, more properly considered from the specific rate of generation of energy in the armature, *i.e.*, the watts per square decimetre of external cylindrical radiating surface, due regard being taken of the available facilities for ventilation. But the consideration of the total armature I^2R loss, as related to the efficiency, may sometimes be a limiting consideration, especially in motors with high armature strengths, as expressed in armature ampere turns per pole piece. It is also conceivable that in motors required to run in both directions, and hence requiring the brush position to coincide with the geometrical neutral position, that a case might arise where the IR drop in the armature—which could not in such a case be offset by armature demagnetisation—might be of such magnitude as to cause an undesirably great difference in speed between no load and full load, and hence that, from a consideration of speed regulation, the current density would need to be chosen low.

§ 12. **Hysteresis.**—Before considering the matter of the armature losses from the standpoint of the specific losses per unit of radiating surface, we must give some attention to the remaining component of the total armature loss, namely, that in the core due to hysteresis and armature currents. This is one of the "constant" losses, *i.e.*, a component of the no-load loss, and can with advantage be kept low. By the method of design with relatively high armature strengths, the core loss is maintained low more in virtue of the greatly reduced amount of material subjected to these losses than by any reduction of magnetic density, which, with the periodicity of reversal of magnetisation, determines the specific rate of dissipation of energy in the core. As a matter of fact, a consideration of the best proportions from the commutating standpoint, which comes in a later section, shows us that it is desirable to run the core at high magnetic density, so as to make it as short between the core flanges as is consistent with transmitting the required magnetic flux, and with providing the necessary means for ventilation.

§ 13. **Magnetic Flux.**—A formula has already been given, $E=4TNM\times10^{-8}$ for determining from the speed, winding, and voltage, the magnetic flux, M, entering the armature per pole. This, divided by the magnetic cross section of the core (taking into

account that, as the magnetic lines flow in both directions after having entered the armature and traversed the projections, this cross section is double the geometrical cross-section of one side below the slots), gives the flux density.

In Fig. 21 are given values for the specific loss in terms of the

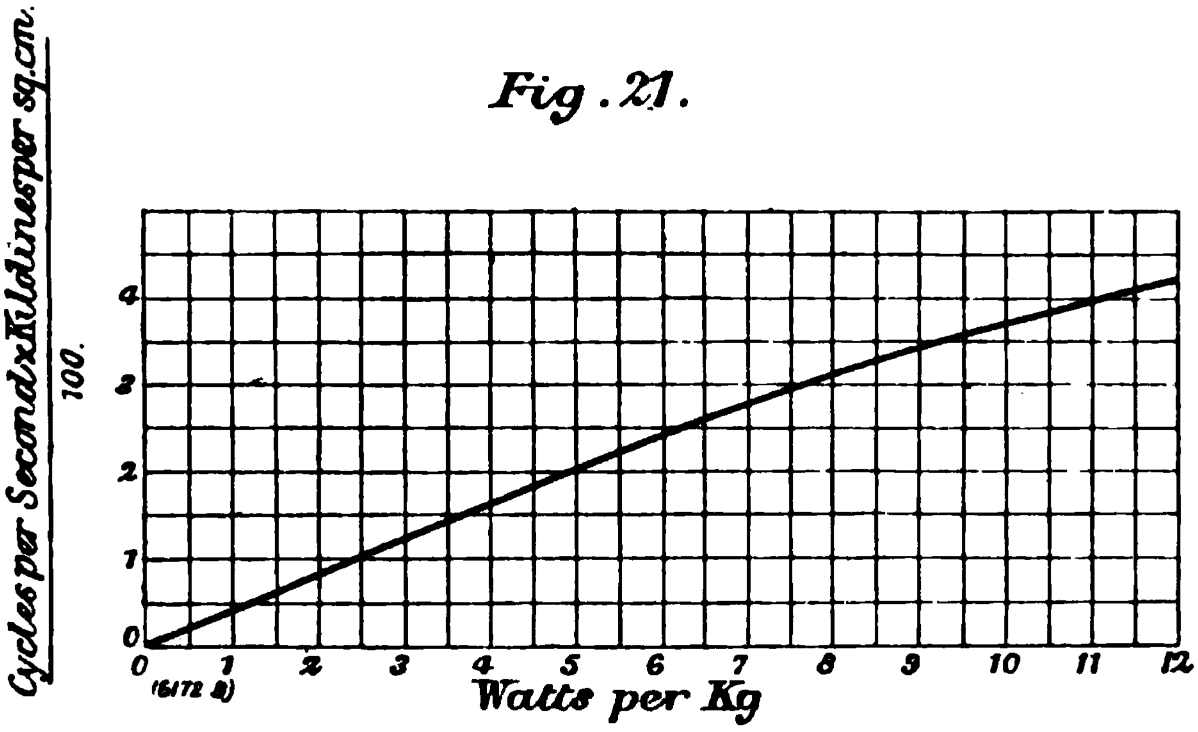


FIG. 21.—Diagram of Core Losses.

product of the periodicity in cycles per second, and the density below slots in kilolines per square centimetre.

Let us, for example, assume the case of an armature in which—

Total weight of magnetic laminations, inclusive of teeth	50 kilogrammes.
Density below slots (=D)	8·0 kilolines.
Periodicity of magnetic reversal in complete cycles per second (=N)	27
$\frac{ND}{100}$	2·2

From Fig. 21 we find corresponding to a value of 2·2 for $\frac{ND}{100}$, that—

Specific loss in iron due to hysteresis and eddy currents	5·4 watts.
Total core loss	$5{\cdot}4 \times 50 = 270$ watts.

One might reduce the magnetic density by decreasing the internal diameter of the armature laminations; but, especially in small machines, the reduction is not so much as in proportion to the increased available section. This is because the increased length of magnetic path required to be followed by that part of the magnetic flux, taking advantage of this additional section,

reduces the amount of such magnetic flux and leads to an unequal distribution of the flux, the density being less the nearer the inner circumference is approached. Also, owing to the increased weight of material subjected to hysteresis and eddy currents, the core loss decreases more and more slowly by each successive reduction in the diameter of the internal circumference. It is also undesirable, on small motors, to encroach much upon the inner air space, as the access of air to the ventilating ducts is thereby impeded. In some types of motors the density below slots has purposely been made extremely high, with a view to reducing the reactance per

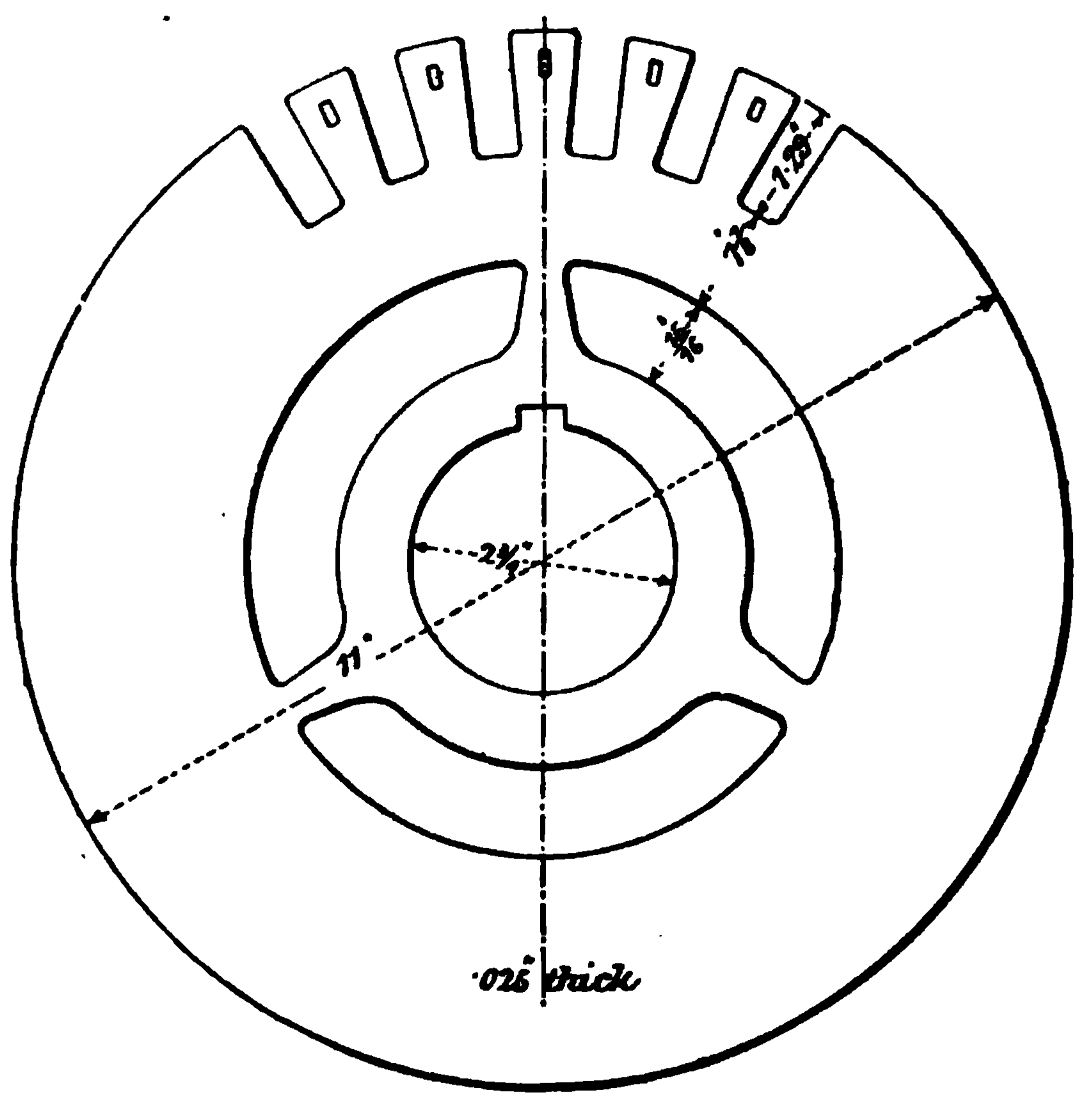


FIG. 22.—Punched-out Armature Lamination.

turn and preventing sparking, but this is a crude way of accomplishing the purpose. An interesting instance of the kind is shown in Fig. 22, where material has been punched out for this purpose, and where the resulting density is excessively high.

§ 14. **Temperature Rise of Armature.**—Suppose, in a given armature, the length over winding parallel to shaft equals 42 centimetres, diameter of armature equals 38 centimetres, therefore cylindrical radiating surface equals $4{\cdot}2 \times 3{\cdot}8 \times \pi$ equals 50 square decimetres; core loss equals 300 watts, armature I^2R loss equals 800 watts, therefore total armature loss equals 1100 watts, therefore watts per square decimetre of peripheral radiating surface equals 22 watts. Then, if this were an open type motor, with

many and wide ventilating passages and fairly high speed, the thermometrically determined increase of temperature above the surrounding air after a full load, run until the attainment of constant temperature, would not be more than 0·8 deg. Cent. per watt per square decimetre, *i.e.*, not more than $0{\cdot}8 \times 22 = 18$ degs. Cent. above surrounding air.

§ 15. **Temperature Rise Measured by Increase of Resistance.**—When determined by increase of resistance, the temperature increase would generally be at least from 20 per cent. to 40 per cent. higher, even in well-ventilated armatures; and with defective ventilation the difference between the two measurements would be correspondingly greater.

The remaining losses relate to the commutator, and the following example of the estimation of the commutator losses for a shunt motor of 4-pole, 5 brake horse-power motor, rated capacity at 110 volts and 900 revolutions per minute, will serve to illustrate the method to be employed.

§ 16. **Commutator Losses.**—Normal brush thickness, 13 millimetres; length of brush contact arc, 13·4 millimetres; width of brush, 38 millimetres; contact surface per brush, 5·1 square centimetres; number of brushes per pole, 1; total number of positive brushes, 2; contact surface of all positive brushes, 10·2 square centimetres; amperes total, 38·6; amperes per square centimetre brush contact surface, 3·78; resistance per square centimetre of brush contact surface, 0·2 ohm; brush contact resistance (positive plus negative) $= \frac{0{\cdot}2}{10{\cdot}2} \times 2 = 0{\cdot}039$ ohm; volts drop at brush contacts (positive plus negative), 1·5; I^2R loss at brush contacts (positive plus negative), 58 watts; total contact surface of brushes (positive plus negative), 20·4; brush pressure in kilogrammes per square centimetre, 0·1; total brush pressure in kilogrammes, 2·0; friction coefficient, 0·3; effective component of brush pressure in kilogrammes, 0·643; diameter of commutator, 16 centimetres; revolutions per second, 15; brush friction in kilogrammetres per second $= 0{\cdot}16 \times \pi \times 15 \times 2 \times 0{\cdot}3 = 4{\cdot}5$; brush friction loss in watts, 41, for 1 kilogrammetre per second $= 9{\cdot}8$ watts; total commutator loss $= 58 + 41 = 99$ watts (brush I^2R plus brush friction loss); commutator circumference, 5·02 decimetres; active length of commutator surface (the "active" surface extending from the armature connections to the outer ends of the segments), 0·65 decimetres; cylindrical surface of commutator, 3·3 square decimetres; watts per square decimetre of cylindrical radiating

surface, 30; peripheral speed of commutator, 7·53 metres per second; temperature increase per watt per square decimetre radiating surface, 1·0 deg. Cent. (this is a higher value than customary, on account of the specially low peripheral speed with the consequent poor ventilation); temperature increase at the peripheral surface, 34 degs. Cent.

There have now been set forth not only methods for calculating all the magnetic circuit proportions, but also all the component losses entering into the determination of the efficiency at various loads, and of the "constant" losses (*i.e.*, those at no load) and the "variable" losses.

§ 17. **Sparking.**—But as yet no reference has been made to the questions relating to sparking; these are, however, of even more importance than any as yet discussed. In the writer's opinion, the best way of designing motors with a certainty of satisfactory commutation is altogether to abandon dependence upon electro-magnetic assistance in reversing the current in the coil undergoing commutation under the brush, and to rely upon having such proportions that the reactance voltage in the short-circuited coil has, at full load, such a low value as in itself to ensure freedom from sparking. This is sometimes objected to as tending, in most cases, to require a much more expensive commutator construction than is generally customary. The writer will admit that the number of segments should preferably be much greater, but complete immunity from sparking contributes in a far greater degree to reduced losses at the commutator, to an improved efficiency, and hence also to a smaller permissible radiating surface than is generally understood to be the case. It is true that, from the manufacturer's standpoint, this is more or less of a doubtful advantage, since, owing to the difficulties attending an exact determination of the efficiency as a whole, the component losses are generally determined partly by indirect estimations and partly by individual measurements, and the commercial efficiency is deduced from these component values. Hence the stray losses at the commutator escape detection in most cases, although they may often be inferred, from the observed temperature rise and the estimated specific rate of generation of heat, to be present in large proportions. In a machine so proportioned that the reactance voltage at full load is thoroughly conservative, a good mechanical construction of shaft, commutator, and brush gear, and a solid frame and foundations, will generally, at fair peripheral speeds and with thoroughly good ventilation, show not over 0·8 deg. Cent. increase

in temperature at the surface, above the surrounding air, per watt per square decimetre of external cylindrical radiating surface, *i.e.*, of the radiating surface corresponding to the active surface of the segment.

§ 18. **Stray Losses in Commutation.**—When, as very frequently occurs, higher values, say from 1·2 degs. Cent. to 2 degs. Cent. per *apparent* watt per square decimetre are observed, then, if the *mechanical* constructions are satisfactory, there is the best of reason for suspecting a heavy stray loss, due to short-circuit currents in the coils under commutation, and to losses of energy in sparking at the brushes. These stray losses are especially undesirable, since their action is detrimental to the commutator, and the effect is cumulative. As the deterioration proceeds with time, the commutator losses and the temperature rise also

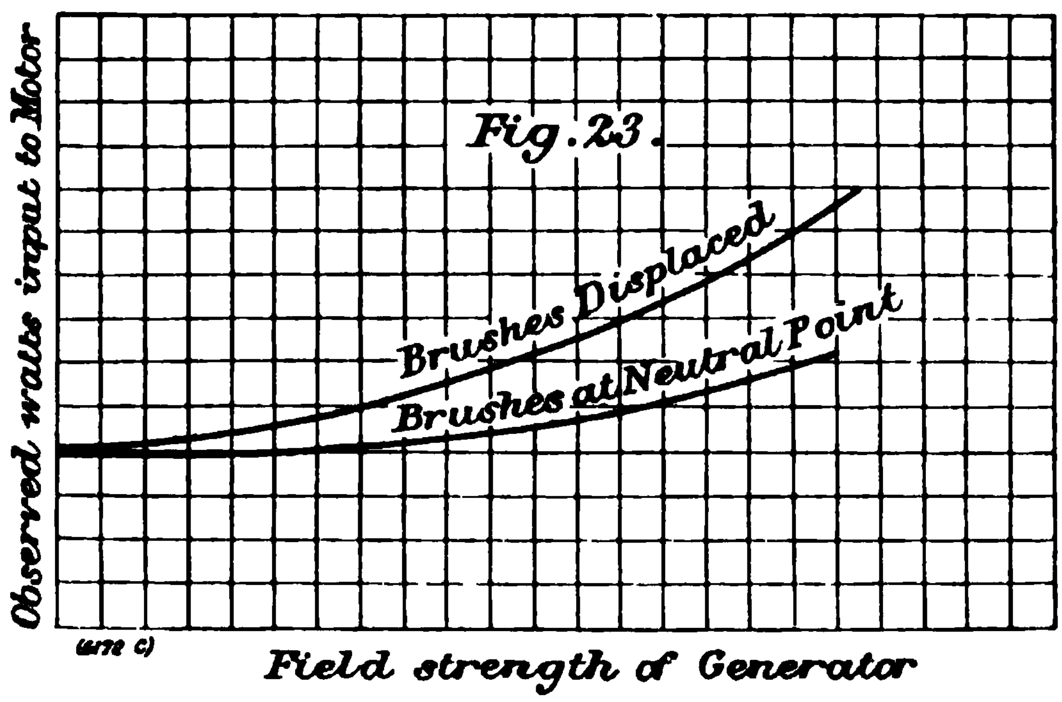


FIG. 23.—Diagram of Motor Tests.

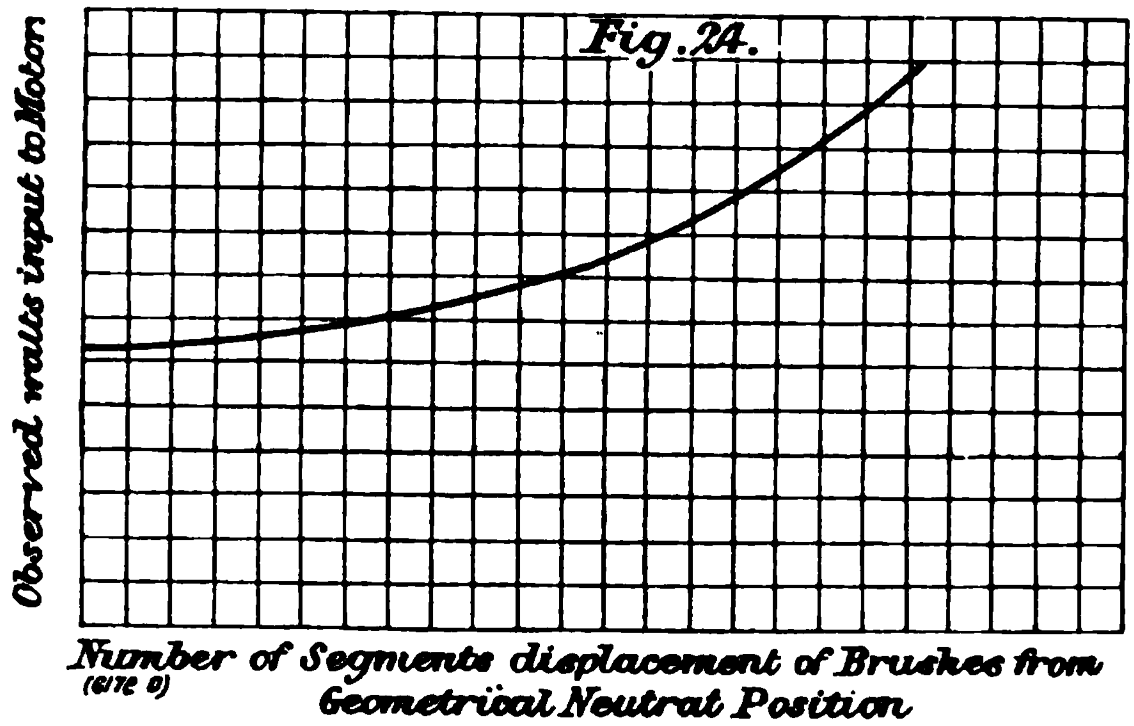
increase. One difficulty about this question is the lack of available data as to the magnitude of commutation losses under various conditions.

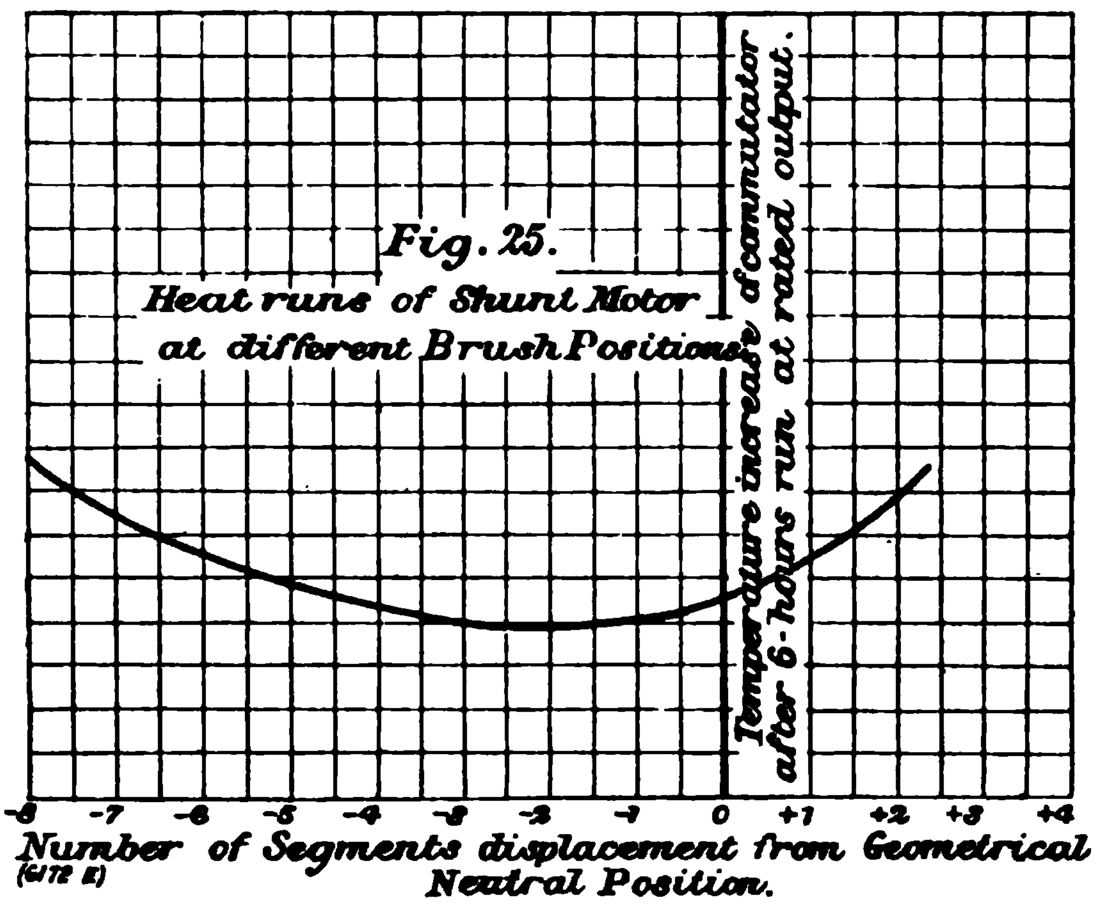
§ 19. **Tests of Magnitude of Commutation Losses.**—Such tests as the following, carried out on several very different machines, would be most instructive.

Test No. 1.—Run a machine at constant speed, and without load, as a generator, by means of a small motor, and measure the input to the small motor, first with the brushes of the generator in the neutral position and with varying field excitation, and afterwards with the brushes considerably displaced from the neutral position, again with varying field excitation. Plotting from the results, the curves of Fig. 23, the difference in values between the two curves is an indication of the amount of the losses in commutation and sparking.

Test No. 2.—Run a motor at no load, separately exciting the fields, and varying the excitation by the small amount necessary to maintain constant speed throughout a considerable range of brush positions. From the observations of the current taken by the motor, plot the curve of Fig. 24.

The value of the watts absorbed corresponding to the neutral position of the brushes corresponds to the core loss and friction;





FIGS. 24 and 25.—Diagrams of Motor Tests.

the difference between this and the other values affords some indication of the general magnitude of the commutator stray losses. One of the slight errors coming into consideration is the variation in core loss caused by the change in field excitation required to maintain constant speed.

Test No. 3.—Tests of the temperature rise of the commutator on long runs with load, with the brushes in different positions

during successive runs, would also be interesting, though in heat runs it is difficult to obtain consistent results for temperature rise. Such observations should be plotted as in Fig. 25.

In such a series of tests as that indicated in Fig. 25 the speed should be maintained the same in the successive tests by the necessary slight alteration in the field strength.

The reason why reversal by means of the impressed magnetic field is so undesirable is that, as the load, and hence also the current, increases, the distorting influence of the armature current distorts this field ever further away from the geometrical neutral point. Hence, while the ideal brush position at no load is at the geometrical neutral point (since there is then but a negligible armature current, and the less the rate of change of the impressed flux linked with the short-circuited armature coil the better), an increase of load and of current in the armature would be best provided for by an increased rate of change of the impressed magnetic field linked with the short-circuited coil to offset the reactance voltage in it, this reactance voltage (or voltage of self-induction) increasing, of course, with the load.

§ 20. **Fixed Brush Position.**—Now, to have a portion of the impressed magnetic flux linked with the coils when short-circuited under the brushes, the brushes must be slightly displaced from the geometrical neutral point. Then, for some small value of the load, the reactance voltage will be just offset by the electro-motive force induced in the coil by the impressed flux. With increasing load we should require a stronger impressed flux (the rate of cutting of the flux remaining constant) to neutralise the higher reactance voltage set up by the increased current. Instead, however, of this being the case, the magnetic flux is distorted, in proportion to the load, further and further away from the geometrical neutral point. Even if it were permissible to alter the brush position as the load increased, there would always be some limit to the load where there could not be found any brush position where the rate of cutting of this impressed flux by the conductors of the short-circuited coil would be sufficient to offset the reactance voltage in that coil.

But any change of brush position with load is undesirable. It is now almost universally required that a motor shall operate sparklessly at all loads without change in the brush position, and in motors for operation in either direction this fixed brush position must be the geometrical neutral point. In the cases where the fixed brush position is, in a motor, that with a few

segments of backward lead, the practice is to set the brushes at no load with as much backward lead as is consistent with freedom from sparking, for at no load, *i.e.*, no appreciable current in the armature, too great a backward lead would (owing to the cutting of the impressed flux by the coil while short-circuited under the brush) set up such heavy induced currents in this short-circuited coil that sparking would result when the segment moved out from under the brush, opening the short circuit. After thus setting the brushes at no load with the greatest permissible backward lead, the correct normal rating for the machine would be, so far as relates to sparking, the greatest load corresponding to which the current could be sparklessly collected at the fixed brush position already determined at no load.

So much for the process of electro-magnetic commutation. By means of carbon brushes, and sometimes, also, by far from satisfactory auxiliary features in the design, electro-magnetic commutation has been employed very extensively. But it is much better to proportion the motor for low reactance voltage at full load, and thus make it at all loads *inherently* incapable of sparking. This leads to commutators of rather large diameter, with many and narrow commutator segments, and hence to increased cost for labour. But the great decrease in the "stray" losses in commutation, ensuring a temperature rise of not over 0·8 deg. Cent. per watt per square decimetre of cylindrical radiating surface, enables the commutator to be proportionately shorter than in cases where there exists a considerable probability of from 40 per cent. to 80 per cent. higher total commutator losses than are attributable to brush friction and contact-resistance loss. Thus a proportional saving in material is effected, as so great a radiating surface is not required as where these "stray" losses exist.

§ 21. **Determination of Reactance Voltage.**—The principle of the method of calculating the reactance voltage is as follows:—

The reactance voltage in the coil short-circuited under the brush, is that voltage due to the lines induced in the coil by the current undergoing reversal, which, being a rapidly-changing current, is accompanied by a rapidly-changing magnetic flux of self-induction through the circuit traversed by it. As the coils successively arrive at the position of short circuit under the brush, they cease for the moment to be a part of the main circuit of the armature winding from negative to positive brushes,

and the current of constant strength which had up to the moment before been flowing through them must stop and reverse, *i.e.*, it must undergo one-half of a complete cycle during the very brief time that the coil is short-circuited under the brush, that is, during the interval of time elapsing between the arrival of the forward edge of a given segment under the brush and the departure from under the brush of the rear edge of the segment immediately in front of it in the direction of rotation of the commutator. If P be the peripheral speed of the commutator in metres per second, and T the thickness of the brush in millimetres (or, more exactly, the length of the arc of brush contact in millimetres), then this interval of time, *i.e.*, that of one-half of a complete cycle is, in seconds, $\frac{T}{1000\ P}$, and, therefore, the average periodicity of reversal in complete cycles per second is, $\frac{1000\ P}{2\ T}$.

Now this change in the value of the current doubtless takes

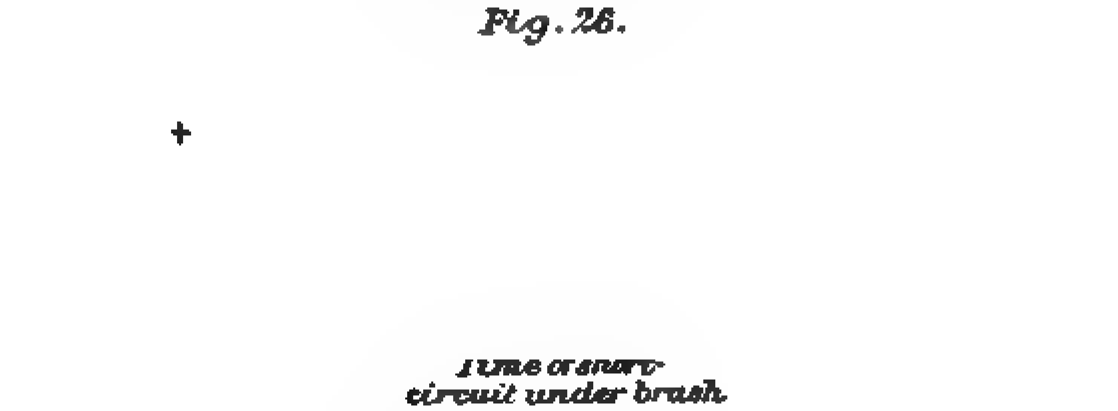


Fig. 26.—Diagram of Short Circuiting at Brushes.

place at a rate very different from that of a sine wave change, but an experimental determination would be very difficult, and no really satisfactory experimental data are available on the subject.

§ 22. **Short-Circuiting Coils in Commutation.**—It is convenient, therefore, in obtaining comparative data, to assume a sine wave rate of change in the value of the current, *i.e.*, to assume that, taking time as abscissæ and instantaneous current strengths as ordinates, the change of current as the coil passes over from the circuit on one side of the brush, through the interval when it is short-circuited under the brush, to the time of its joining the circuit on the other side, is as represented in Fig. 26, the width of the shaded rectangle representing the time during which the coil is short-circuited under the brush, and its height the complete change in current strength which it undergoes. As

a matter of fact, the above mode of representation might be fairly correct for the case of brushes correctly set to make—for a given value of the current—the best use of the impressed magnetic field, this impressed field being of just the right strength, acting in opposition to the coil's reactance voltage, to first reduce to zero the current originally present in the coil, and then, still in opposition to the reactance voltage, to induce a current in the

FIGS. 27, 28, and 29.—Diagrams of Short Circuiting at Brushes.

opposite direction which, just as the coil leaves the position of short circuit, shall have become just equal in strength to the current in the circuit of which it then becomes a part.

But with fixed brush position for all loads, the perfect commutation can only occur at some one intermediate value of the load. At no load the occurrence will be somewhat as represented in Fig. 27, the rate of building up of the current in the short-

circuited coil being somewhat greater than in Fig. 26, because, there being no armature current, there is no distortion of the impressed field. The magnitude of the abrupt change in current strength when the segment leaves the brush must not be so great that the resulting reactance voltage, due to this change of current acting through the inductance of the coil, shall cause sparking. This abrupt change in current strength may, without causing sparking, be greater the less the inductance of the coil, because the reactance voltage is proportional to the product of the current and of the inductance, as also of the periodicity, which latter is, however, assumed, although without sufficient justification, to be independent of the values of the inductance and the current.

At half load, as shown in Fig. 28, there is no tendency to sparking, for the voltage induced by the conductors of the short-circuited coil cutting through the impressed field is just sufficient to neutralise the reactance voltage at the instant the coil leaves the position of short circuit under the brush.

At full load, represented by the diagram of Fig. 29, the rate of decrease of the current originally present in the coil is still slower because of the greater reactance voltage, corresponding to a given decrease in the greater current concerned, as also to the weaker impressed flux (which has undergone further distortion, due to the greater armature reaction), and when the segment leaves the brush the current, though reversed, is represented to be still of less than half the strength of the current in the circuit it is to join. Hence the moment of leaving the brush is accompanied by a sudden rush of current equal to the required deficiency, and this, owing to the inductance of the coil in which it takes place, gives rise to a reactance voltage which must not be so great as to cause sparking. The brush position giving best commutation at half load might perhaps give a deficiency in current at full load about equal to the induced current at no load; at any rate, some such occurrences as those indicated in the diagrams are taking place.

§ 23. **Small Reactance with Wide Neutral Zone.**—By not making use of the impressed magnetic field, and merely making the value of the reactance voltage small at full load, and by having a wide neutral zone (that is, a liberal distance between pole tips—a small pole arc), so that the conductors of the short-circuited coil are not cutting through any impressed magnetic flux during the time of short circuit under the brush, the corresponding diagrams at no load, half load, and full load will be somewhat

as represented in Figs. 30, 31, and 32, reliance being placed upon the small inductance of the coil for obtaining sparkless commutation at the instant of abrupt change in current strength as the coil leaves the brush. The diagrams show clearly that the resistance of the brush plays a part in reducing the current strength, but not in building up the reversed current, which latter step in the process may be said to occur by "forced commutation."

In the light of these explanations it will be agreed that, until

FIGS. 30, 31, and 32.—Diagrams showing Effect of Brush Resistance.

further experimental data are forthcoming, it is as well to abandon much pretence to precision in the matter of the law of the rate of change of the current strength during reversal under the brush, and to employ, for uniformity's sake, the assumption that it follows a sine rate of change.

On this assumption, letting n equal the frequency (complete cycles per second), l the mutual inductance of *one* coil between adjacent segments with relation to *all* the coils simultaneously undergoing commutation, and I the current per coil, then the

reactance voltage and the sparking will be respectively equal and proportional to

$$V = 2\pi\ n\ l\ I.$$

§ 24. **Mutual Inductance.**—Further explanation is required as to the mutual inductance l. *All* the coils simultaneously undergoing commutation are active in setting up lines through any one short-circuited coil. Hence we must estimate the total magneto-motive force set up by all these simultaneously short-circuited coils, and, ultimately, the flux resulting from the sum of their magneto-motive forces.

From fairly satisfactory experiments, there have been deduced, as rough average values for open slot windings, that for the "embedded" portion of the coil there will be set up 4 C.G.S. lines per ampere turn and per centimetre of "embedded" length. For

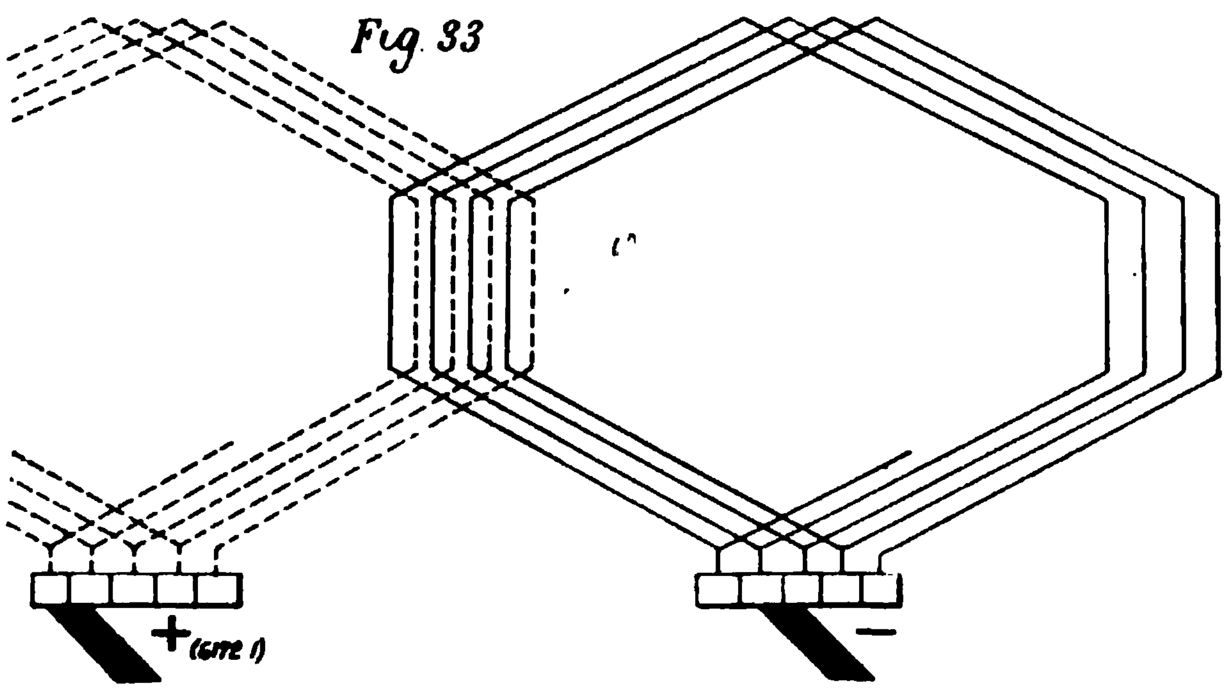


Fig. 33.—Effect of Short-Circuited Coils.

the "free" length, meaning thereby the end connections and that portion of the rest of the turn corresponding to the space occupied by the core ventilating ducts, there will be set up 0·8 C.G.S. lines per ampere turn and per centimetre of "free" length. It must further be noted that if each side of the short-circuited group of coils consists of n conductors, then n ampere turns are concerned, per centimetre of "embedded" length, in setting up lines through any one of the short-circuited coils, whereas for the "free" length, as will be seen from Fig. 33, only $\frac{n}{2}$ ampere turns per centimetre of "free" length are concerned. Noting, furthermore, that the "linkage" of magnetic lines and short-circuited turns per coil between segments is equal to the above deduced total flux set up through any one short-circuited coil between adjacent segments, multiplied by the number of turns in one such coil, the following

example of the calculation of the reactance voltage for a 4-pole, 5 brake horse-power, 900 revolutions per minute, 220-volt shunt motor, will give all the further guidance required for carrying through corresponding calculations on other motors.

§ 25. **Calculation of Commutator Sparking Constants.** —Number of poles, 4; number of segments, 81; number of segments per pole, 20·3; pressure, 220 volts; voltage per segment, 10·9; number of slots, 27; number of conductors per slot, 30; armature turns per pole, 101; amperes to commutator, 19·2; style of winding, 2-circuit, single; amperes per circuit, 9·6; armature ampere windings per pole, 975; commutator diameter, 16 centimetres; revolutions per second, 15; commutator peripheral speed metres per second, 7·5; length of brush contact arc, 10 millimetres; frequency of commutation cycles per second, 375; width at circumference of commutator segment plus insulation, 6·2 millimetres; length of a single armature turn, 68 centimetres; effective length of armature laminations in centimetres, 9·0; number of coils short-circuited per brush, 2; number of turns per coil, 5; maximum number of simultaneously commutated conductors per group, 20; embedded length per turn, 18 centimetres; free length per turn, 50 centimetres; lines per ampere turn for embedded length, $4{\cdot}0 \times 18 = 72$; lines per ampere turn for free length, $0{\cdot}8 \times 50 = 40$; total lines per ampere for embedded length, $72 \times 20 = 1440$; total lines per ampere for free length, $40 \times 10 = 400$; total lines linked with short-circuited turn, 1840. Then inductance of one 5-turn coil, with relation to all the turns simultaneously short-circuited, equals $1840 \times 5 \times 10^{-8} = 0{\cdot}000092$ henrys; reactance equals $2\pi \times 375 \times 0{\cdot}000092 = 0{\cdot}216$ ohm; reactance voltage per segment equals $0{\cdot}216 \times 9{\cdot}6 = 2{\cdot}07$ volts.

CHAPTER IV

TYPES OF WINDINGS

§ 1. Two-circuit and Multiple-circuit Windings.—In interpreting the results from this method of estimating the

Fig. 34.

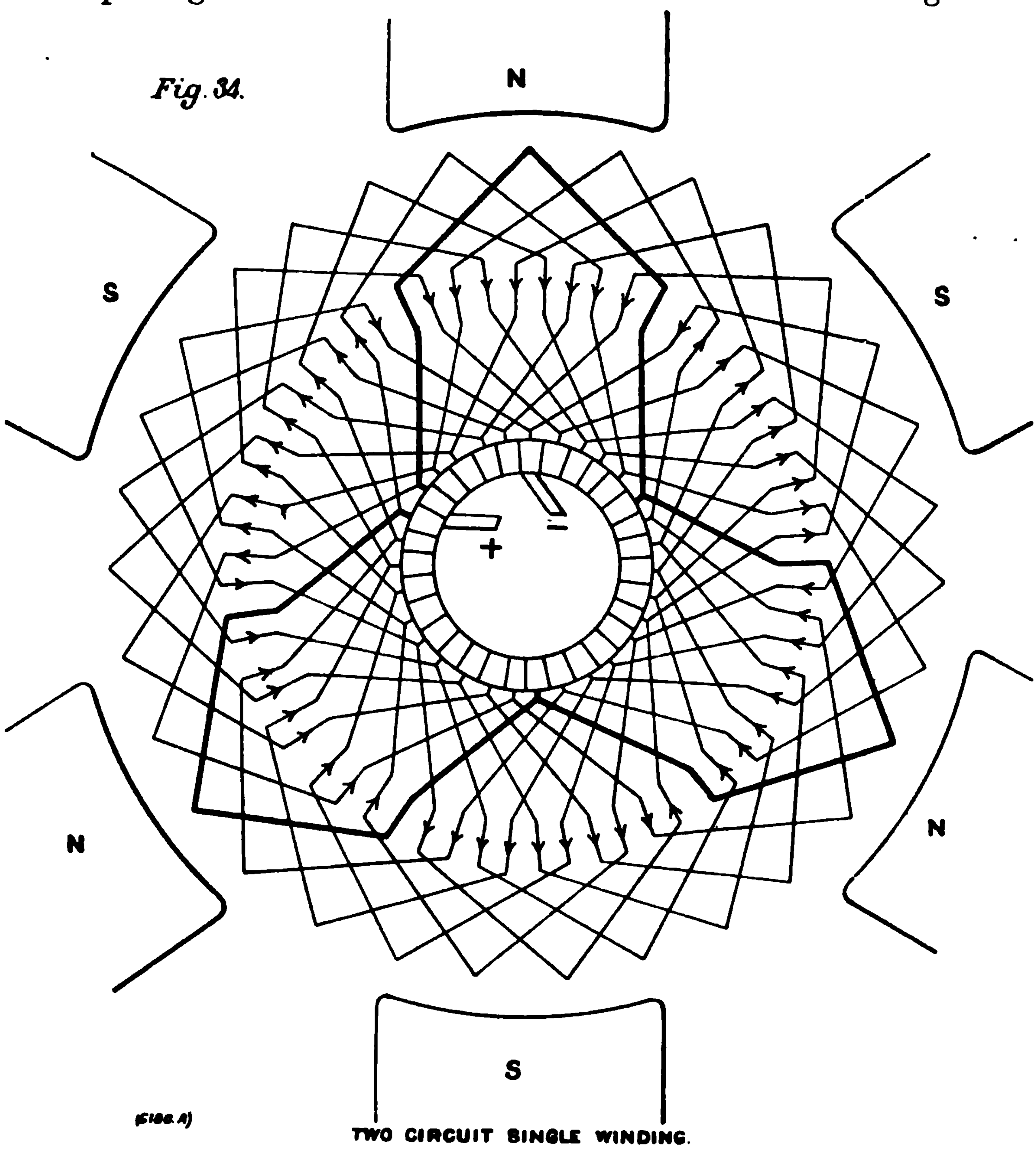
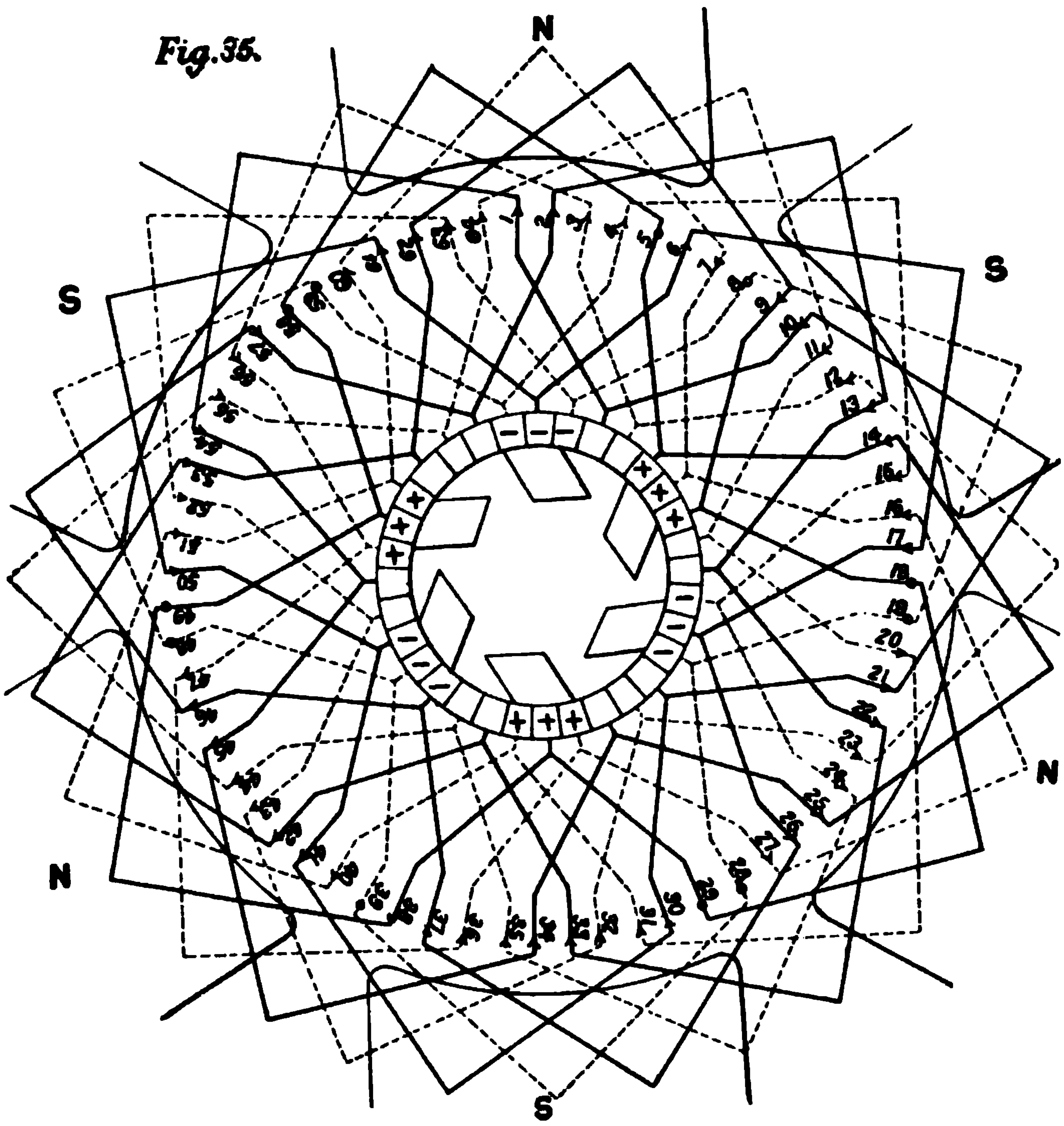


Fig. 34.

reactance voltage, it is essential to distinguish between two-circuit windings and multiple-circuit windings, as well as between

single and multiple windings. Any useful explanation of winding nomenclature is too elaborate to be given here[1]; it will suffice to refer to the winding diagrams shown in Figs. 34, 35, 36, and 37, representative respectively of a two-circuit single, a two-circuit double, a multiple-circuit single, and a multiple-circuit double winding. The two double windings (Figs. 35 and 37) are given as representing the simplest instances of multiple windings,



TWO CIRCUIT, DOUBLE WINDING.

FIG. 35.

which may also be triple, quadruple, etc. It may further be explained that multiple-circuit windings, in this system of nomenclature, are generally of the type sometimes termed lap

[1] For detailed treatment of armature windings, reference may be made to Arnold's *Ankerwicklungen und Ankerkonstruktionen*, 3rd edn., 1898, Julius Springer, Berlin; and Parshall and Hobart's *Armature Windings*, 1895, D. Van Nostrand Co., New York. See also pp. 60 to 70 of *Electric Generators*, by Parshall and Hobart, Engineering Pub., 1900; also ch. v., p. 78, of *Design of Dynamos*, by Dr Thompson, E. & F. N. Spon, 1903, where different colours are used adding greatly to clearness.

windings, whereas two-circuit windings are often described as wave windings. Two-circuit windings may be single or multiple (Fig. 34 is a two-circuit single winding, and Fig. 35 a two-circuit multiple winding). Multiple-circuit windings may also be of either class; thus, Fig. 36 is a multiple-circuit single winding, and Fig. 37 a multiple-circuit multiple winding.

The method of estimating the reactance voltage, of which an example has been given, lends itself to the case of the multiple-circuit

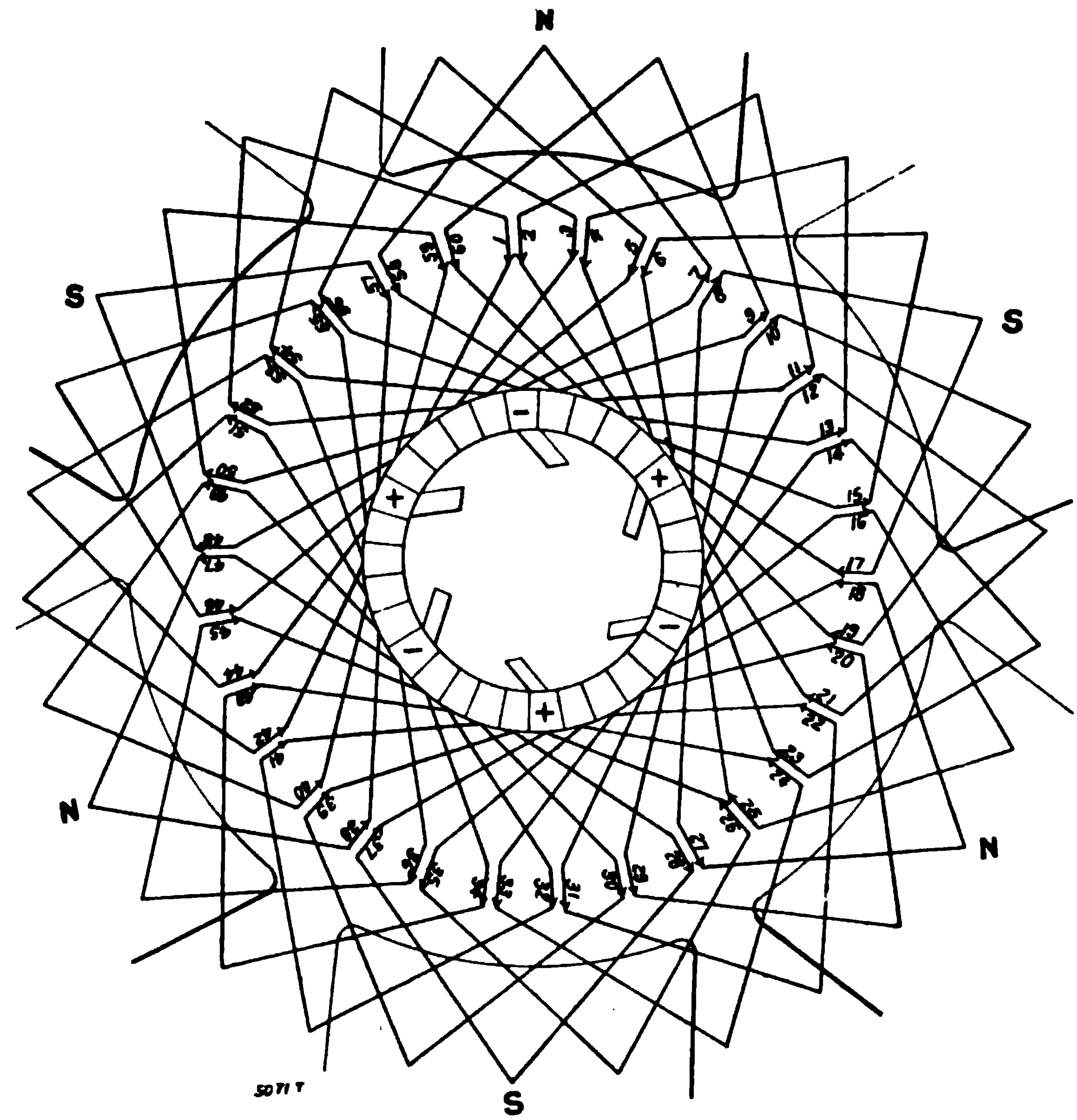


FIG. 36.—Multiple-circuit Single Winding.

single winding without further comment. For a multiple-circuit double winding one must take care to employ the right value for the current per conductor, *i.e.*, one-half that for a multiple-circuit single winding, and also to note that the time during which a coil is short circuited under a brush, is, for a given width of brush, less than for a multiple-circuit *single* winding. Thus in Fig. 38 are represented two adjacent coils of a multiple-circuit double winding and the four commutator segments to which the ends of the coils

are connected. A brush is also shown diagrammatically below the segments, and the arrow indicates the direction of motion of

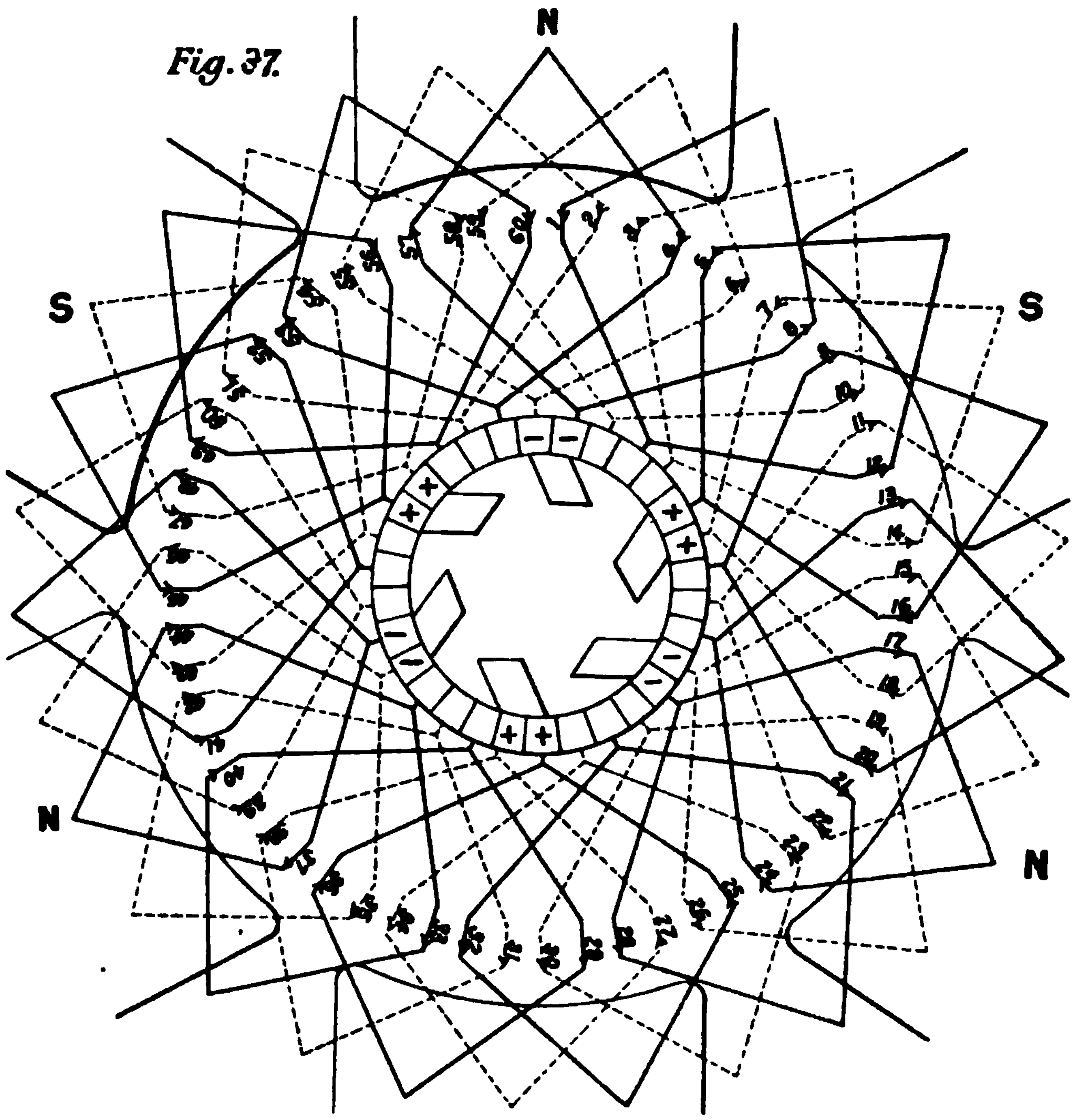


Fig. 37.—Multiple-circuit Double Winding.

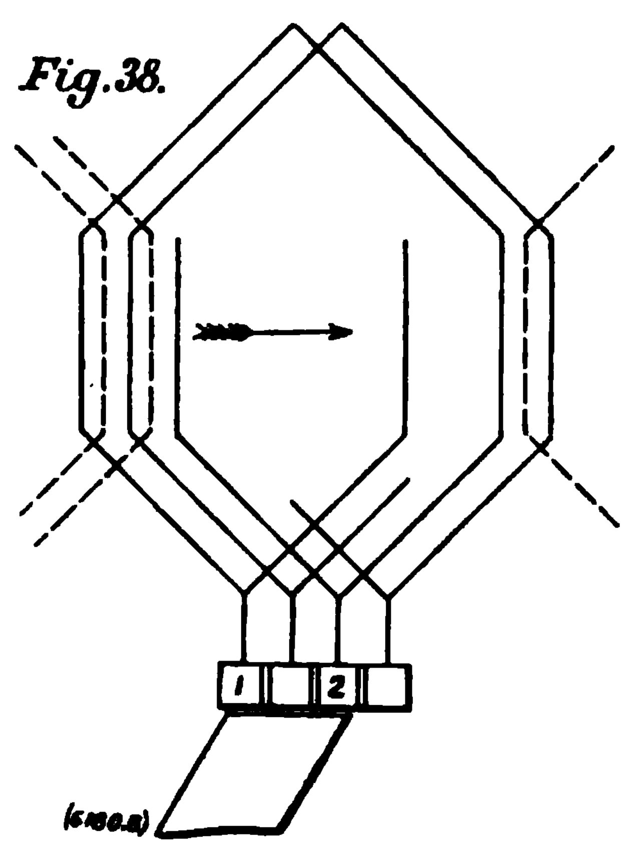


Fig. 38.

the armature past the brush. Obviously the coil connected to segments 1 and 2 is short circuited under the brush for a shorter

period than would be the case if the coil were connected to adjacent segments, as would have been the case in a multiple-circuit single winding. This must be taken into account in estimating the frequency of reversal, *i.e.*, the term n in the formula—

$$V = 2\pi n l I.$$

for the reactance voltage of the short-circuited coil.

§ 2. **Reactance Voltage of Two-circuit Windings.**—Those portions of the winding undergoing commutation have, in two-circuit windings, the choice of two short-circuited conducting paths in cases where as many sets of brushes are employed as there are

Fig. 39.

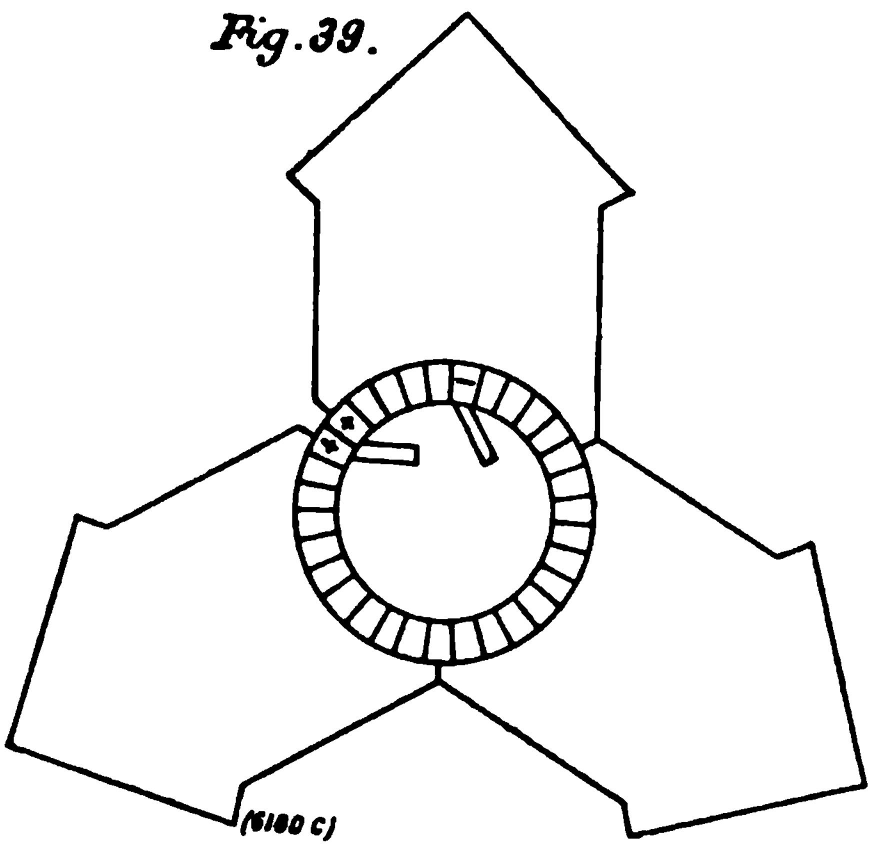
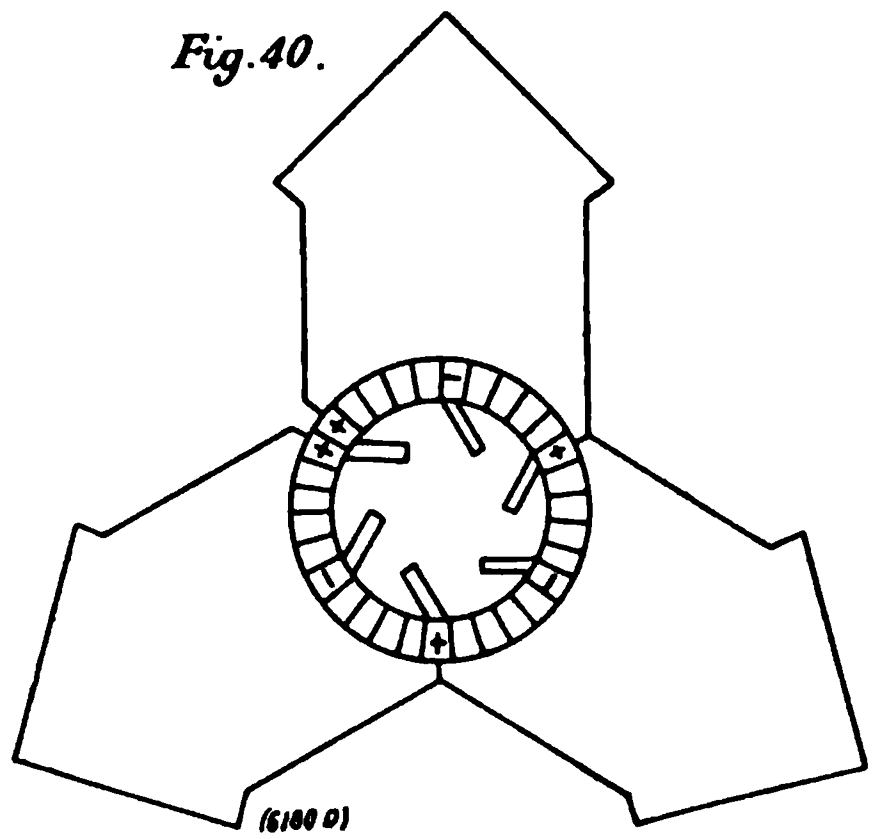


Fig. 40.



FIGS. 39 and 40.—Reactance Voltage of Two-circuit Windings.

pole pieces in the motor. To illustrate by the case of a six-pole motor, there is drawn in Fig. 39 a diagram showing those conductors of Fig. 34 temporarily undergoing short circuit at the positive brush. In this electric circuit there are six conductors, or the equivalent of three turns, in series, and if there is, at full load, a reactance voltage of 1 volt per turn, the reactance voltage in the short-circuited coil amounts to 3 volts.

But if, as in Fig. 40, all three sets of positive brushes are employed, each pair of conductors is short circuited by another path, consisting of two positive brushes and the cable interconnecting them. In each of these independent circuits the reactance voltage is but 1 volt, and the commutation should be much better than when but one set of positive brushes is used.

In practice the occurrences are doubtless a combination of these two alternatives.

Fig. 41 indicates diagrammatically an analogous system of circuits, in which R_1 represents the contact resistance of a brush bridging two adjacent segments, R_2 the contact resistance from a segment to the second brush, plus the resistance of the brush itself and its contact with the brush holder, and R_3 the value for the third brush. C_1, C_2, and C_3 represent the resistances of the interconnecting cables, and the reactance voltage in the three turns is represented by the symbol for a voltaic cell. Since R_1, R_2, and R_3 are dependent upon the adjustment of the brush pressure and the extent to which the brush surface is worn in, it is useless to attempt to estimate the division of the current between the alternative paths. These considerations and others relating to

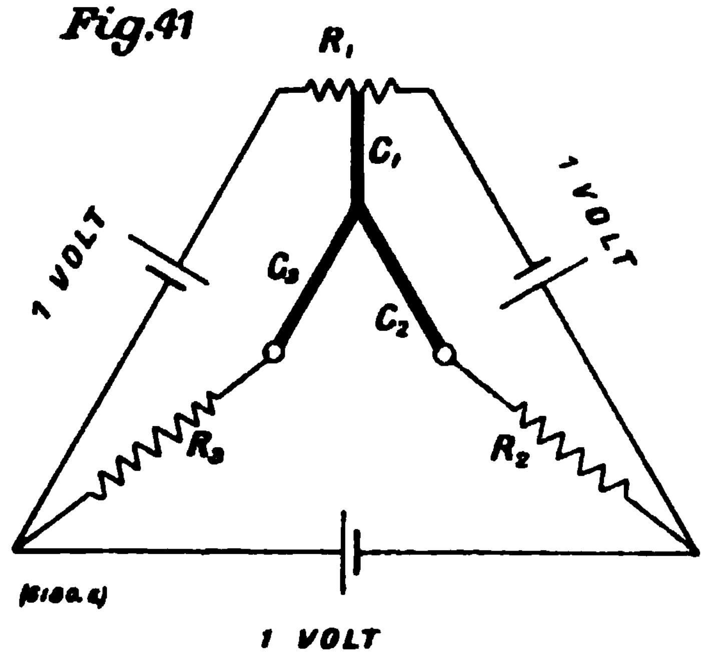


FIG. 41.—Reactance Voltage of Two-circuit Windings.

the property of the two-circuit winding, in virtue of which the current passing to the external circuit may be very unevenly divided amongst the different sets of brushes of the same polarity,[1] afford ample explanation of the results of experience with two-circuit windings, namely, that they must be employed with caution, and must have more conservative constants than multiple-circuit windings. These considerations are, however, no justification for abandoning the use of such windings. There are great ranges of outputs, voltages, and speeds, where they may be proportioned

[1] For this reason there is introduced the additional indefiniteness that the potential in the three circuits of the diagram of Fig. 41 will not necessarily be equal and of the value of 1 volt. An unequal distribution of the current amongst the three sets of positive brushes would lead to a reactance voltage of more than 1 volt in one or two of the three circuits and a lower voltage in the remaining circuit or circuits, the sum of the reactance voltages in the circuits always, at full load, amounting to 3 volts.

with such low reactance voltages between consecutive segments as to give excellent results. Manufacturers often take up these windings too confidently, and employ them without regard to their special fitness for any particular machine. Obtaining naturally very unsatisfactory results in many cases, they go to the other extreme and employ multiple-circuit windings exclusively, even in those cases where two-circuit windings would be much more suitable.

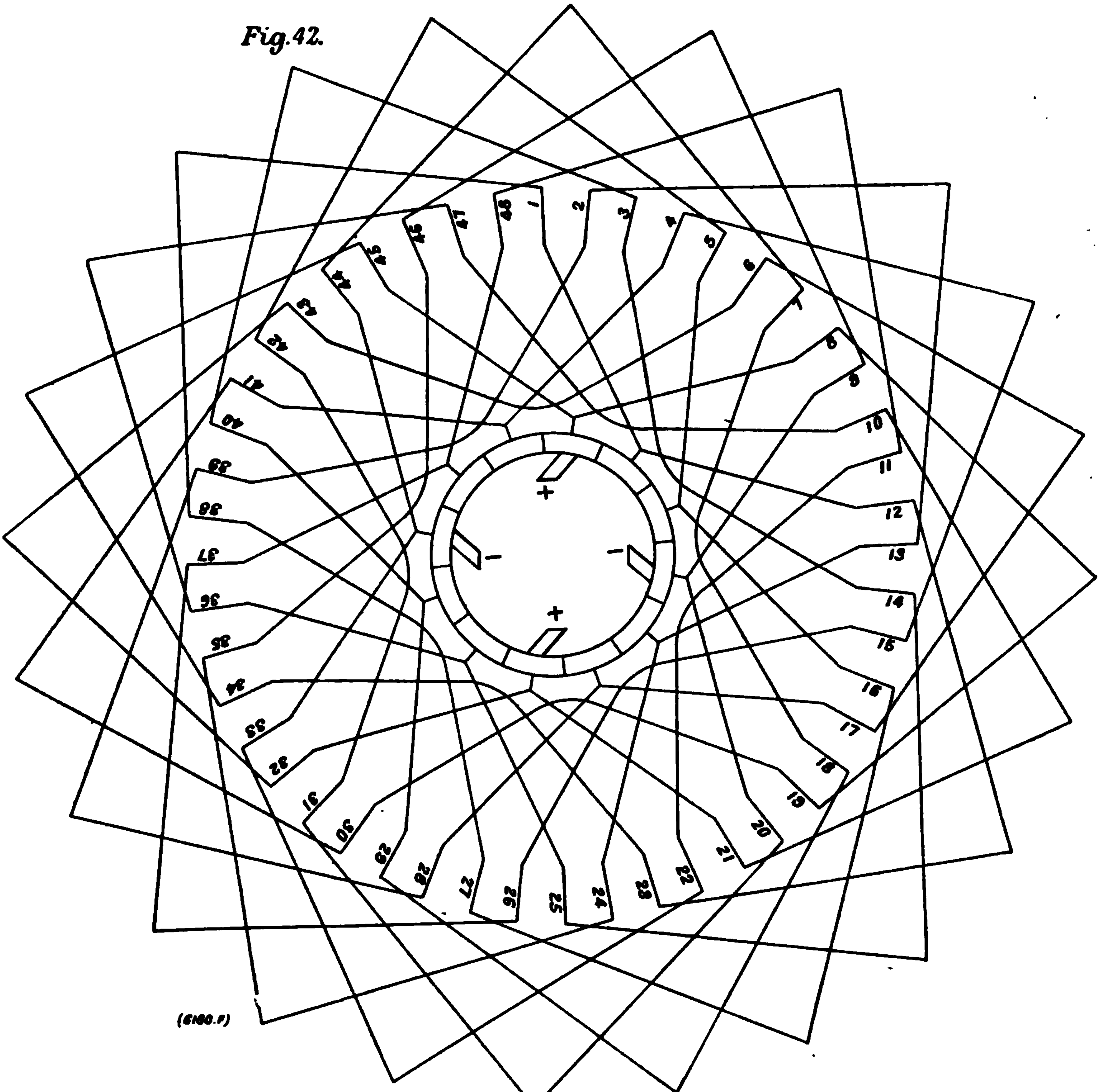
Excellent results, especially with small motors, may attend the correct use of two-circuit windings. They have one very important property, which is, that equal electromotive forces are induced in the two circuits through the armature, irrespective of any eccentricity of the armature in the field, and thus irrespective of wear in the bearings. In multiple-circuit windings the armature must be very accurately centred, in order to avoid unequal electro-motive forces in the different branches and the consequent heating and sparking due to cross currents.

§ 3. **Two-circuit Single Winding.**—The reactance voltage of two-circuit single windings is readily derived, either by considering the case where but one set of positive and one set of negative brushes are used, and dividing the result by the number of pairs of poles, when there is a set of brushes for each pole; or else by carrying through the calculation just as for a multiple-circuit winding; remembering, in taking the final step of obtaining the reactance voltage from the product of the reactance and the current per conductor, that each conductor carries one-half of the full current input to the motor, irrespective of the number of poles. By this latter method the reactance voltage corresponds to the minimum obtainable by the use of as many sets of brushes as there are poles, and it should be multiplied by the number of pairs of poles where only two sets of brushes are employed.

§ 4. **Two-circuit Multiple Windings.**—For two-circuit *multiple* windings there is the further difference that the periodicity is greater by the amount of the width of the intermediate segments, similarly to the case illustrated in Fig. 38 for multiple-circuit multiple windings. Two-circuit multiple windings should, however, seldom be used. In them is found, in far greater degree, the indefiniteness just referred to for two-circuit single windings. Two-circuit double windings may occasionally be desirable; higher multiples should be avoided. As, however, the advantages alleged for them by their advocates occur chiefly in large generators, no special explanation of their properties will be given here.

§ 5. **Four-Circuit Multiple Winding.**—In Fig. 42 is shown a four-circuit single winding, which has the property of the two-circuit winding of having in the four branches a symmetrical distribution of the potential, even when the armature is eccentric

Fig. 42.



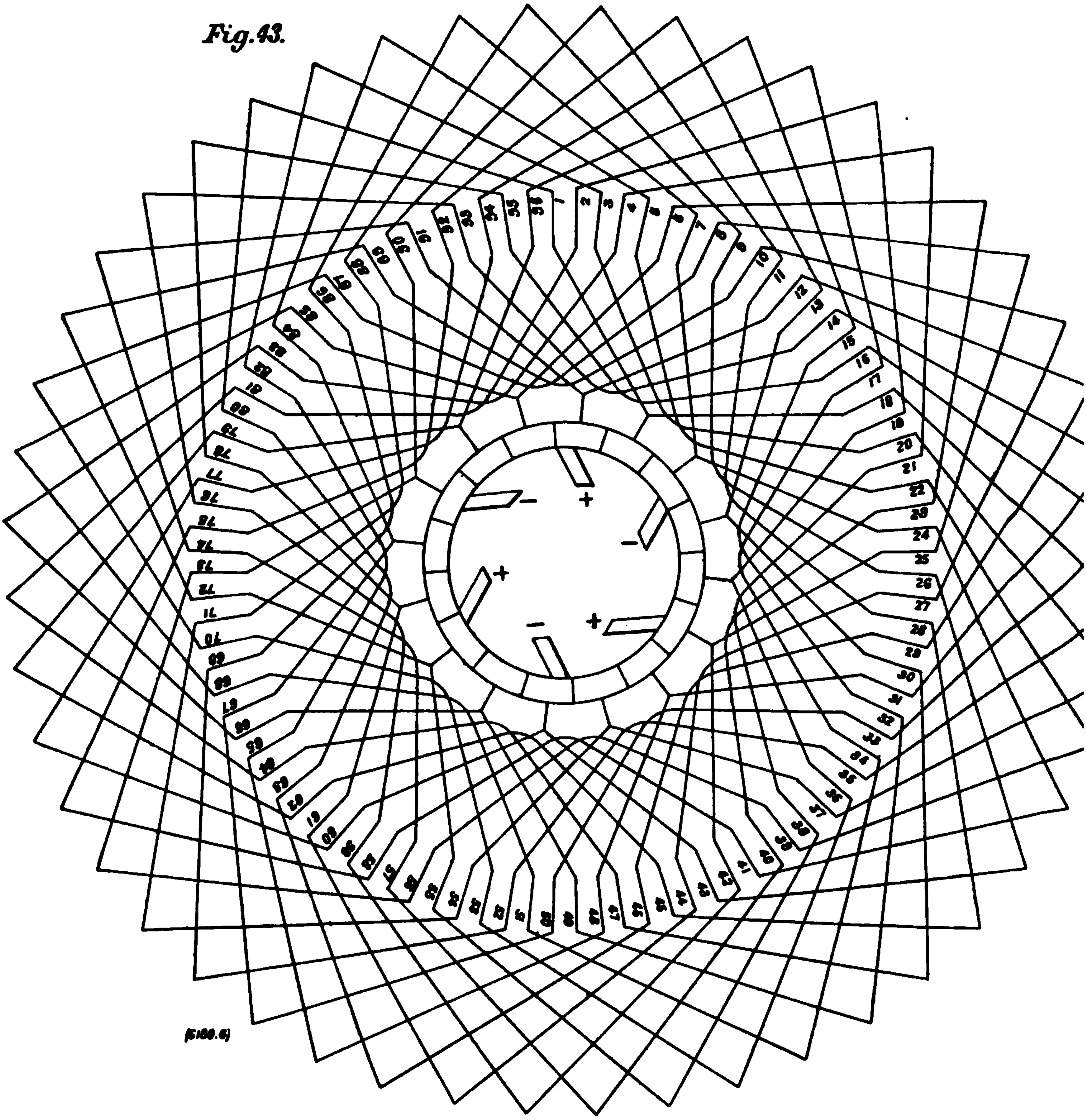
4 CIRCUIT SINGLE WINDING.
Derived from a 2-Circuit, double winding of the formula $c = ny + 2m$ $(48 = 4 \times 11 + 2 \times 2)$ by omitting every other segment.

FIG. 42.

in the field. This winding is derived from a two-circuit double winding by leaving out every alternate commutator connection. Because of the above-mentioned property, it should have a field of usefulness in motors of those ratings requiring several turns per

segment, and it has the feature of superiority over the two-circuit winding that it ensures an equal division of the current between all sets of brushes of the same polarity. Very numerous similar

Fig. 43.



6 CIRCUIT SINGLE WINDING,
Derived from a 2-Circuit, triple winding of the formula $c = ny + 2m$ $(96 = 6 \times 17 - 2 \times 3)$ by leaving out every second and third segment.

FIG. 43.

windings may be derived. One other of this class is given in Fig. 43.

§ 6. **Compound Wound Motors.** — Compound wound motors are of two classes. In the one case, the series winding

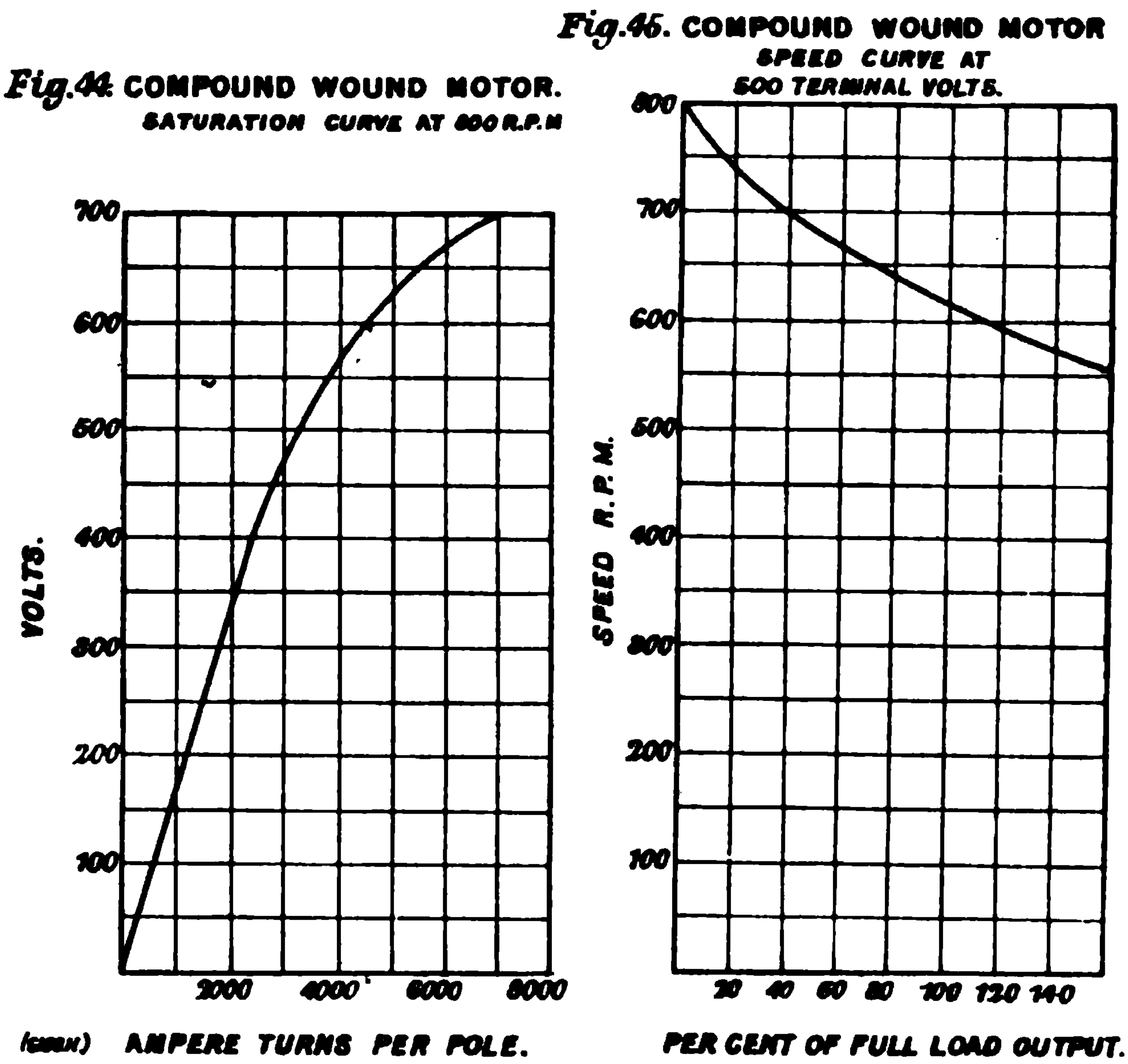
opposes the shunt winding, with the object of compensating for fall in speed with load or for obtaining speeds increasing with the load. Such motors are rarely required, and there is, furthermore, the objection to this arrangement that special provision has to be made, such as means for cutting out the compound winding at starting, otherwise the series coil, with its lower inductance, would rise to its full strength much more quickly than the fine wire shunt winding, and the motor would start in the wrong direction In the rare cases where the speed must not decrease as the load increases, the brushes may, in the plain shunt motor, be set with such a backward lead as to maintain practically constant speed, through the influence of armature demagnetisation.

Much more useful is the compound wound motor with the series coil reinforcing the shunt. Motors of this type obviously decrease in speed with increasing load, and the extent of the decrease is dependent upon the relative strengths of shunt and series fields. If, for example, the shunt winding of such a motor supplies 2000 ampere turns per pole, and the series coil at full load supplies 1000 more ampere turns, then, neglecting magnetic saturation and IR drop in armature, series coils, and brush contacts, the speed at full load will be 66·7 per cent. of the no load speed, the saturation of the magnetic circuit increasing this, in an average case, to 80 per cent.

Of course, much more extreme proportions may be used, and this is very desirable in the case of motors required to handle intermittent and heavy loads where there is no objection to the heaviest work being performed at much slower speeds. This is of great advantage to the whole supply system; the generating plant is more evenly loaded, and the pressure regulation is better. Moreover, it is of considerable advantage as regards good commutation that a motor should carry its heaviest loads at reduced speeds. This is because the periodicity of commutation, and therefore the reactance voltage for a given current, is proportional to the speed. The compound wound motor has in this respect, to a certain extent, the advantage obtained in a series motor. For some work it is preferable to the series motor, the shunt winding acting as a check on its speed at light loads.

§ 7. **Compound Motor's Speed Curve.**—The chief point wherein the calculations relating to a compound wound motor differ from those for a shunt motor, relates to its speed at various loads. The estimation of a compound motor's speed curve may be illustrated by the case of a 500-volt, 30 horse-power motor. It

has a shunt winding giving a constant magneto-motive force of 3000 ampere turns per spool. When developing 30 horse-power the motor consumes 50 amperes, which pass through a series field winding with fifty turns per pole. Therefore the series excitation is $50 \times 50 = 2500$ ampere turns per pole. The saturation curve of the machine at its no load speed of 800 revolutions per minute is given in Fig. 44. From the saturation curve the speeds for



FIGS. 44 and 45.

various loads and 500 terminal volts may be calculated as indicated in the following table :—

CHARACTERISTICS OF 30 HORSE-POWER MOTOR AT VARIOUS LOADS.

	Per Cent. of Full Load.					
	0	25	50	75	100	125
Amperes input	4	15	26	38	50	63
Series excitation per pole	200	750	1300	1900	2500	3150
Shunt excitation per pole	3000	3000	3000	3000	3000	3000
Total excitation per pole	3200	3750	4300	4900	5500	6150
Voltage for 800 revs. per minute ...	500	550	585	620	650	680
Speed (revs. per min.) for 500 volts	800	727	682	645	615	588

The speeds thus derived are plotted against the corresponding

loads in the curve of Fig. 45. In this illustration it is convenient to assume—for the sake of simplicity—that the brushes are so placed that armature reaction compensates for the IR drop. Were this not the case it would be necessary to calculate the internal voltage from the current input and the resistance, and use these values in deriving the speeds from the saturation curve.

In motors with a wide neutral zone, low reactance voltage per segment, and high armature strength at full load (the latter expressed in armature ampere turns per pole), the brushes could be given a large permanent forward lead, and a speed decreasing

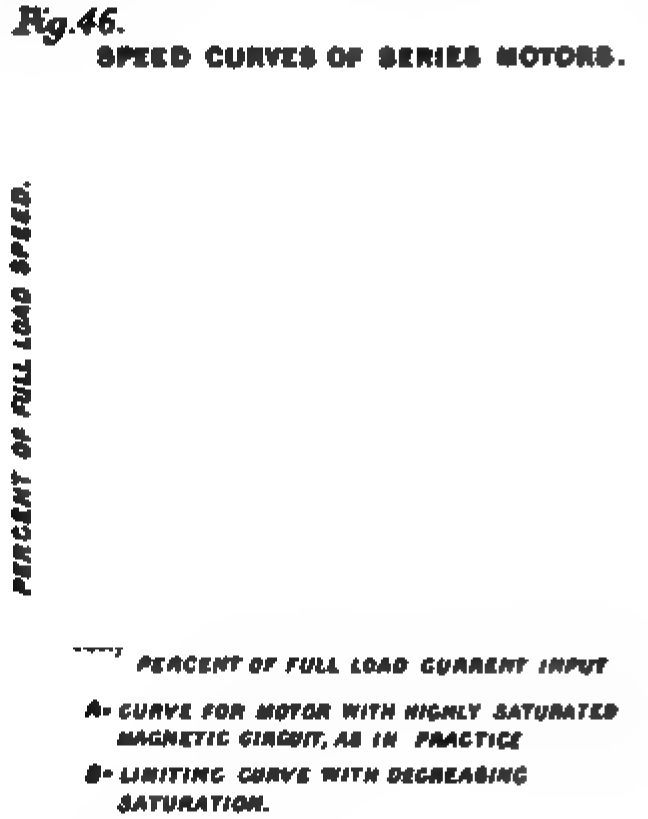


FIG. 46.

rapidly with increasing load would be obtained without the necessity for a compound winding.

§ 8. **Series Motors.**—In the series motor the field excitation is supplied by the main current, which passes through the field coils, connected in series with the armature. The excitation is consequently proportional to the input to the motor—practically also to the load—and were it not for the saturation of the magnetic-circuit and internal resistance drop, the speed would, at constant terminal voltage, be inversely as the amperes input, as shown in the dotted line curve *B* of Fig. 46, where, at 50 per cent. of full load, for instance, the speed is double that at full load.

But series motors are, in practice, generally proportioned to have a highly saturated magnetic circuit at full load current, and

the speed curve assumes the form of the full line curve *A* of Fig. 46.

Series motors are chiefly used for locomotion, hoisting, and other work where a high starting torque is required, and, as the load is generally of an intermittent nature, they are proportioned with much higher current densities in armature winding and field spools for their normal rated load than would be suitable for motors designed to carry their full load during the entire period of service. An arbitrary basis of rating for tramway motors which has now been in generally-accepted use for a number of years defines the nominal capacity as the horse-power output, giving 75 degrees Cent. thermometric rise of the hottest accessible part after one hour's continuous run on a testing stand at rated voltage. Tramway motors in actual service are required to carry an average load of only some 35 per cent. or less of their rated load, and this shows the great importance of designing them for high efficiency at light loads. And they are inherently capable of being proportioned to give this result, for the loss in field excitation, instead of being a component of the "no load" loss, increases from a negligibly small amount at no load with the square of the load, and hence is a component of the so-called "variable losses." In motors for light work, however, the gearing loss comes in and increases the "no load" losses considerably; but large, direct connected series motors are inherently of very high efficiency at light loads. These considerations have reference exclusively to motor and gearing, but it should be kept in mind that series motors require auxiliary controlling apparatus, in which, when starting, very considerable losses take place in external resistances.

§ 9. **Methods of Motor Control.**—In the simplest form the control is affected by a resistance in series with the motor, which, according to the load and the required speed, is varied in amount. A much greater economy is obtained by the method of "series parallel" control, which requires either two or more motors, or two independent armature windings and commutators per motor. By this method the two motors are connected in series at starting and for low speeds, and less external resistance and waste of energy are required than by the pure rheostatic method. For the higher speeds the motors are thrown in parallel, first with and then without resistances in series. Even by these series parallel methods of control no really high efficiencies during the period of starting are obtained. These methods have from time to time been modified by the addition of the element of independent

control of the field strength by a diverting shunt to the series winding. This has come to be considered as objectionable in the belief that a weakening of the field is inconsistent with satisfactory commutation. With the better understanding of the occurrences during commutation and the accumulated experience in motor design, a reversion to this feature of motor control, and to compound wound motors, in connection with the series parallel system, would probably be attended with satisfactory results as regards commutation, and would permit of a further increase in efficiency.

Two-circuit armature windings exclusively are employed in tramway motors—firstly, because the excessive wear in the bearings requires satisfactory operation with unequal air gaps above and below the armature; and secondly, because, with their rather

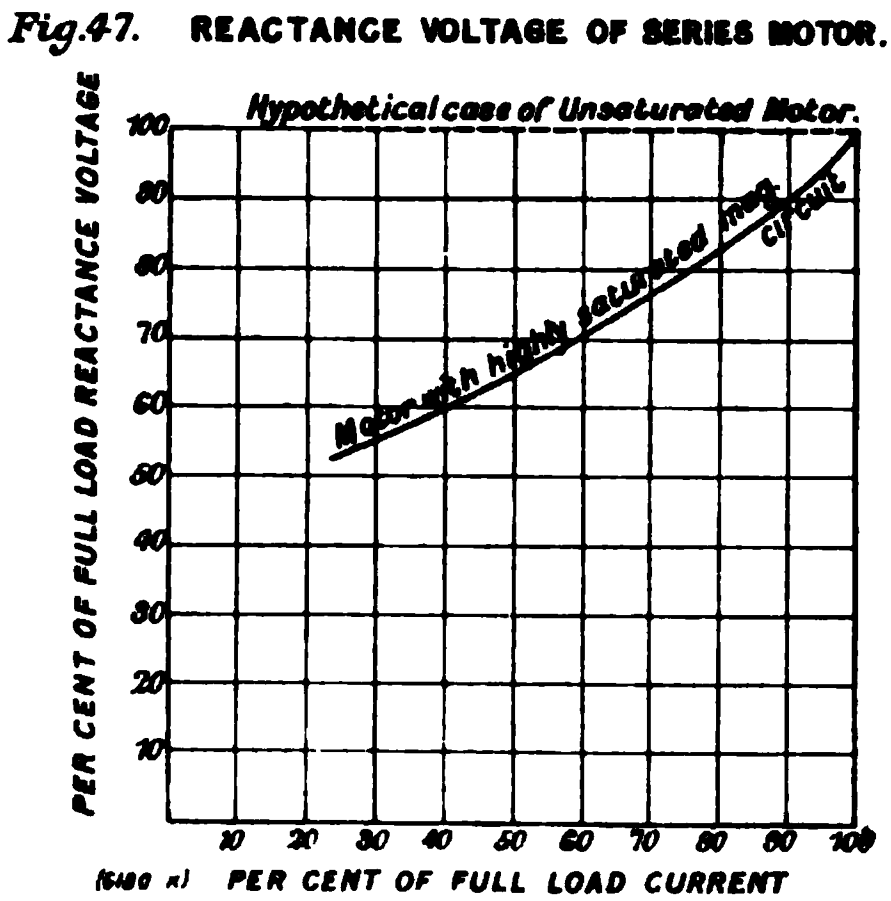


FIG. 47.

inaccessible location, it is convenient to use but two brushes on the commutator, which are made as accessible as practicable generally by a single suitably-located cover in the casing.

§ 10. **Reactance Voltage in Series Motor.**—The reactance voltage per commutator segment does not, in series motors, increase proportionally to the load, as in shunt motors, for the reason that the periodicity n in the formula $V=2\pi n l I$ for the reactance voltage decreases with the decreased speed attending the increase in the current, I. Let us assume the case of a motor having the full line speed curve of Fig. 46. The frequency of commutation corresponding to full load speed may be taken as 100, and the full load current also as 100. The inductance, l, between

consecutive segments of the winding may, for practical purposes, be taken as constant. The reactance voltage, V, varies therefore with the load, as the product. $n\ I$, *i.e.*, as the product of abscissæ and ordinates of the full line curve of Fig. 46. This product is plotted in the full line curve of Fig. 47, in terms of the current input, I.

According to the degree of saturation, the rate of change of the reactance voltage with the load will vary in position by a curve lying between the two curves of Fig. 47, the dotted line curve representing the limit approached with decreasing saturation of the magnetic circuit.

Evidently the broad assertion may be made that a series motor should show far less increasing tendency to spark with increasing load than would a shunt motor.

CHAPTER V

MOTOR CHARACTERISTICS

→§ 1. **Comparison of 8½ H. P. Shunt Compound and Series Motors.**—A saturation curve is given in Fig. 48, which may be taken to represent that of three motors—the first a shunt motor with constant speed at all loads, the second a compound wound motor with a speed decreasing to one-half at full load (of

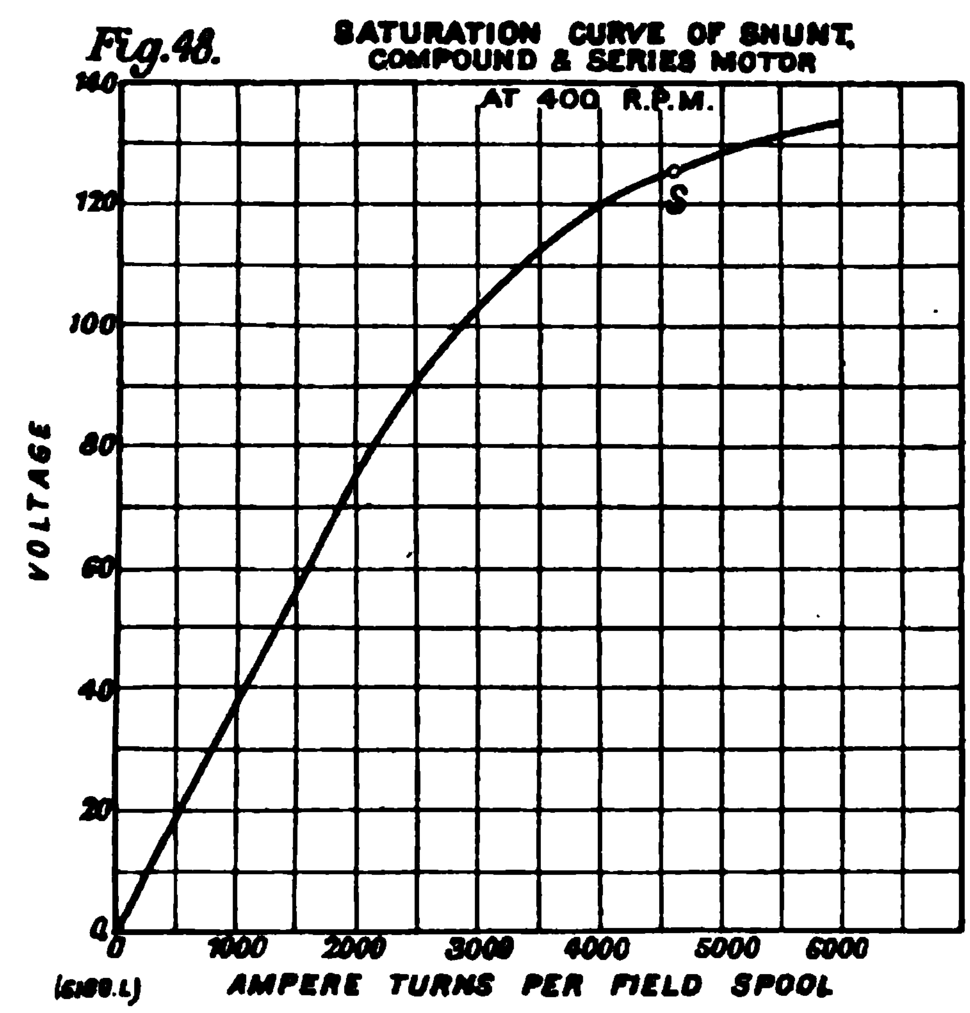


Fig. 48.

its 4600 full load ampere turns, 1700 are shunt and 2900 are series), and the third a series motor, with a speed decreasing at full load to one-half its value at 37 per cent. of full load amperes input. The motors are all of the same capacity and speed at full load, and are, at full load, all worked at the point *s* of the saturation curve. The three speed curves are shown in Fig. 49. Identical armatures are assumed to be employed in

the three cases, and the inductance per segment is 0·000020 henrys, the periodicity of commutation 580 cycles per second at the full load speed of 400 revolutions per minute, and the full load current in the winding is 30 amperes. Therefore the full load reactance voltage, $V = 2\pi\, n\, l\, I$, is

$$V = 2 \times \pi \times 580 \times 0{\cdot}000020 \times 30$$
$$= 2{\cdot}2 \text{ volts}$$

in all three motors. The motor is for 8·5 rated horse-power output at 84·5 per cent. efficiency. The armature winding is

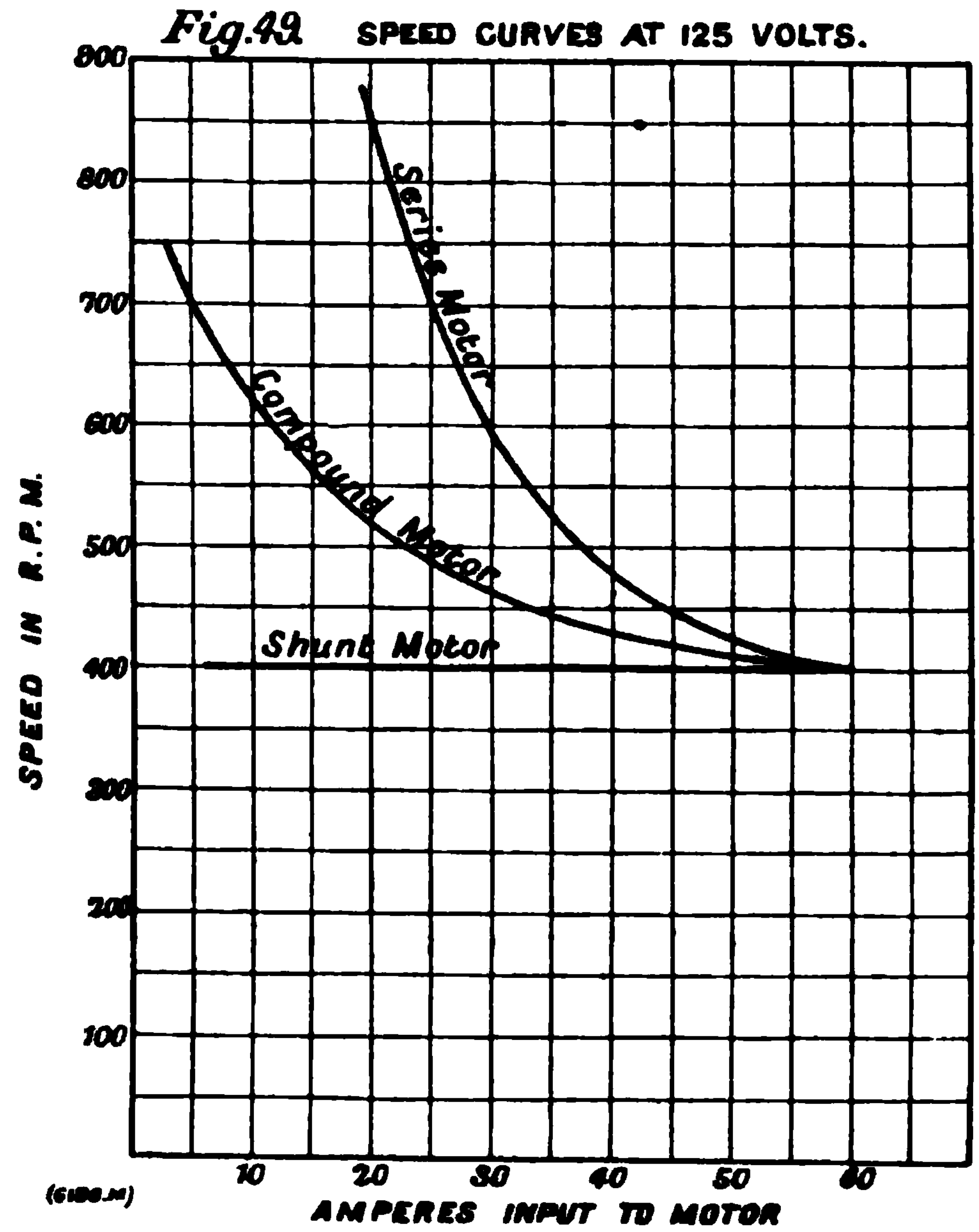


FIG. 49.

two-circuit single, and the full load current input is 60 amperes. The normal voltage is 125.

The three curves representing the change in reactance voltage with the load are given in Fig. 50. In deriving these curves, the assumption has, for convenience, been made that the efficiency remains the same at all loads considered, and the internal IR drop has been neglected. While these assumptions are quite widely departed from in practical cases, the errors do not materially affect the point under consideration—the relative reactance voltages of shunt, compound, and series motors.

§ 2. **Torque Curves.**—From the saturation and speed curves (Figs. 48 and 49) of these three motors we may derive curves of torque expressed in kilogrammes at 1 metre radius. A *shunt* wound motor is almost always operated (except at starting up) with constant potential at its terminals. The magnetic flux is therefore constant at all loads, and the torque increases in direct proportion to the horse-power output, and (except at very low loads) approximately in proportion to the amperes input. In a *compound* wound motor the torque at first increases considerably more rapidly than the output, to compensate for the rapidly decreasing speed. At higher outputs, where the speed decreases but slowly—owing to saturation—the torque increases at only a

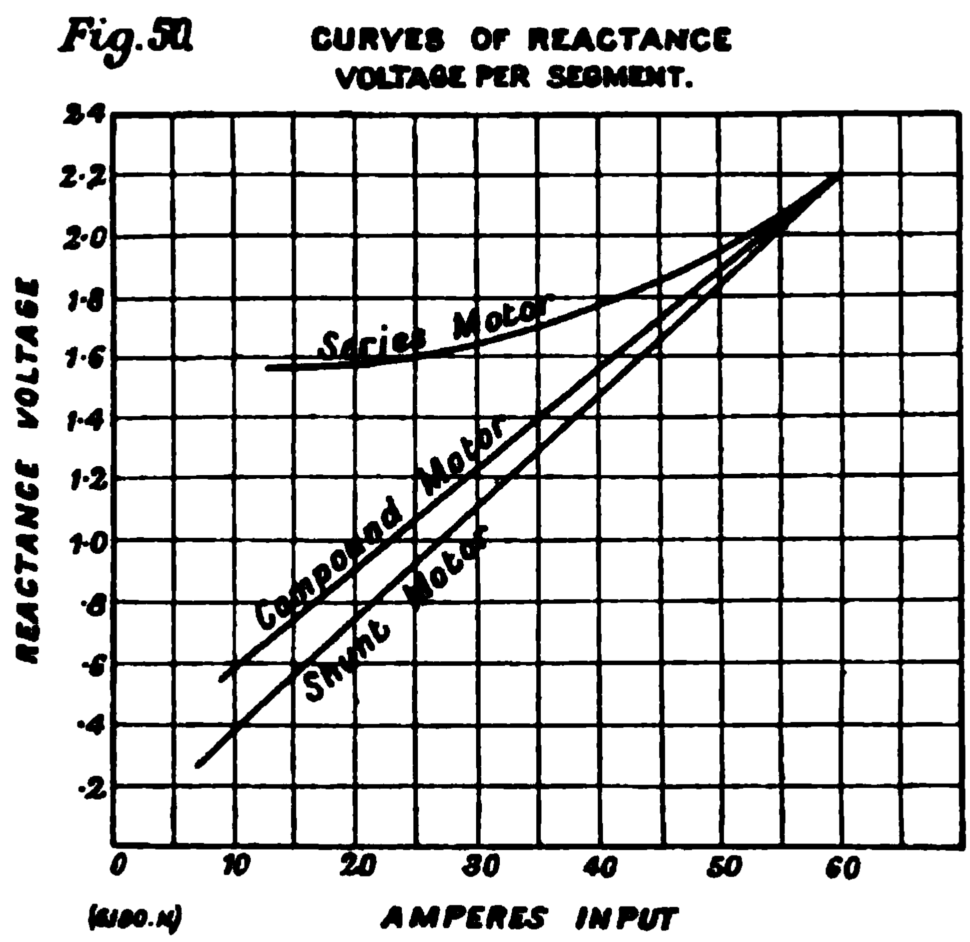


FIG. 50.

slightly higher rate than the output. Looked at from the standpoint of the actual electro-magnetic occurrences, the more rapid increase in torque with increasing load than occurs in a shunt motor is due to the increasing magnetic flux caused by the compounding coil in addition to the increasing armature current, which latter *alone* increases with the load in the shunt motor. Where *series* motors are operated under this same condition of constant potential at their terminals at all loads, we find them endowed with the same property, as regards rate of increase of torque with load, and the check in this rate of increase which saturation causes, as the compound motor, but in a greater degree. The curves of torque and speed for the three types of motor when

operated at constant terminal voltage are given in Fig. 51, and are derived from the saturation and speed curves of Figs. 48 and 49. In deriving these curves it has been considered sufficiently exact for the purpose to assume the same efficiency curve for the three motors, although, in practice, the series motor's efficiency curve would be higher at light, and lower at heavy, loads. The *British* horse-power is employed, equal to 76 kilogrammetres per second.

§ 3. **Torque Curves of Series Motors.**—Series motors are, however, rarely operated at constant terminal voltage for all loads. Either the terminal voltage is cut down to a greater or less extent by resistances in series with the motor, or two motors are for some loads and speeds connected in series, thus each receiving

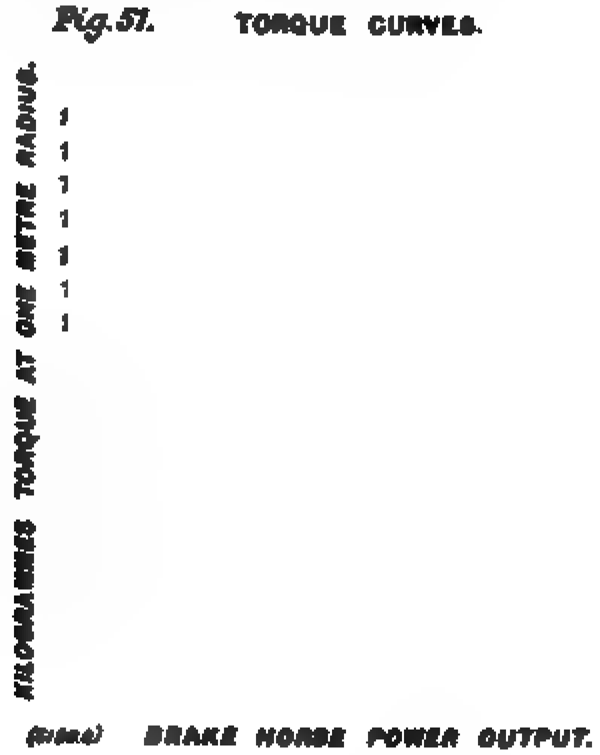


Fig. 51.

half the voltage, and in parallel for other loads and speeds, having then each the full voltage at their terminals, or, and most generally, by combinations of such series parallel connections with series resistances. Hence the study of the torque of a series motor is better taken up from the standpoint of the torque as function of the current, regardless of the terminal voltage. The torque in kilogrammes at 1 metre radius is, for this same series motor, plotted in the full line curves of Fig. 52, in terms of the amperes input, and corresponds to the saturation curve of Fig 48. The current input required to start the motor without load depends upon the no load loss, which is made up of the core boss (which is next to nothing at starting), the friction loss in bearings,

gearing, and brushes (which together often make up a very considerable total friction loss), and the almost negligible loss due to the I^2R of this small starting current flowing through the resistances of brush contacts and windings. Hence the product of the current input and the voltage at the terminals of the motor is a fairly good measure of the mechanical friction at starting. The modern tramway motor, with single reduction gearing, generally requires, when the gearing is in good condition, from 10 per cent. to 15 per cent. of the rated full load current to supply these losses.

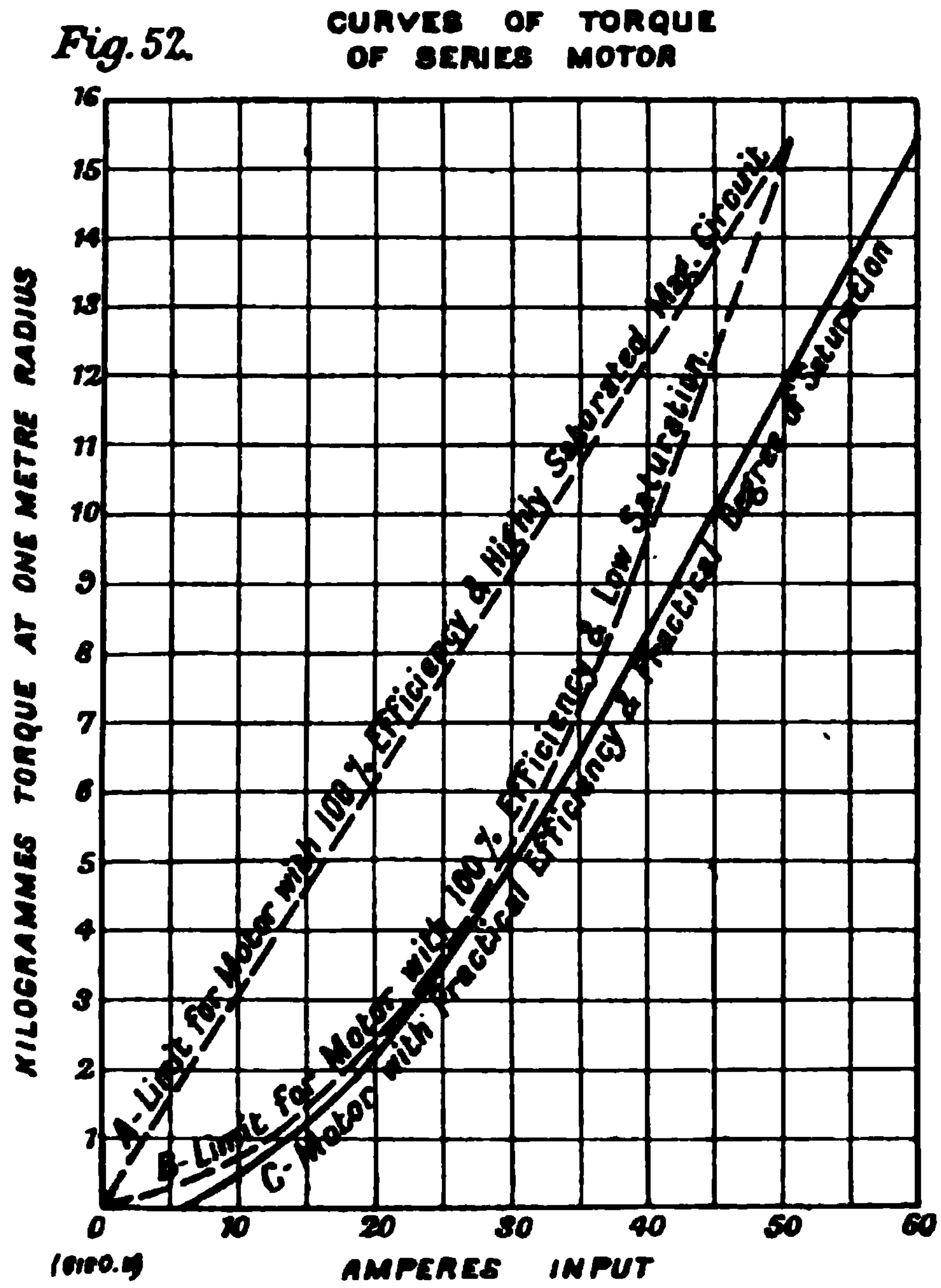


FIG. 52.

Hence, when not connected to any external load, it will start from rest with from 10 per cent. to 15 per cent. of the full load current.

There have also been drawn in Fig. 52 two dotted curves, intersecting at a point representing a torque of 15·4 kilogrammes at 1 metre radius at 50·6 amperes, whereas the actual motor, with 84·5 per cent. full load efficiency, requires $\frac{50{\cdot}6}{84{\cdot}5} = 60{\cdot}0$ amperes. These two dotted curves represent the limiting conditions of two motors for the same horse-power, speed, and voltage as the actual motor (8·5 horse-power, 400 revolutions per minute, 125 volts),

but having 100 per cent. efficiency. In the motor corresponding to the straight line torque curve the magnetic circuit is imagined to be saturated for all currents, giving a constant magnetic field at all loads, as in a shunt motor. This is the limit approached by the modern tramway motor, which is generally designed with a magnetic circuit of small cross section and highly saturated. The lower dotted curve represents the opposite limit which would be approached with motors of high efficiency and low saturation

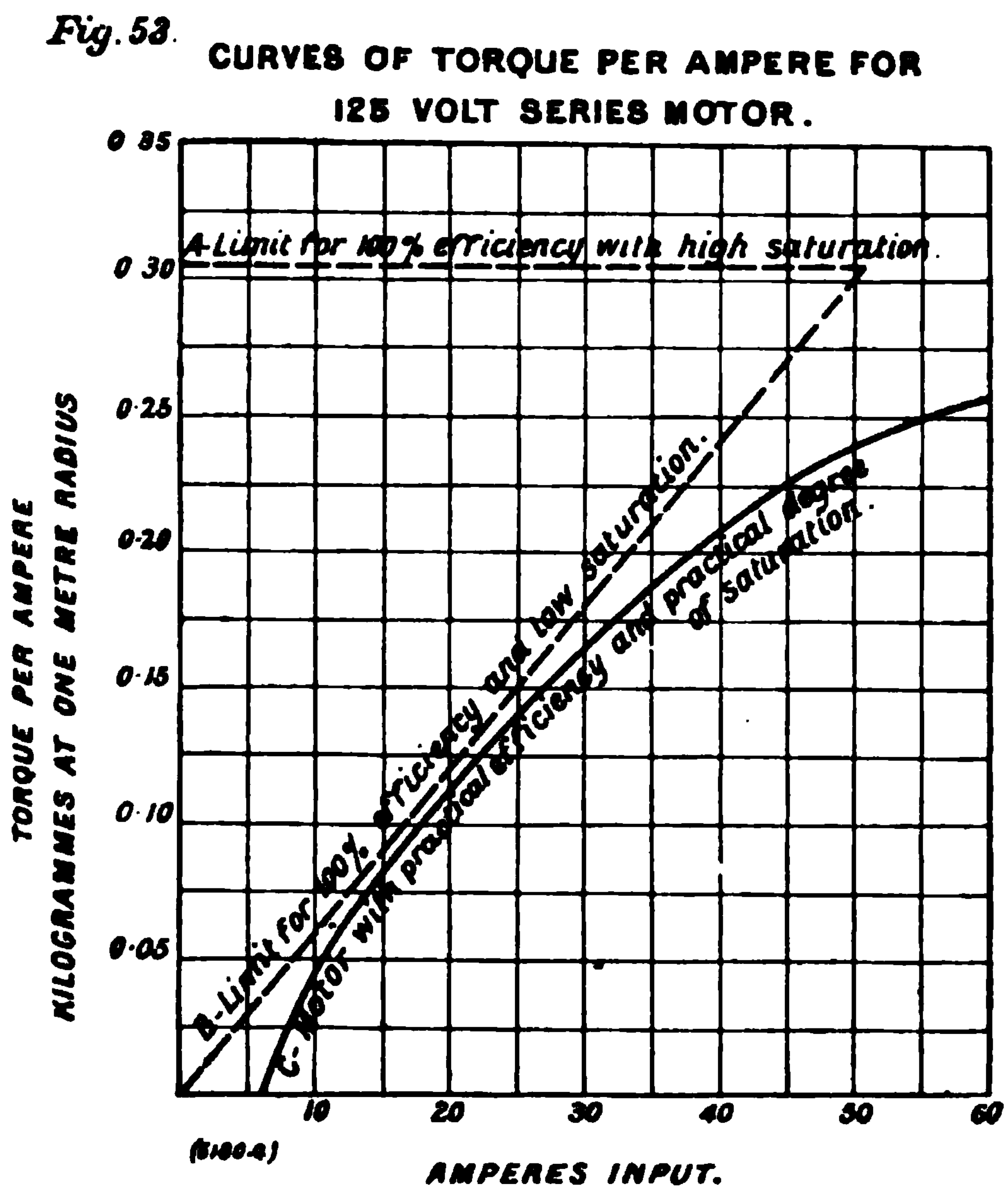


FIG. 53.

—an essentially impracticable direction toward which to work with motors for purposes of locomotion.

The curves A, B, and C of Fig. 53 have been derived from the corresponding curves of Fig. 52 (see page 63), and show the torque in kilogrammes at 1 metre radius per ampere input in terms of the total amperes input. For a motor of a given rated capacity at a given voltage and full load speed, the torque per ampere constitutes in a certain measure a criterion of the quality of the design.

§ 4. **Tramway Motor of 27 H.P.**—For series motors employed for locomotion when we have a knowledge of the diameter of the car wheel and of the ratio of gearing between motor and axle, the torque is conveniently expressed in pounds pull at

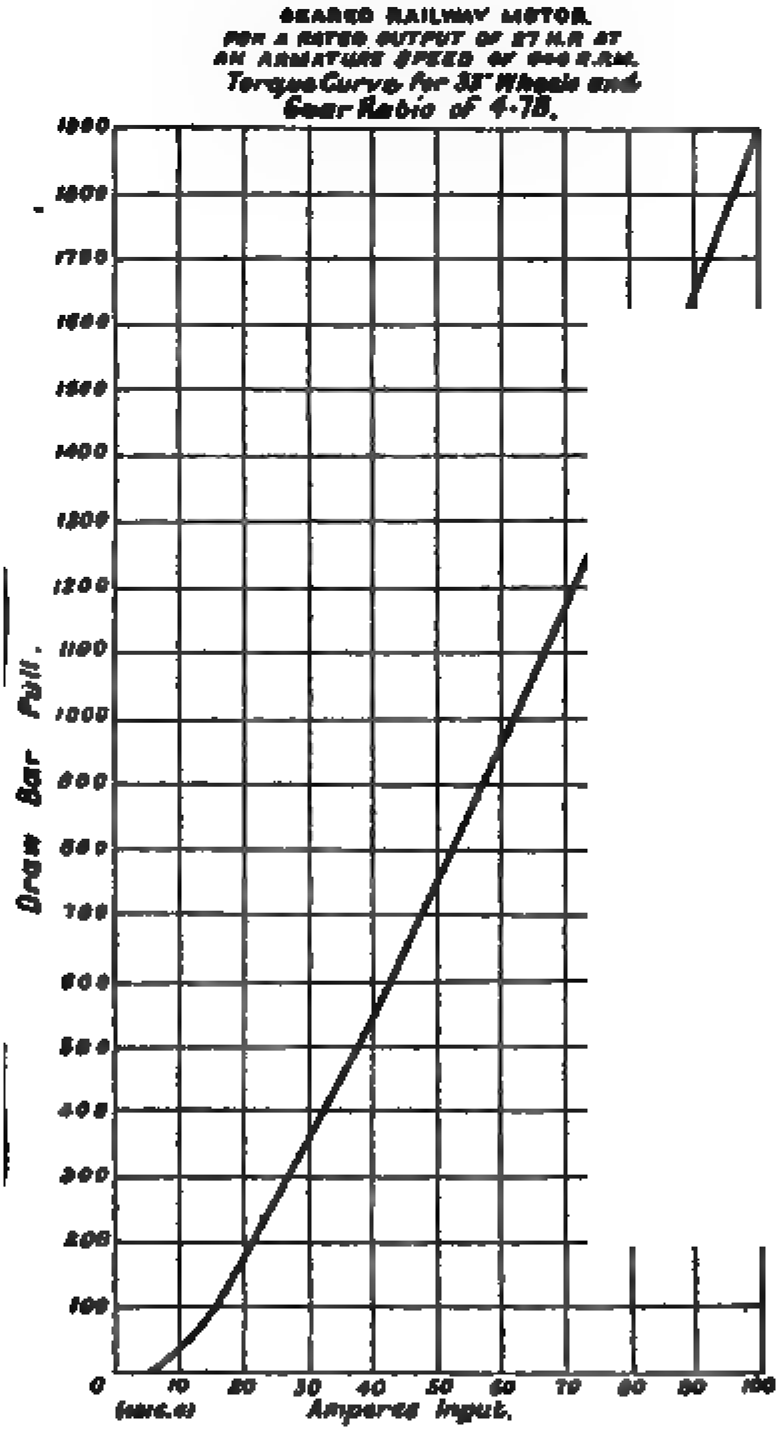


Fig. 54.

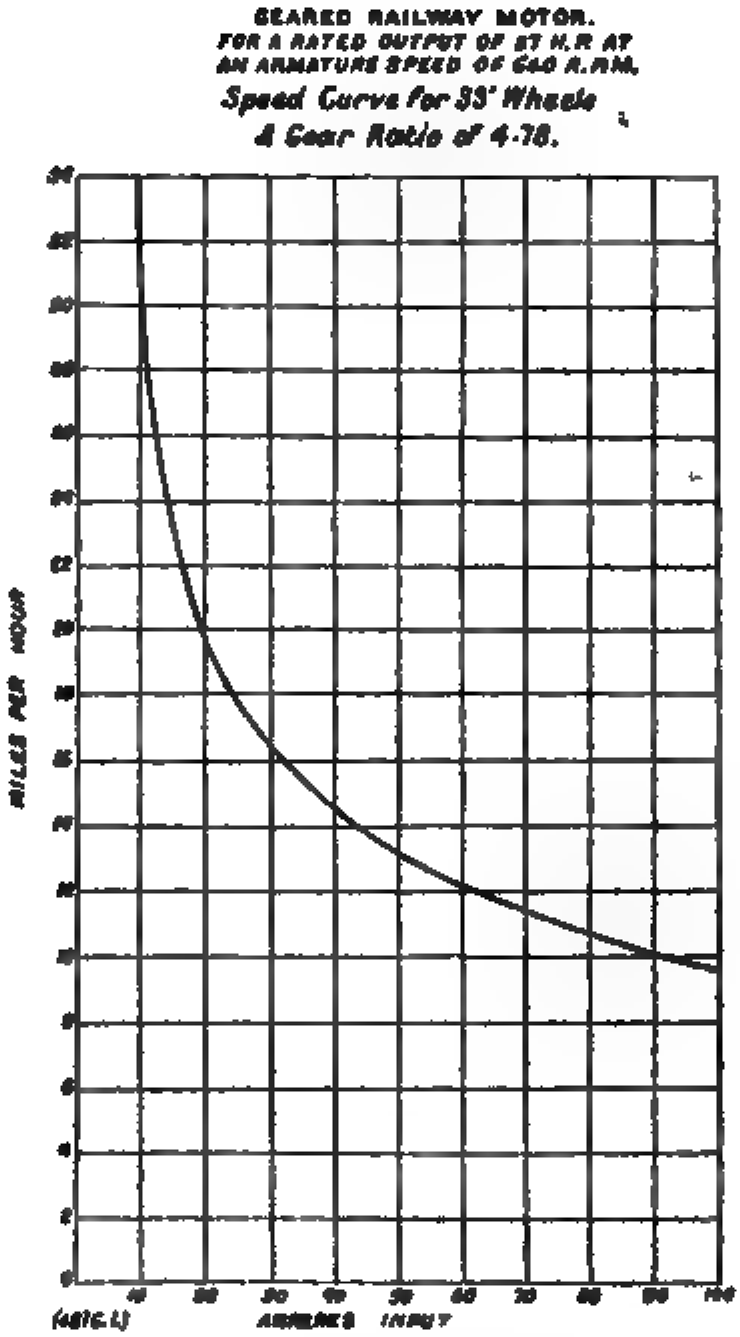


Fig. 55.

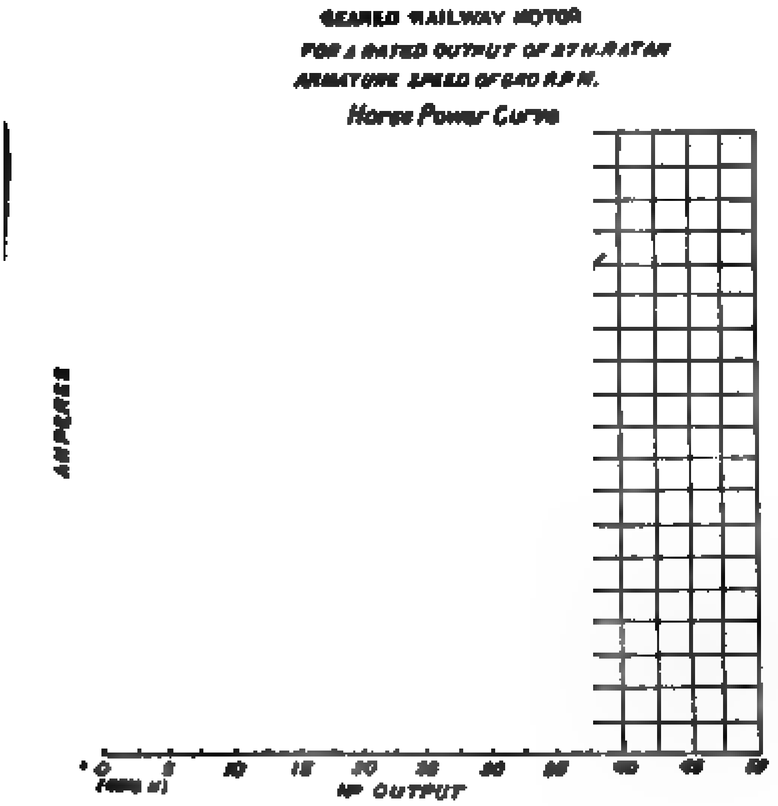


Fig. 56.

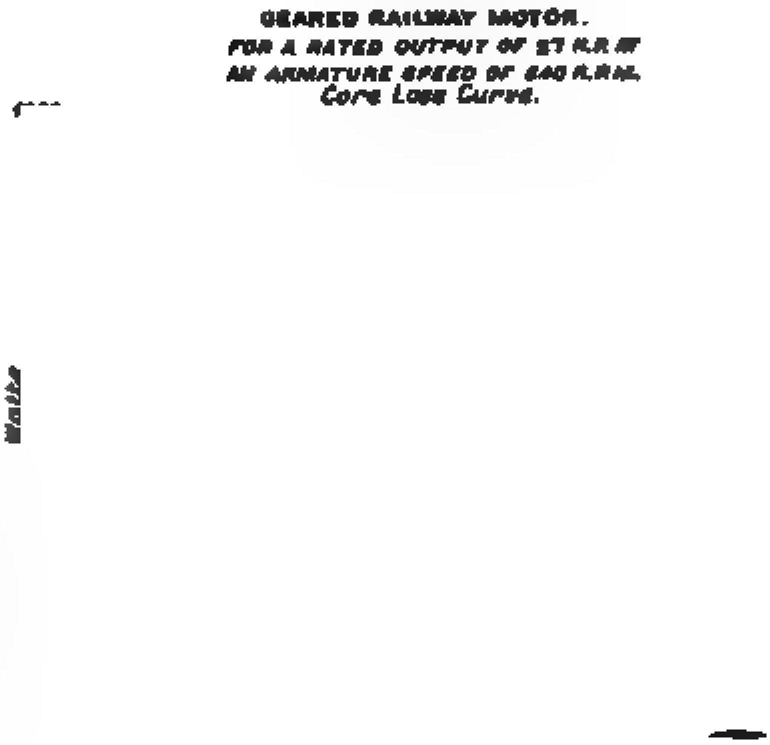


Fig. 57.

the draw-bar, and the corresponding speed of locomotion in miles per hour.

As practical examples of the principles which have now been set forth with relation to the series motor, there are given in Figs. 54 to, 59, pp. 65, 66, curves of the properties of one of the most widely used light tramway motors. Its rated capacity is 27 horse-power, at 500 volts, and an armature speed of 640 revolutions per minute.

§ 5. **Railway Motor 117 H.P.**—In Figs 60 to 63, page 67, another set of curves are given

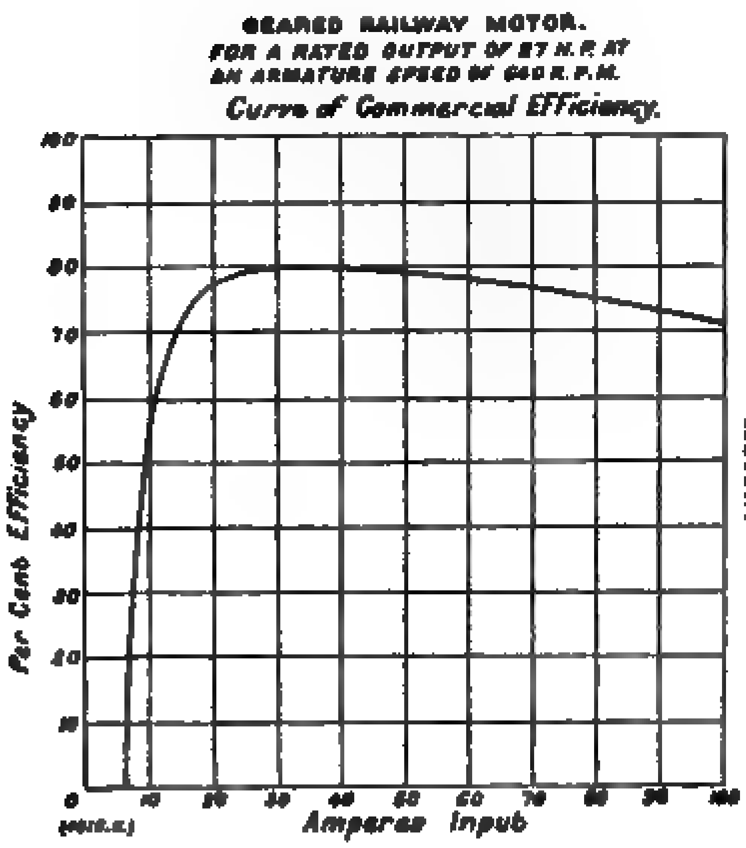


FIG. 58.

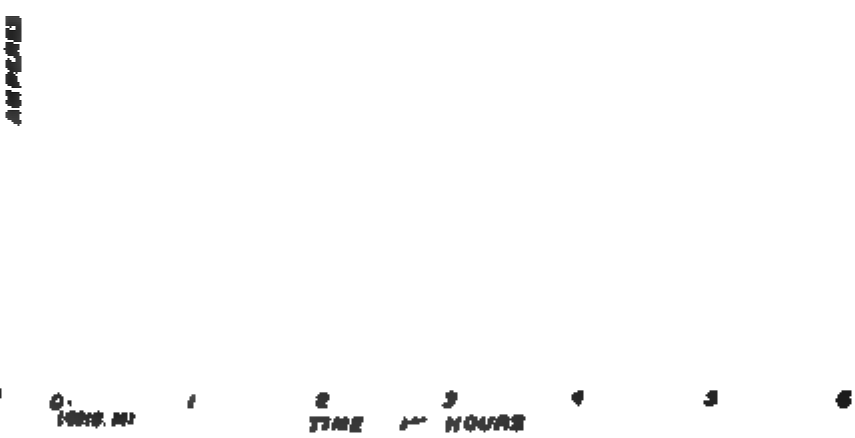


FIG. 59.

direct connected motors, which were used in equipments of four motors apiece on many of the locomotives of the Central London Railway. The high maximum efficiency of this 117 horse-power motor—94 per cent.—as compared with the maximum efficiency of only 80 per cent. in the 27 horse-power motor, is partly due to the large size of the former unit, but in great measure arises from the gearing loss in the small motor.

§ 6. **Tramway Motor of 38 H.P.**—In a recent contribution to the *Proceedings of the American Institute of Electrical Engineers*, Mr W. B. Potter gives a list of schedule speeds for a 38 horse-power motor. It is rated at 38 horse-power on the nominal basis already stated, of one hour's run, and 75 degs. Cent.

DIRECT CONNECTED RAILWAY MOTOR.
Fig 60 — SATURATION CURVE —
When driven on open circuit at 130 r.p.m, field

Fig. 60.

DIRECT CONNECTED RAILWAY MOTOR.

Fig 61.

Fig. 62.

DIRECT CONNECTED RAILWAY MOTOR.
— CURVE OF COMMERCIAL EFFICIENCY. —

Fig. 63.

rise. The table shows clearly to what a wide variety of speeds, weights of car, and nature of service it may be applied by means of different gear ratios. From Mr Potter's table the curves

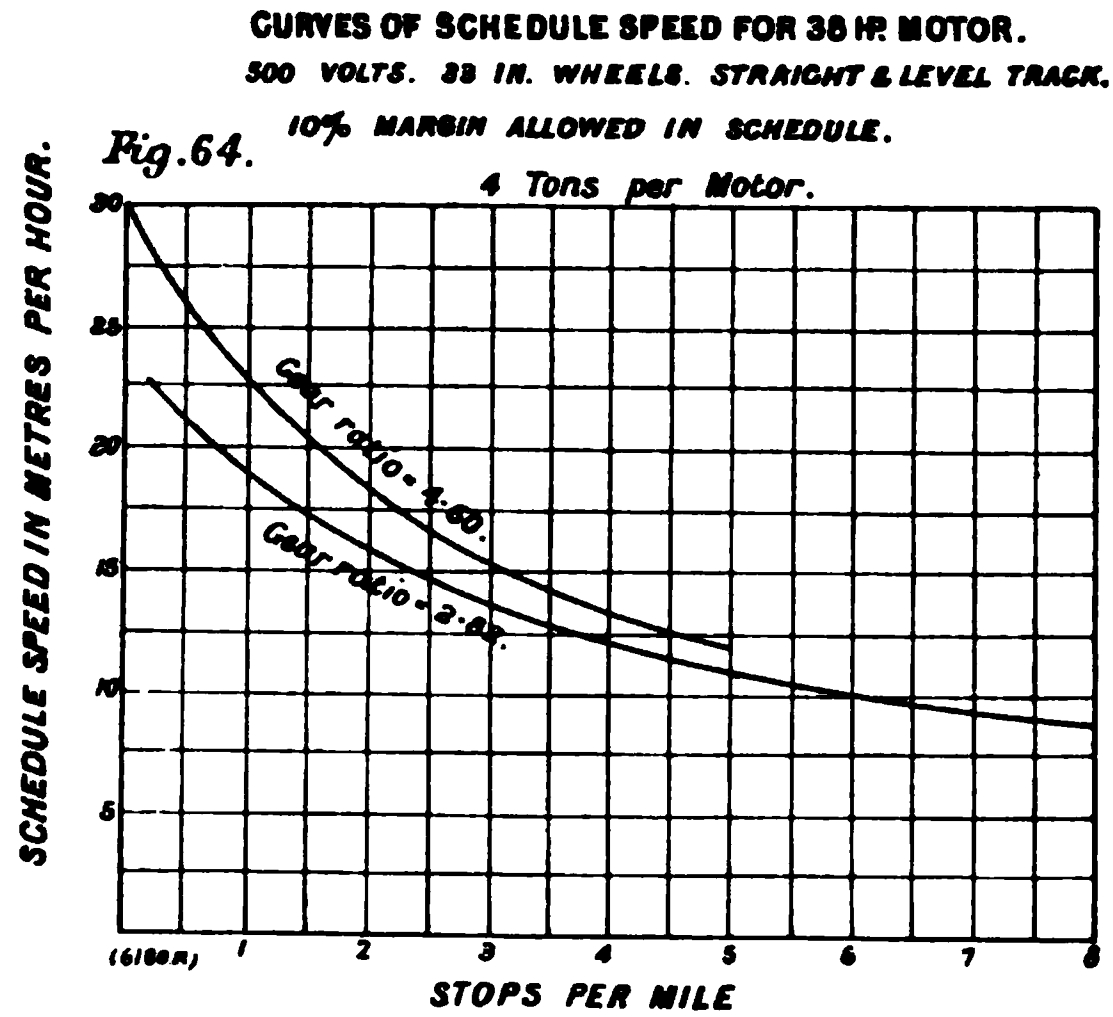


Fig. 64.

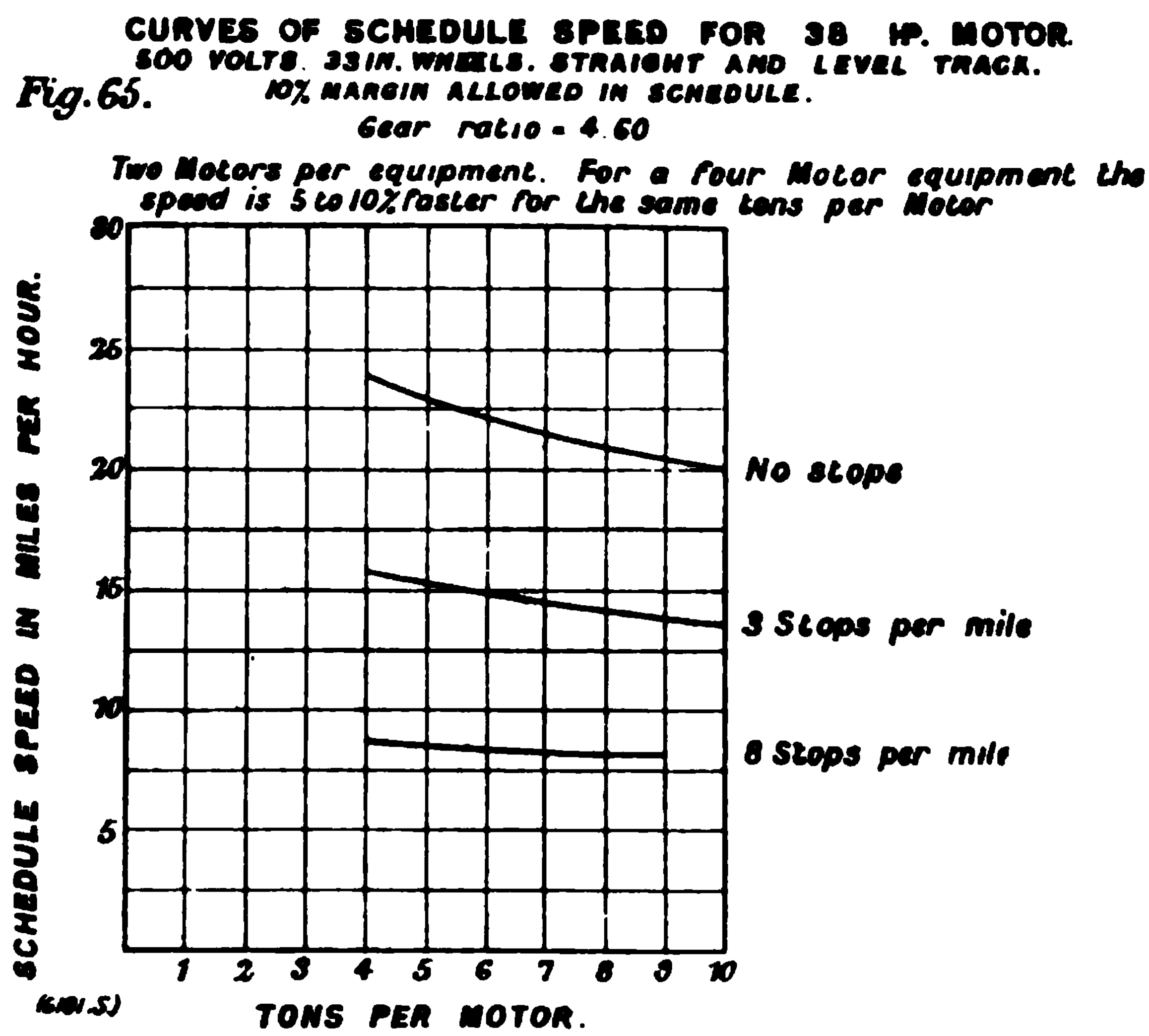


Fig. 65.

of Figs. 64 and 65, page 68, have been derived. The curves show the conditions of service corresponding to a given basis of temperature rise.

§ 7. **Full Load Commercial Efficiency of Shunt, Compound, and Series Motors.**—The full load commercial efficiency of a series motor has but little significance; greater interest attaches to its maximum efficiency, and the percentage of the rated load to which it corresponds. In Fig. 66 is given a curve fairly representative of the maximum efficiency obtained in geared series motors of various rated capacities. The maximum efficiency is generally obtained at from 50 per cent. to 70 per cent. of the rated full load capacity. Series motors for tramway purposes could with advantage be designed to have the maximum efficiency occur at still lower loads. The chief difficulty in the way of obtaining this result is the loss in the transmission gearing,

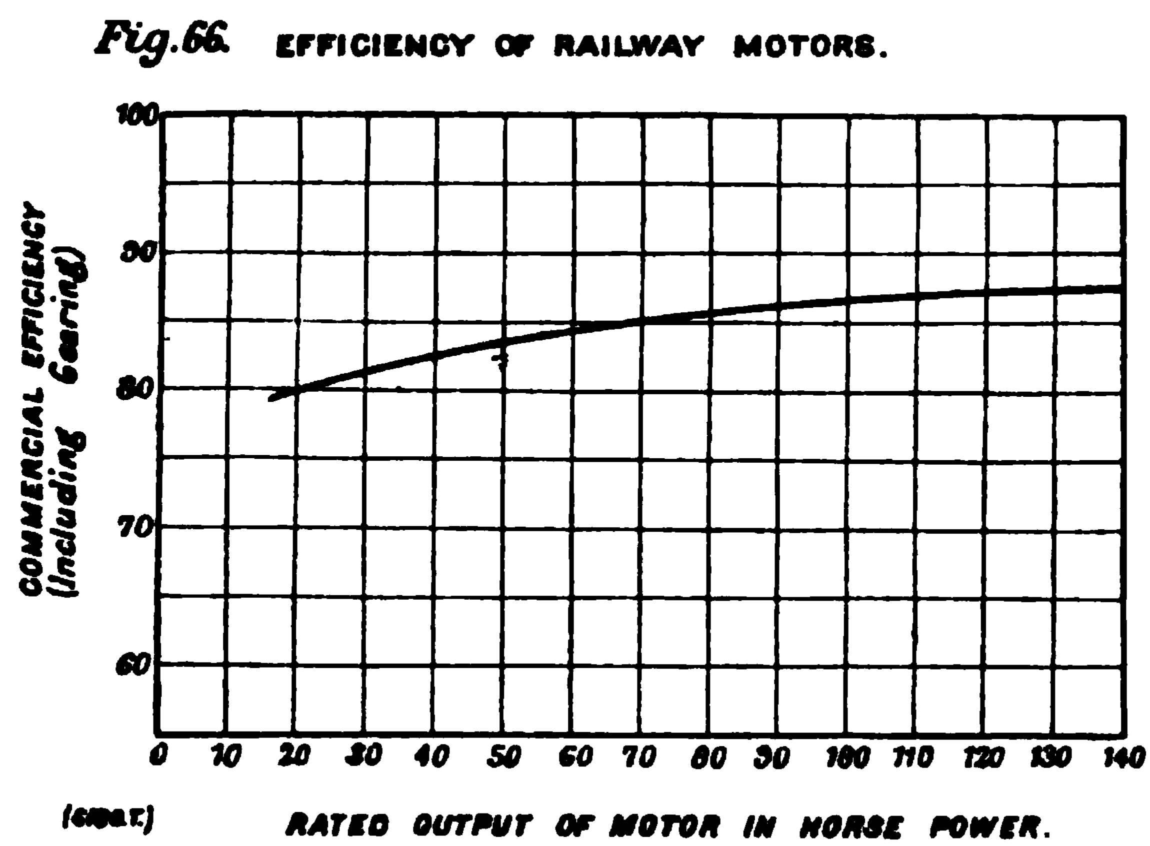


Fig. 66.

which, though not part of the electrical design, nevertheless has to be taken into account in obtaining the commercial efficiency.

In shunt wound and compound wound motors the full load commercial efficiency varies but little in different designs. Although the component losses—and hence the efficiency at light loads—may be very greatly modified at the will of the designer, the total of the losses is a very uniform quantity for a given rated capacity, and this is true for motors of very different rated speeds. Even the normal voltage need not greatly affect the efficiency, though the increased commutator losses generally result in giving slightly lower total efficiency the lower the normal voltage. This tendency is partly offset—especially in small motors—by the increased losses in the shunt winding of motors for higher voltages In Fig. 67 are plotted the efficiencies of shunt and compound

wound motors of the ordinary ranges of speeds and of capacities up to 100 horse-power.

Small fan motors of from 1200 to 1600 revolutions per minute, consuming 50 watts, yield, probably, some 0·02 horse-power (or

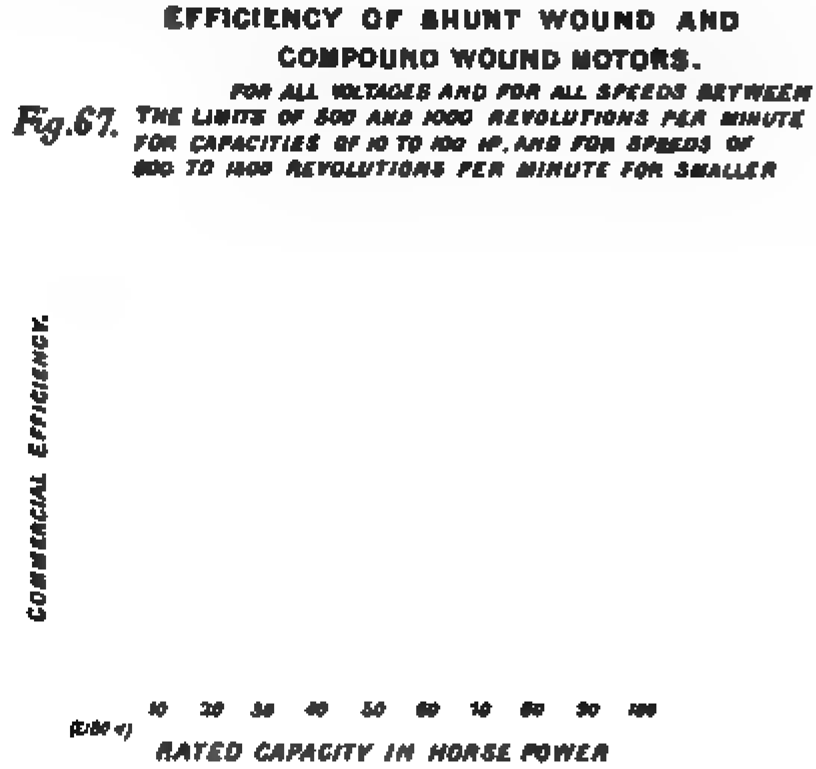


FIG. 67

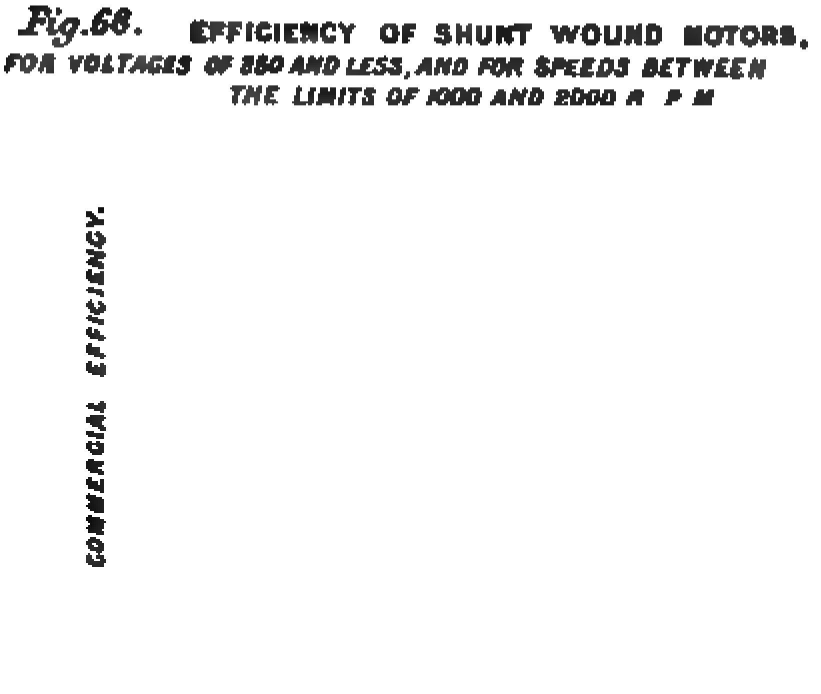


FIG. 68.

15 watts), thus having an internal loss of 35 watts, and about 30 per cent. efficiency; 0·05 horse-power motors, for the same range of speed, consume about 90 watts, their efficiency being about 42 per cent., and the losses somewhat over 50 watts. The efficiencies of small motors are plotted in Fig. 68.

§ 8. **Absolute Values of the Losses.**—Although it is customary to consider the percentage efficiency, it is in some ways more useful to consider and compare the percentage of losses, and the absolute values of the losses, in watts. Thus in two machines

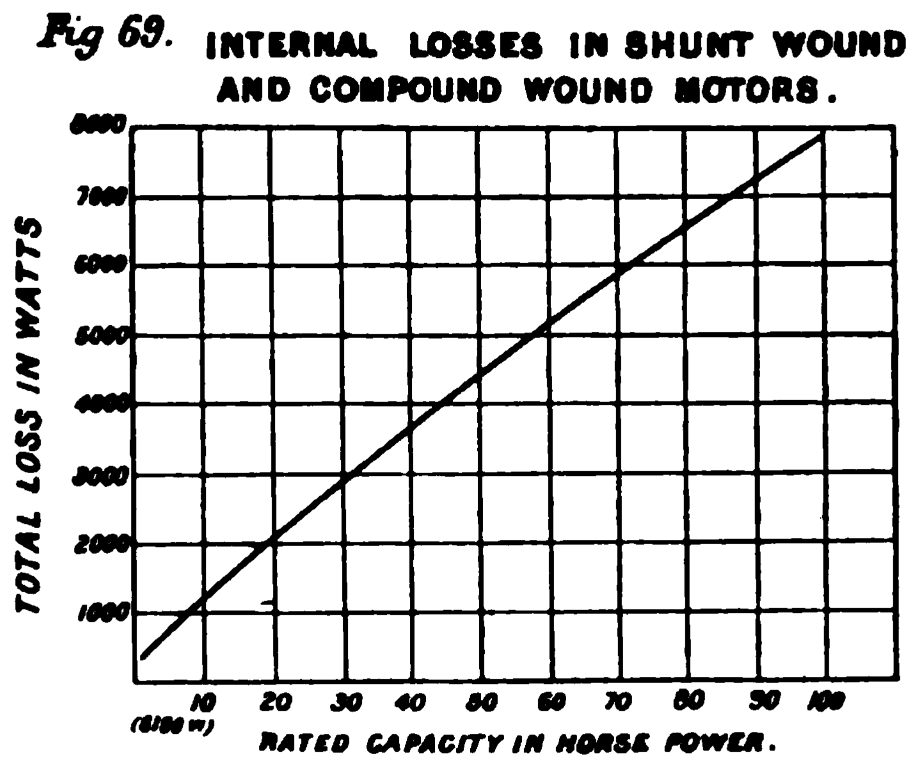


Fig. 69.

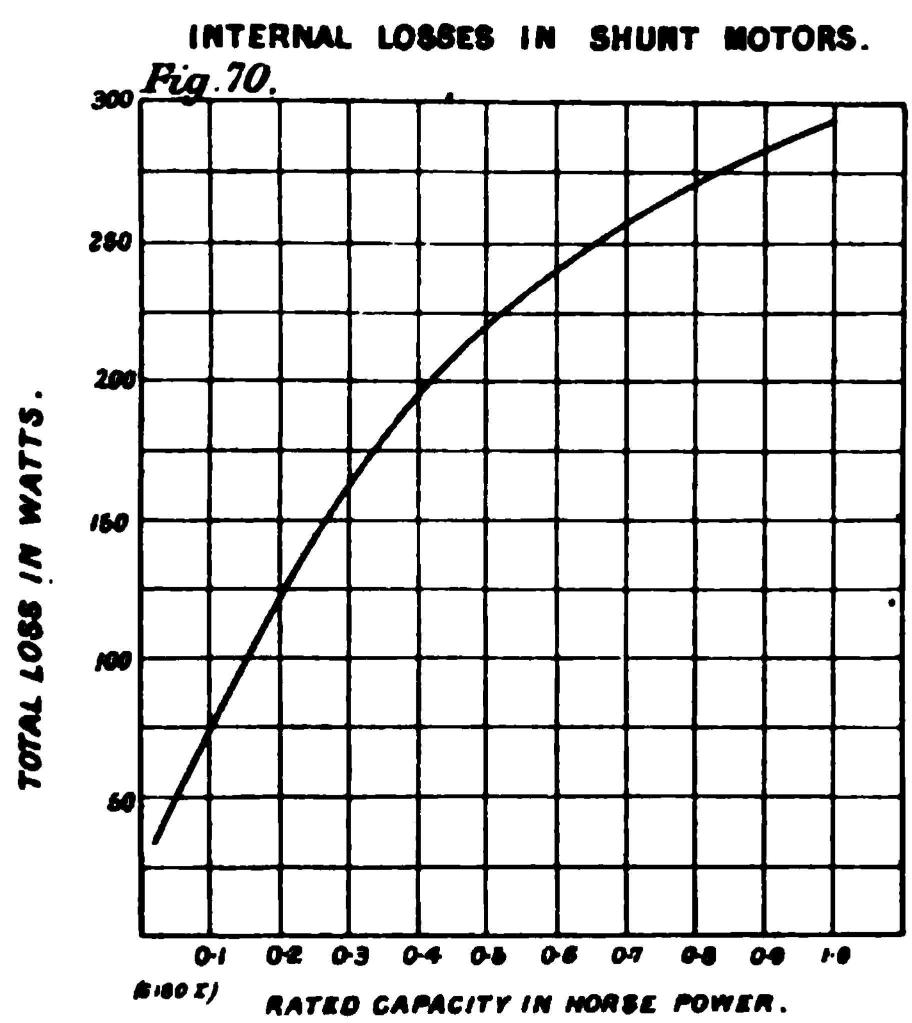


Fig. 70.

having efficiencies of 89 per cent. and 90 per cent. respectively, differing therefore but 1 per cent. in efficiency, the losses are in the first case 10 per cent. greater than in the second.

In Figs. 69 and 70 are plotted the losses in watts, corresponding to the two curves of commercial efficiencies in Figs. 67 and 68.

CHAPTER VI

MOTOR DESIGNING AND TESTING

§ 1. **"Space Factor" of Armature Slots.**—The ratio of the total cross section of copper in an armature slot or field spool to the cross section of the total available space is termed the "space factor." It is a very variable quantity, and depends not only upon the normal voltage and output of the motor, and upon the shape of the slot or field spool, but—and very largely—upon the care and skill employed in the winding and insulating, and upon the insulation tests to which the motor is subjected. Many manufacturers prefer to subject the apparatus to very severe insulation tests, with a view to reducing to a negligible percentage the number of subsequent breakdowns. Others reason that such large factors of safety are unnecessary, and that for a given expenditure a motor of inferior qualities in other respects must be the result of such lavish insulation. The writer is of the opinion that the very highest factors of safety which it has yet been customary to employ in any country in the design of the insulation of electrical machinery are none too liberal. The subjecting of electrical machinery to severe insulation tests does not as a rule lead to any undue sacrifice of space to insulating purposes, but rather to a careful study of the properties of insulating materials and their correct treatment. The skilful use of high grade and durable, though relatively thin, insulation, will go further towards producing a motor capable of undergoing severe insulation tests than lavish space allowances less skilfully utilised. The properties of magnetic and conducting materials entering into the construction of the motor have been carefully investigated with a view to their economical use, but although a very great deal of experimental work has been carried out on insulating materials, the results are not yet accessible in such form as to permit of their ready and correct application in motor construction, and

the choice and treatment of the insulating materials are hardly given the attention they deserve, even by the very largest manufacturers.

A consideration of the relative proportions of the winding space, devoted respectively to copper and insulation, emphasises the importance of a study of the question of a better use of insulating materials.

In Fig. 71 are given two curves limiting a shaded area within which the "space factors" of the armature slots of small motors

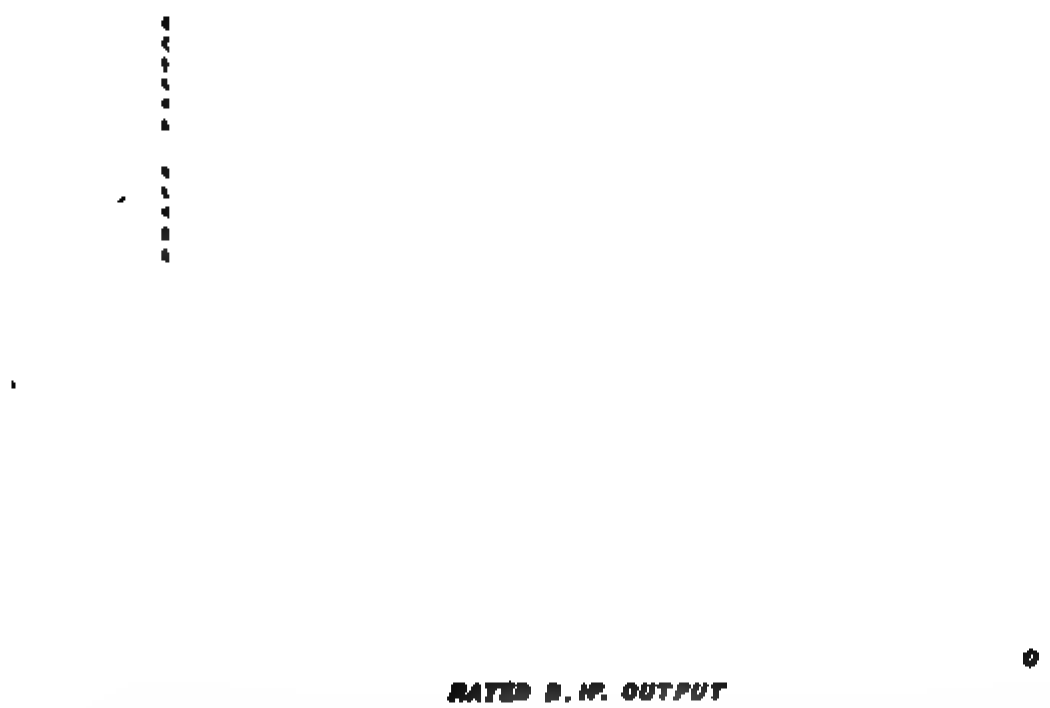


Fig. 71.—Diagram showing Space Factors of Armature Slots.

lie. The range of voltage is from 100 to 600 volts. The values approach the lower limit, or even fall below it, the higher the voltage, the greater the number of slots, and the less the skill and knowledge employed in the processes of winding and insulating, and in the selection and testing of materials. Under fair conditions the lower limiting curve for 600 volts, and the upper for 100 volts, and for motors of the ranges of speeds corresponding to the data of Figs. 67 to 70, should be consistent with subjecting the motors to the following insulation tests:—

Rated Voltage.	Guaranteed Insulation Tests from Copper to Iron at 60° Cent. for One Minute.
100	2000 R.M.S. volts.
600	3600 R.M.S. volts.

In Fig. 72 the shaded portions of the six rectangles show the proportions of the total slot area occupied by copper and insulation respectively. These results are extremely significant. The art of insulating, in motor manufacturing, is still extremely crude, and there is every reason to look for great improvements. In cable manufacturing[1] it is well understood that the cost of the copper is, for all but the lowest voltage, but a small part of the cost of the

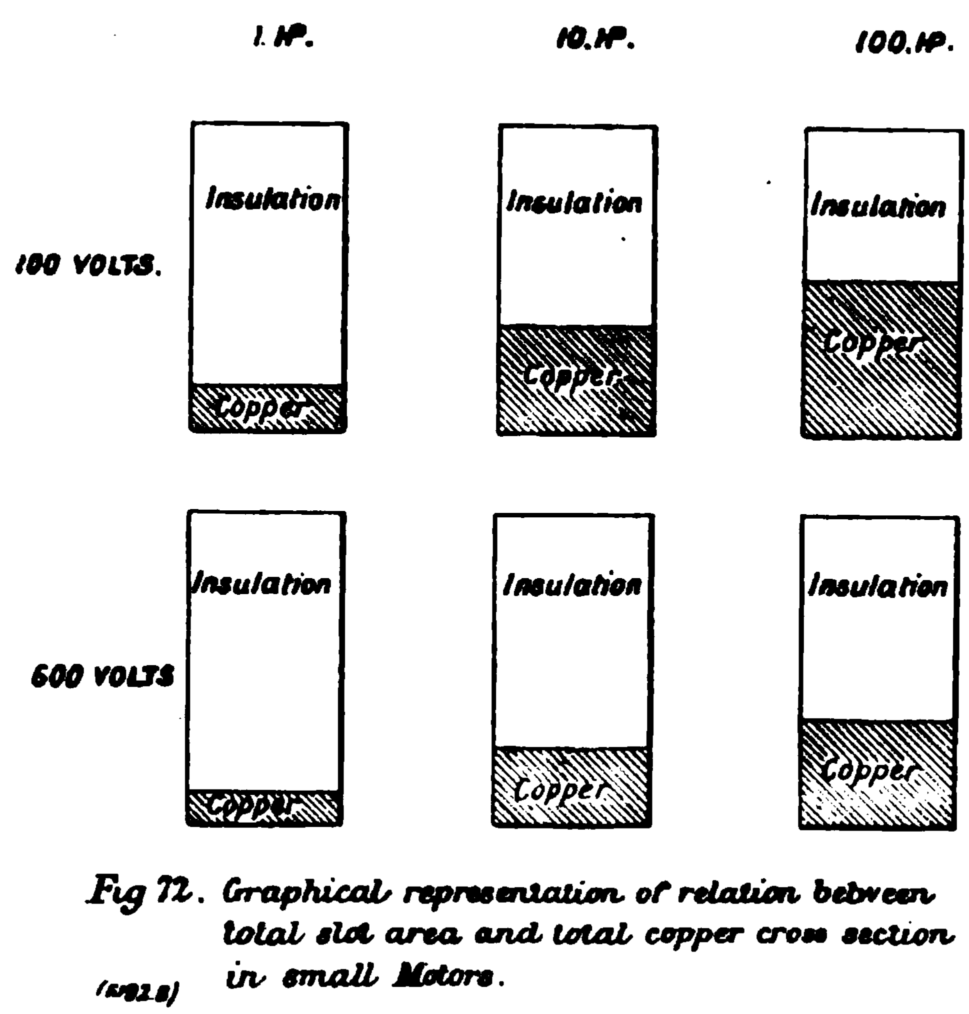


FIG. 72.—Diagram showing Proportions of Slot Area and Copper and Insulation.

complete cable, and the design and construction of the insulation is reduced to a science, whereas in motor building the insulation is regarded as one of the less important details. But the diagrams of Fig. 72 show that the possible gain, either in decreased dimensions of motors or in increased output (which amounts to nearly the same thing), is not a matter of a few per cent. merely.

The lower the voltage and the larger the capacity, the greater is the percentage of the total insulation devoted to insulating the

[1] Much of the information on insulating materials contained in the paper, by O'Gorman, on "Insulation of Cables," *Proceedings of the Institute of Electrical Engineers*, 1901, vol. xxx., page 608, could be employed to great advantage in the design of dynamo electric machinery.

conductors from the armature core. In small motors for the higher voltages, on the other hand, the greater part of the insulation space consists in the coverings of the individual wires. Hence, in spite of the increased cost of insulation, it often, in very small motors, becomes economical to employ silk-covered wire. A great deal of the space lost in these smaller sizes for higher voltages is due to the multiplicity of wires required. Hence two-circuit windings should be employed wherever these small sizes are built multipolar. The mistake should, furthermore, never be made of using two wires in parallel in small motors for high voltages for the purpose of using the same slot as for the lower voltages of the same size. Such mistakes are, nevertheless, constantly being made, and this is the result of a widespread failure to appreciate the proportions actually existing between amounts of insulation and copper, even when employed to the best advantage possible, in the light of the experience at present available. If the same slots must be used for all voltages, then the size should be chosen with due regard to the high voltage requirements. The choice of width and depth, and number of slots also, of course, have a great influence upon the "space factor."

§ 2. **Space Factor of Field Spools.**—In field spools, higher "space factors" may generally be used, but this varies even more with the nature of the design. Thus, for a given rating a motor with many poles requires relatively few turns of relatively large cross section, as compared with a machine for the same output at the same voltages and speeds with few poles. The former has, consequently, a high, and the latter a low, "space factor." Moreover, the field-spool "space factor" is of much less importance than the armature slot "space factor," because space on the field poles is less restricted. The following example (Table III.) of the application of a method for calculating a spool winding proceeds from the standpoint of an assumed "space factor," and has the advantage of bringing out clearly the bearing of this factor on the result, and the consequences in relation to weight of copper employed and watts required in excitation, of changes in the "space factor," or in the overall spool dimensions:—

TABLE III.—TABULATED CALCULATIONS FOR SPOOL WINDINGS FOR 25 BRAKE HORSE-POWER MOTORS FOR 100, 200, AND 400 VOLTS.

	Voltage.		
	100	200	400
Internal diameter of spool (centimetres)	13	13	13
Depth of winding (centimetres)	5	5	5
External diameter of spool (centimetres)	23	23	23
Mean diameter of spool (centimetres)	18	18	18
Mean length, one turn (metres) (a)	·565	·565	·565
Ampere-turns per spool (b)	4200	4000	3800
a × b	2380	2260	2150
$\cdot000176 \times a^2b^2$	1000	900	810
"Space Factor" (s)	·45	·40	·35
Axial length of spool (centimetres)	13	13	13
Cross section spool winding (t) (square centimetres)	65	65	65
Total cross section copper in spool ($m = s \times t$) (square centimetres)	29·3	26·0	22·8
Cubic centimetres copper in spool (100 a s t) ...	1660	1470	1290
Weight copper in spool (kilogrammes) (1 cubic centimetre of copper weighs ·0089 kilogramme) ...	14·8	13·1	11·5
Watts per spool at 60° Cent. (w) $\text{Watts} = \frac{\cdot000176 \times a^2b^2}{\text{kilogrammes}}$	67·5	69·0	70·5
Square decimetres of external cylindrical surface per spool	9·4	9·4	9·4
Watts per square decimetre of external cylindrical surface	7·20	7·35	7·50
Number of poles	4	4	4
Volts per spool (v)	25	50	100
Amperes per spool ($i = {}^{w}/_{v}$)	2·70	1·38	0·705
Turns per spool $\left(= \frac{b}{i}\right)$	1560	2900	5400
Cross section copper per turn $\left(\frac{m\ i}{b}\right)$ (square centimetres)	·0188	·0090	·0042
Current density in amperes per square centimetre ...	144	153	168
Watts in all spools	270	276	282
Kilogrammes copper in all spools	59·2	52·4	46·0
Diameter bare spool copper (millimetres)	1·54	1·07	0·73

§ 3. **Spool Windings for Different Voltages.**—This 25 brake horse-power motor is designed for the three voltages, 100, 200, and 400, on the plan of employing somewhat lower magnetic densities the higher the rated voltage. This is done in order that, with the lower field spool "space factor" of the higher voltage designs, spools of the same external dimensions shall not require appreciably more energy for excitation, and hence shall not have a greater temperature rise. Since the magnet frame is the same for all three voltages, this requires either the adoption of higher speeds for the motors with the higher-rated voltages, or else the employ-

ment of armatures of greater strength as expressed in armature ampere turns per pole piece. The latter plan is disadvantageous, since one encounters in the armature design the difficulty that the "space factor" of the armature slot is also much lower the higher the voltage. In view of these facts it is more consistent to widen the magnetic system, and in some cases to decrease the number of field poles for the higher voltage motors, and thus use less armature ampere turns. This leads to extremely practical results, since the width required for the commutator is, for a given output, almost in inverse proportion to the rated voltage. In machines designed on this plan, not only may due regard be paid to the variations in the "space factors" of both armature slots and field spools, but the commutators may be given proportions suited to the duty required of them. The overall dimensions are precisely the same for all voltages; in fact, all essentially mechanical parts —base, stands, bearings, and shaft—as well as distance between bearings—are entirely independent of the voltage, so that the same drawings, patterns, and castings are used for all voltages, the patterns being constructed to be adjustable in width to the extent necessary for the different voltages. The diameters of magnetic yoke, whose dimensions must be chosen with reference to mechanical as well as magnetic considerations, and of armature and commutator, are the same for all voltages, and the magnetic yoke could, in many cases, be made the same section for all voltages, different magnet cores in number and section being used, according to the voltage of the motor.

§ 4. **Designs for a Group of Motors.**—In Fig. 73 are given outline drawings for a group of machines designed by the writer on this plan. The machines were originally designed for generators. Their ratings for normal output, speed, and voltage as motors, and their dimensions, are set forth in Table IV.[1] (page 78).

In the field spools of machines with many poles the volts per spool become relatively low, and also the volts per turn, thus permitting of but a thin layer of insulation on the conductor. This materially increases the "space factor," but requires that additional care be taken in winding. In the

[1] The designs shown in Fig. 73, and in Table IV., are adapted from a paper on "Modern Commutating Dynamo Machinery," published in the *Journal of Proceedings of the Institute of Electrical Engineers*, 1901, Vol. XXXI., page 170, where this plan of designing is discussed in much greater detail.

TABLE IV.—PARTICULARS OF 110, 220, AND 525-VOLT MOTORS (see Fig. 73).
(DIMENSIONS IN MILLIMETRES.)

Number of Poles.	Brake Horse-power Output.	Revolutions per Minute.	Voltage.	A	B	C	D	E	F	G	H	I	J	K	L	Length of Laminations between Flanges.	Number of Ventilating Ducts.	Width of each Ventilating Duct.	Effective Length of Armature Laminations.	Diameter of Magnet Core	Radial Length of Magnet Core.	Number of Armature Slots.	Width of Slot.	Depth of Slot.	Number of Commutator Segments.	Thickness of Segments at Surface (excluding Insulation).	Thickness of Segment from Commutator Connection to outer end of Commutator.	Armature Conductors per Slot.	Height of Armature Conductor.	Width of Armature Conductor.
6	100	500	110	1366	1650	1300	740	610	1325	340	660	620	330	215	1350	125	3	10	80	155	190	60	16·2	28·0	240	7·2	290	8	11·1	2·7
6	100	500	220	1366	1650	1300	740	610	1325	390	610	620	430	250	1350	170	5	10	108	170	190	96	10·4	29·5	384	4·2	190	8	12	1·2
6	100	500	525	1366	1650	1300	740	610	1325	420	580	620	500	400	1350	282	6	10	200	225	190	102	11·5	19·0	510	3	120	10	6·3	1
8	120	460	110	1585	1900	1500	900	710	1450	370	730	710	395	190	1470	125	3	10	80	130	210	102	14·5	30·0	408	4·7	315	8	12·3	2·3
6	120	460	220	1585	1900	1500	900	710	1450	420	680	710	470	260	1470	165	5	10	104	175	210	108	11·8	30·0	432	4·4	230	8	12	1·6
6	120	460	525	1585	1900	1500	900	710	1450	460	640	710	550	420	1470	248	6	10	169	232	210	114	11·4	22·5	570	3·2	160	10	8	1
8	150	420	110	1813	2210	1740	1070	790	1650	395	875	720	370	200	1660	125	3	10	80	152	235	112	14·0	30·0	336	6·7	350	6	11·7	3·1
6	150	420	220	1813	2210	1740	1070	790	1650	445	825	720	460	300	1660	160	5	10	99	195	235	130	11·0	30·0	390	5·6	250	6	11·7	2·1
6	150	420	525	1813	2210	1740	1070	790	1650	485	785	720	550	435	1660	220	6	10	144	235	235	156	10·6	25·0	624	3·1	170	8	8·6	1·2
8	180	380	110	2114	2470	2030	1200	840	1735	390	940	770	350	275	1740	125	4	10	77	154	250	120	14·0	32·5	360	6·9	400	6	12·2	3·7
8	180	380	220	2114	2470	2030	1200	840	1735	450	880	770	475	300	1740	160	5	10	99	165	250	144	12·0	33·0	576	3·7	275	8	12·5	1·8
6	180	380	525	2114	2470	2030	1200	840	1735	500	830	770	580	330	1740	220	6	10	126	235	250	168	11·4	26·0	672	3·2	170	8	9	1·4

Fig 73.

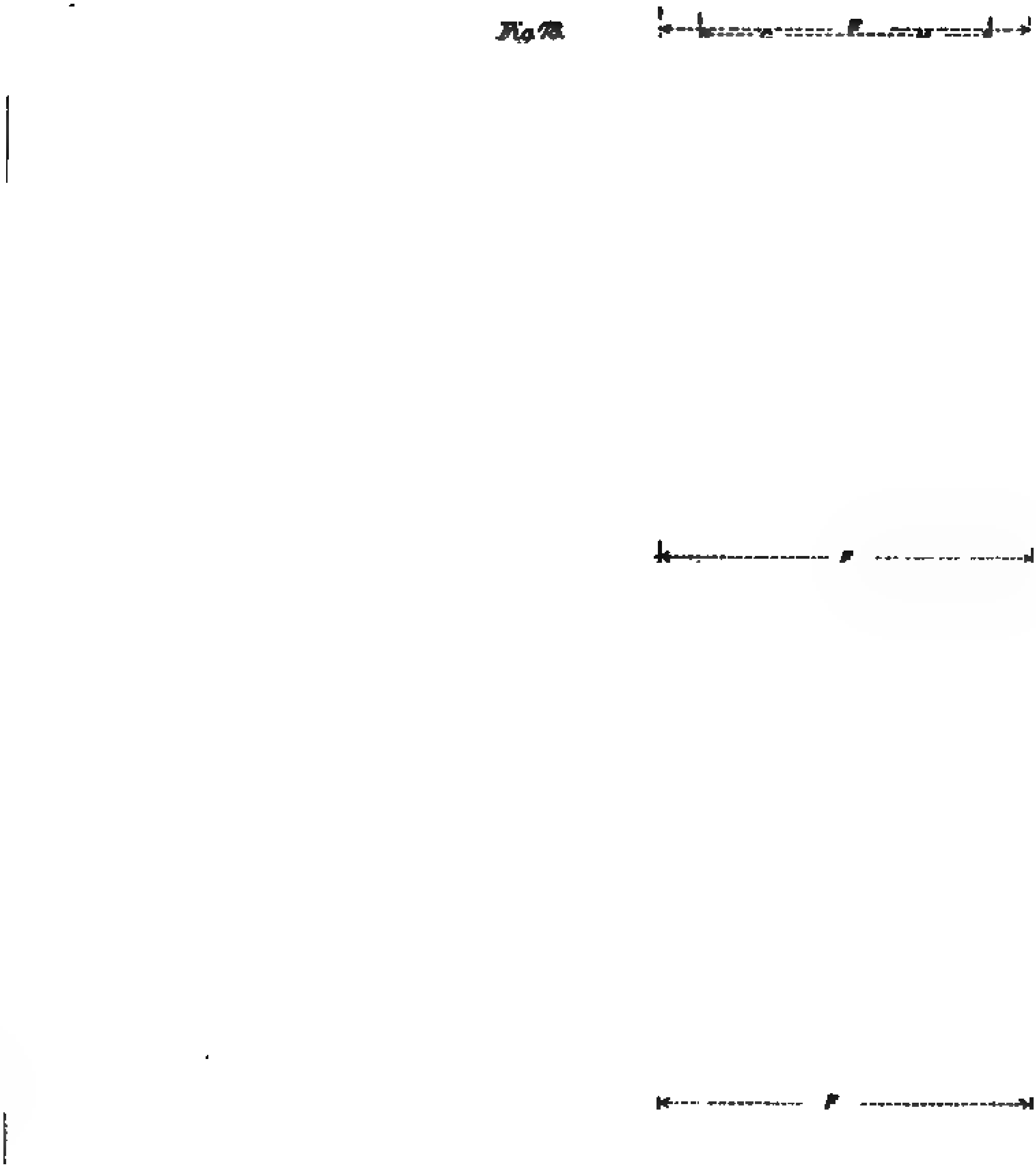


Fig. 73.—Diagrams of Motors for 110, 220, and 525 Volts.

interests of a high "space factor," field spools should all be connected in series.[1]

Series spool windings and low-voltage shunt windings may be constructed from thin strip, wound on the thin edge, and in many cases such construction leads to a much higher "space factor" than can otherwise be obtained. Thus "space factors" of 0·75 are sometimes obtainable in the series spools of moderate sized machines[2] by this construction. There is the additional advantage that the heat is readily conducted, by the continuity of the solid copper, to the exterior surface of the spool, where the circulating air currents quickly dissipate it. Thus, in this type of spool winding, higher values may, for a given temperature increase, be permitted for the "watts per square decimetre of external spool surface." It is of some further advantage to leave the edges of such edge-wound conductors bare, merely insulating between adjacent conductors. When the surface is then polished, the general appearance of the machine is greatly improved.

§ 5. **Hopkinson Method of Testing Motors.**—Motors are generally tested in pairs, one running as a motor from the supply of power, and driving the second as a generator, the current from the latter being returned to the source of supply. By this means a considerable saving is effected, since only sufficient power is required to supply the losses in the two machines. This method of testing was first devised by Hopkinson, and is generally designated as the Hopkinson method, although numerous modifications have subsequently been devised by Kapp, Swinburne, and others.

It is diagrammatically represented in Fig. 74, in which A and B are the armatures of the two machines to be tested, F and G representing their field magnet systems. The two machines are mounted upon the same shaft, or otherwise directly coupled together. S is the main switch leading to the source of supply, and T is a double-pole, double-throw switch leading to the field circuits, and V is merely a single-pole switch, inserted for the purpose of enabling the field circuits to be excited independently of the armature circuits. As is shown, T is so connected to the

[1] In machines with many poles, but for low voltage, the thickness of the insulation on the conductor may exert but a negligible influence on the "space factor," and a more convenient size of conductor may occasionally, in such cases, be obtained by a series parallel connection of the field spools.

[2] This is too high a figure to be attained in motors of less than 100 horsepower rated capacity, unless of very low speed or voltage.

field circuits of the machines A and B, that when it is thrown down, a suitable resistance, R, is inserted in the field circuit of machine A, and when thrown up, the resistance is in B's field circuit. When switches S, T, and V are closed, the machine having the resistance, R, in its field circuit, runs as a motor, its counter electro-motive force being the least, and the other, thus driven mechanically, acts as a generator in virtue of its higher excitation and electro-motive force, and returns to the line as electrical energy, a portion of the electrical energy absorbed by

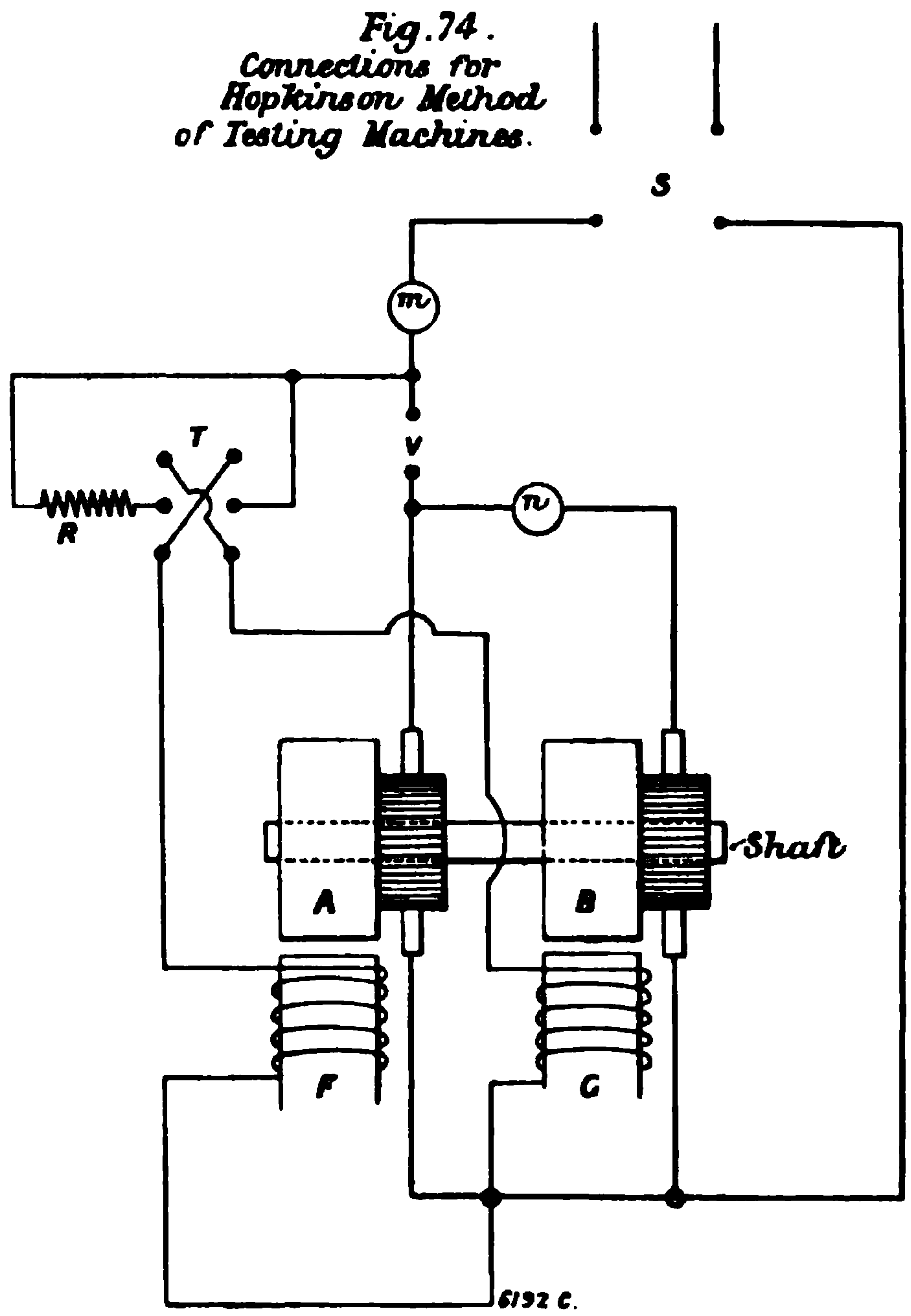


FIG. 74.—Hopkinson Method of Testing Motors.

the motor, this energy having thus undergone a double transformation, first by the motor into the mechanical energy transmitted by the shaft to the generator, and by the latter into electrical energy delivered from its commutator.

It is customary during a heat run to reverse T at, say, half-hour intervals, so that both machines are equally subjected to motor and generator loads, for the motor armature winding is more heavily, and its field winding more lightly, loaded than the generator's.

Ammeter m measures the current supplied to the system to cover the losses, *i.e.*, a current equal to the motor current minus the generator current. Ammeter n measures the motor current when switch T is in the "up" position, in which case the generator current equals $n-m$; and in the "down" position of switch T it measures the generator current, and the motor current is then $n+m$. The generator current is sometimes designated the "circulating current."

§ 6. **Test and Data of Williamson Motors.**—Mr A. D. Williamson has kindly furnished the writer with the results of such a Hopkinson test on two 5 brake horse-power motors of his own design, and built by him for Messrs Vickers Sons & Maxim. These results, together with the observed temperature increase, are given in Tables V. and VI.

TABLE V.—TEST OF TWO 5 BRAKE HORSE-POWER, 600 REVOLUTIONS PER MINUTE, 220-VOLT, SHUNT-WOUND, STANDARD, VENTILATED, ENCLOSED VICKERS MOTORS. (HOPKINSON CONNECTIONS.)

Revolutions per Minute.	Volts.	Auxiliary Current Amperes.	Circulating Current Amperes.	Brake Horse-power.	Square of Efficiency.	Efficiency.
683	220	6·5	19·5	5·75	·751	·866
690	203	5·7	19·0	5·17	·770	·877
685	204	4·3	15·0	4·10	·777	·881
681	204	4·2	14·0	3·82	·770	·877
665	203	3·4	10·0	2·72	·747	·864

TABLE VI.—TEMPERATURE RISE AFTER FIVE AND A HALF HOURS' RUN ON FULL LOAD.

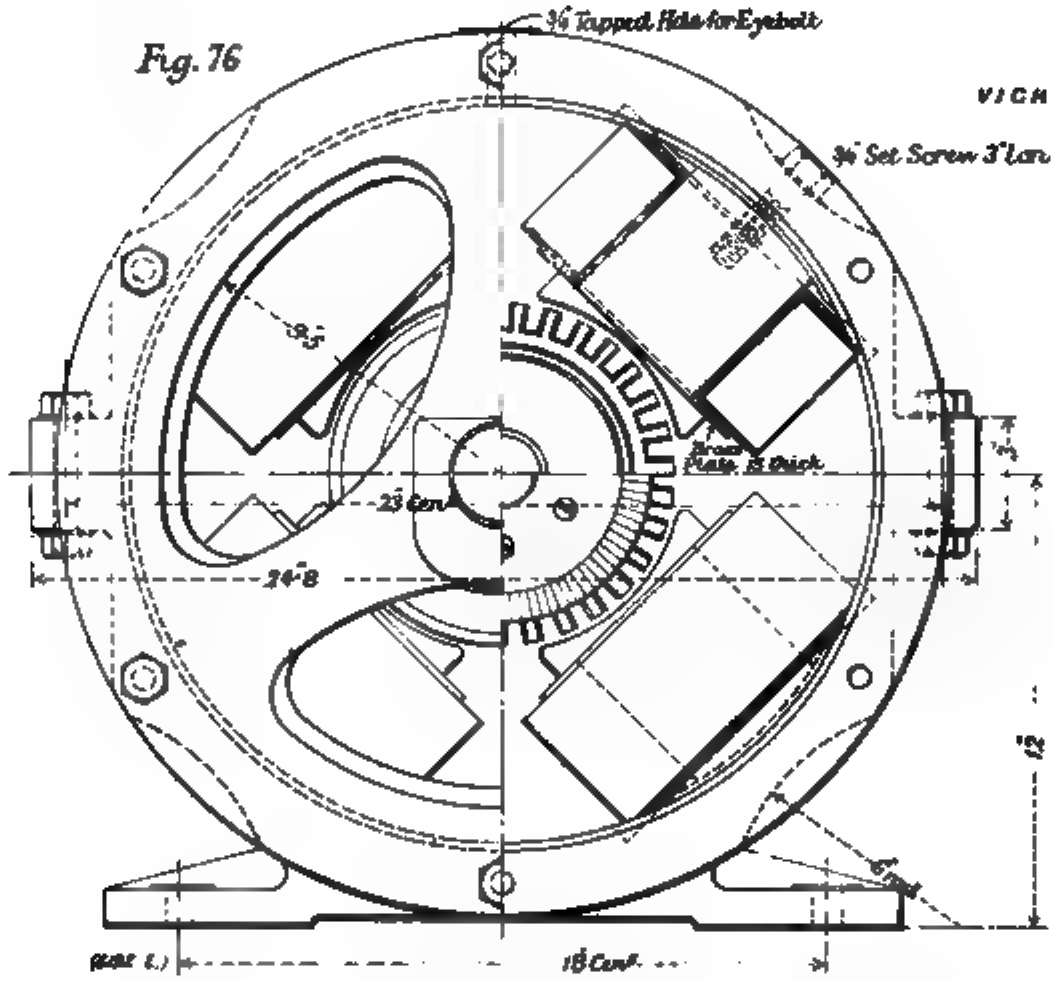
Date of test, July 21st, 1902. Temperature of engine room, 32 degs. Cent.

	Ultimate Temperature. Degrees Cent.	Rise in Temperature. Degrees Cent.
Commutator	73	41
Core	68	36
Shunt coils	61	29

An engraving of this 5 horse-power motor, assembled and in parts, is given in Fig. 75, and the general arrangement is shown in the drawings of Figs. 76 and 77, Plate 1. The machine may be used up to its full rated capacity as a reversible motor, the commutator being excellent at that load, with the brushes in the neutral position.

Another interesting machine by the same designer is shown in the drawings of Figs. 78 and 79 (page 83). It is rated as a

Fig. 76

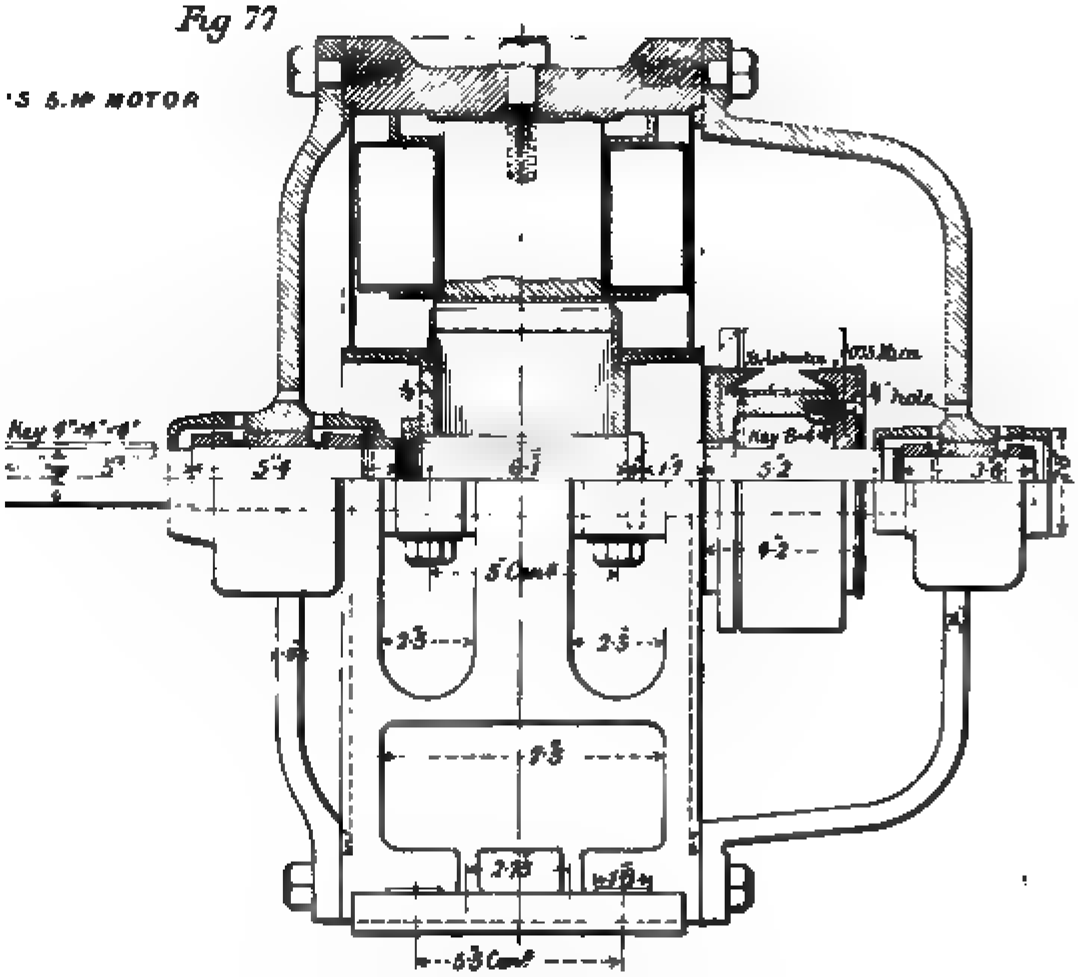


FIGS. 76 and 77.—5 Horse-power Motor b

FIG. 75.—Separate Parts of

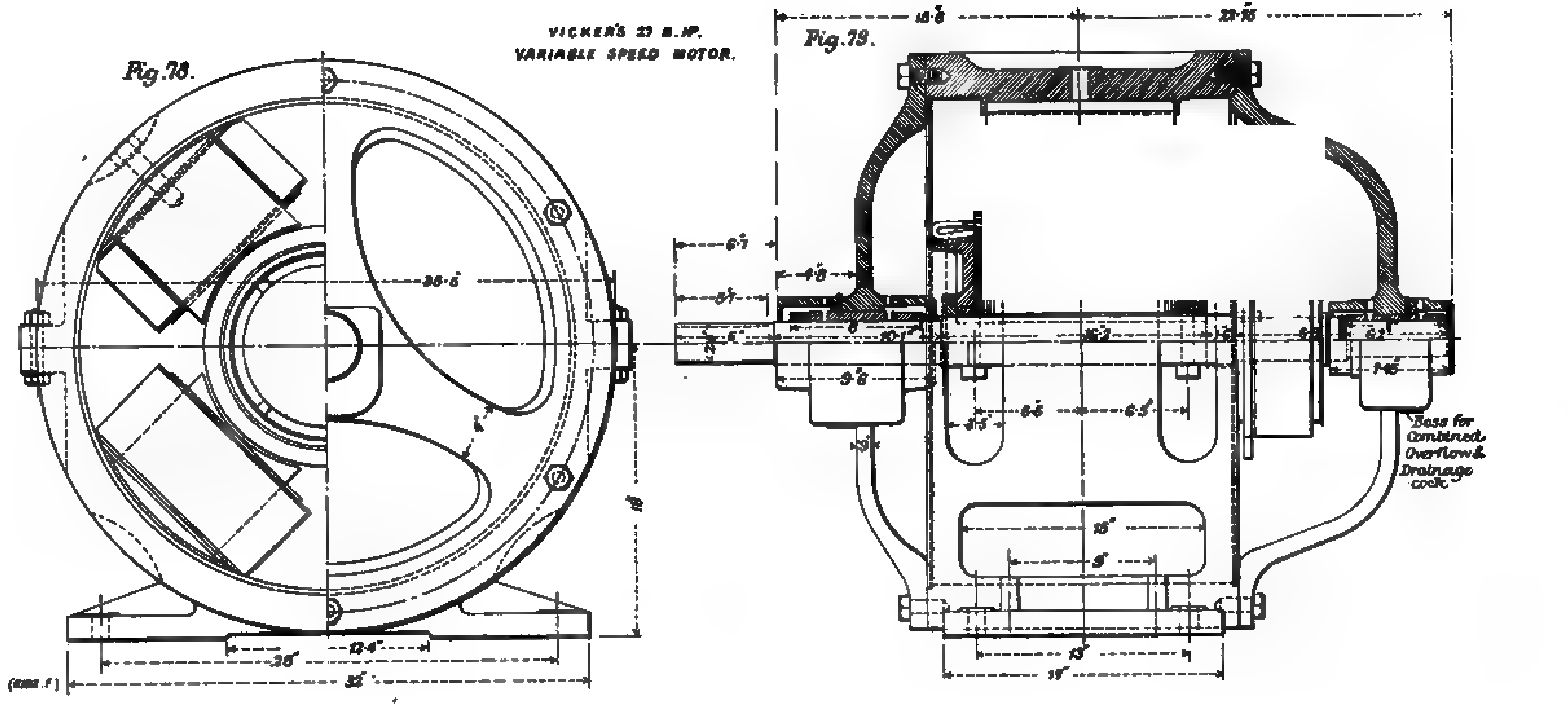
[Plate 1.

Fig 77



Messrs Vickers Sons & Maxim (see page 82).

Horse-power Motor (see page 82).



Figs. 78 and 79.—27·5 Brake Horse-power Variable Speed 220-volt Motor.

27·5 brake horse-power variable speed 220-volt motor, and gives 25 horse-power at a range of speed from 300 to 1000 revolutions per minute on shunt excitation, and 30 horse-power from 300 to 800 revolutions per minute. The motor is quite reversible up to about 800 revolutions per minute at 27·5 horse-power, *i.e.*, it operates satisfactorily under these conditions with the brushes in the neutral position.

The following test results (Table VII.) of one of these motors are of interest:—

TABLE VII.—TESTS OF WILLIAMSON MOTOR.

Date of Test, May 7th and 8th, 1902.

Speed.	Volts.	Total Current.	Shunt Current.	Brake Horse-power.	Efficiency.
324	224	96·5	4·72	24·65	·862
386	222	103·0	3·48	26·75	·88
498	224	101·5	2·78	26·70	·884
502	227	168·7	2·37	45·60	·882
718	221	100·7	2·10	25·73	·872
750	227	123·5	1·20	32·55	·875
950	222	106·3	0·97	26·80	·863
960	222	96·5	0·97	24·85	·875
960	222	106·7	0·97	27·30	·869
970	230	109·4	0·97	28·80	·868

The three runs at different loads, giving the following results, were each of six hours' duration:—

TABLE VIII.—RESULTS OF SIX HOURS' RUN.

Date.	Speed. Revolutions per Minute.	Field Current.	Mean Current.	Mean Voltage.	Mean Brake Horse-power.	Temperature Rise of			
						Commutator Degs. Cent.	Field Coil. Degs. Cent.	Armature Core. Degs. Cent.	Armature Winding. Degs. Cent.
May 12th, 1902 ..	349	3·54	97	222	25	36	42	48	38
May 13th, 1902 ..	353	3 32	116·8	221·4	30	52	42	65	54
May 14th, 1902 ..	610	..	116·8	221·4	30	43	42	54	..

Further data of these two machines is set forth in Table IX. :—

TABLE IX.—GENERAL DATA OF 5 HORSE-POWER AND 27·5 HORSE-POWER MOTORS.

	5 Brake Horse-power.	27·5 Brake Horse-power
Speed	600	300 to 900
Voltage	220	220
Weight in lbs.	840	3470

	5 Brake Horse-power.	27·5 Brake Horse-power.
Material of yoke	Cast iron	Steel
Material of poles	Steel	Steel
Type armature winding	2 circuit	2 circuit
Armature Dimensions :—		
External Diameter	9 ins.	15 ins.
Axial length core between flanges	4·75 ins.	13 ins.
Effective length core between flanges ...	4·5 ins.	12·5 ins.
Number of ventilating ducts	None	None
Depth slot	1 in.	·95 in.
Width slot	·33 in.	·35 in.
Number of slots	31	47
Internal diameter laminations	2 ins.	3 ins.
Height of bare conductor	·072 in.	·4 in.
Width of bare conductor	diameter	·06 in.
Number of conductors per slot	30	6
Slot, "space factor"	·37	·33
Magnet Core :—		
Length of pole face parallel to shaft ...	4·75 ins.	13 ins.
Length of pole arc	5·5 ins.	9 ins.
Radial length of magnet core	5 ins.	7 ins.
Diameter of magnet core	4·3 ins.	...
Length of air-gap	·15 in.	·25 in.
Commutator :—		
Diameter	8 ins.	11·8 ins.
Number of segments	93	141
Length of segment from external end to commutator connection	2·8 ins.	3·7 ins.
Thickness of brushes	·75 in.	1 in.
Width of brushes	1 in.	1 in.
Number of brushes per set	2	3
Number of sets used	2	2
Ampere turns per armature pole	1140	1900
Shunt ampere turns per spool	3150	7300 [1]
Weight copper per shunt spool	15·5 lbs.	83 lbs.
Core loss	170	450 [1]
Shunt loss	210	944 [1]
Commutator loss	50	300 [1]
Armature I^2R loss	230	750
Bearing friction loss	80	250 [1]
Mean length one armature turn	28 ins.	56 ins.
Armature turns per segment	5	1
Mean length one spool turn	19 ins.	40 ins.

[1] At 300 revolutions per minute.

§ 7. **Choice of Number of Armature Slots.**—The choice of the number of armature slots is a matter requiring comment. In the first place the use of the two-circuit winding, which, as has been explained, is especially desirable for small motors, limits the available numbers. Table X. gives, for the case of the plain two-circuit single winding, the number of conductors or sides of coils (groups of conductors) which are possible per slot for motors with from four to sixteen poles.

TABLE X.—DATA FOR APPLYING TWO-CIRCUIT SINGLE WINDINGS FOR DRUM ARMATURES.

Number of Poles.	Possible Numbers of Conductors or Groups of Conductors per Slot, for Symmetrical Windings								
4	1	2	...	6	...	10	...	14	...
6	1	2	4	...	8	10	...	14	16
8	1	2	...	6	...	10	...	14	...
10	1	2	4	6	8	...	12	14	16
12	1	2	...	...	...	10	...	14	...
14	1	2	4	6	8	10	12	...	16
16	1	2	...	6	...	10	...	14	...

Other numbers of conductors per slot can be used. For instance, four conductors, or groups of conductors, per slot in a four-pole motor, by the device of having one "dummy" coil, that is, one coil not connected to the winding or the commutator, or a space block in its place. But this, while often employed, introduces, in the writer's opinion, a very undesirable lack of symmetry, and should only be used where the sparking constants are very conservative. All limitations relating to the possible numbers of groups of conductors per slot lead to difficulties in employing the same punchings for windings for different voltages. Frequently, in spite of the lower "space factor" and other undesirable properties for small motors, multiple-circuit windings, which are free from these limitations, are used for some voltages, in order to use one standard punching for all voltages. The use of conductors of rectangular cross section sometimes enables a given slot to be filled up more efficiently when employed for a winding of a different voltage, speed, or rating from that for which it was designed. Most manufacturers, however, prefer to use round wires for small motors, and on page 318 of the *Electrotechnische Zeitschrift* for April 11th, 1901, Rothert (see also Rothert's U.S.A. patent, No. 660659) describes some novel types of winding. In one of these, with six groups of conductors per slot, and four

conductors per group—Fig. 80—the conductors may lie four wide and six deep, there being twenty-four conductors per slot. In another—Fig. 81—with four groups of conductors per slot, and four conductors per group, the width of the slot would correspond to three conductors, and the depth to six, this arrangement corre-

FIGS. 80, 81, 82, 83.—Different Types of Winding.

sponding to sixteen conductors per slot. The more customary, and, of course, the most effective arrangement corresponding to these two cases would be those shown in Figs. 82 and 83, where, however, the slots are rather deep. The arrangements suggested by Rothert are capable of considerable extension, and increase the flexibility of the two-circuit winding, and, indeed, of slot type

windings in general. The ideal from the manufacturing standpoint is, of course, to have one standard punching suitable for as many windings as possible, and while a too close adherence to this plan of procedure leads to sacrifices much greater than the advantage striven for, it is, nevertheless, well to give careful consideration to all methods leading to reduction in number of standard punchings.

Thus, the cases shown in Figs. 80 to 83 may be taken as representing four arrangements for a motor requiring, with a given total number of conductors, four turns per commutator segment, of the size wire shown in the figures. By adopting the arrangement shown in Fig. 80, a certain number of slots would be required. The use of the arrangement shown in Fig. 81 would require 50 per cent. more slots.

It will be seen from Table X. that, generally, for four or eight poles, Figs. 80 and 82 would be the most suitable with three commutator segments per slot, whereas Figs. 81 and 83, with two

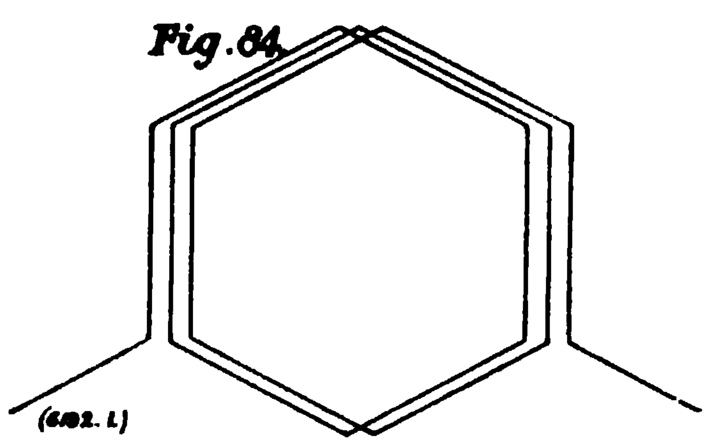


FIG. 84.—Coil Winding.

commutator segments per slot, are suitable for six or ten poles. None of them would do for twelve poles (for a symmetrical two-circuit single winding), and any one of them could be used for fourteen poles. The choice between Figs. 80 and 81, as against Figs. 82 and 83, would be greatly influenced by the sizes of slot found most suitable for designs for other rated voltages for the same motor.

§ 8. **Bar *versus* Coil Windings.**—Wherever at all practicable, the writer prefers bar to coil windings, in spite of the greater number of connections to be made. Thus, for a four-pole machine with three turns per commutator segment, there would generally be employed a coil winding with three turns per coil. This could be diagrammatically represented as shown in Fig. 84 above. For some sizes of slots, and especially for small conductors, this would be the best arrangement, but there often arise cases, even with conductors of fairly small cross section, where it would be

Fig 85. 4.Pole, 2-Circuit, S
with

Fig. 85 (se

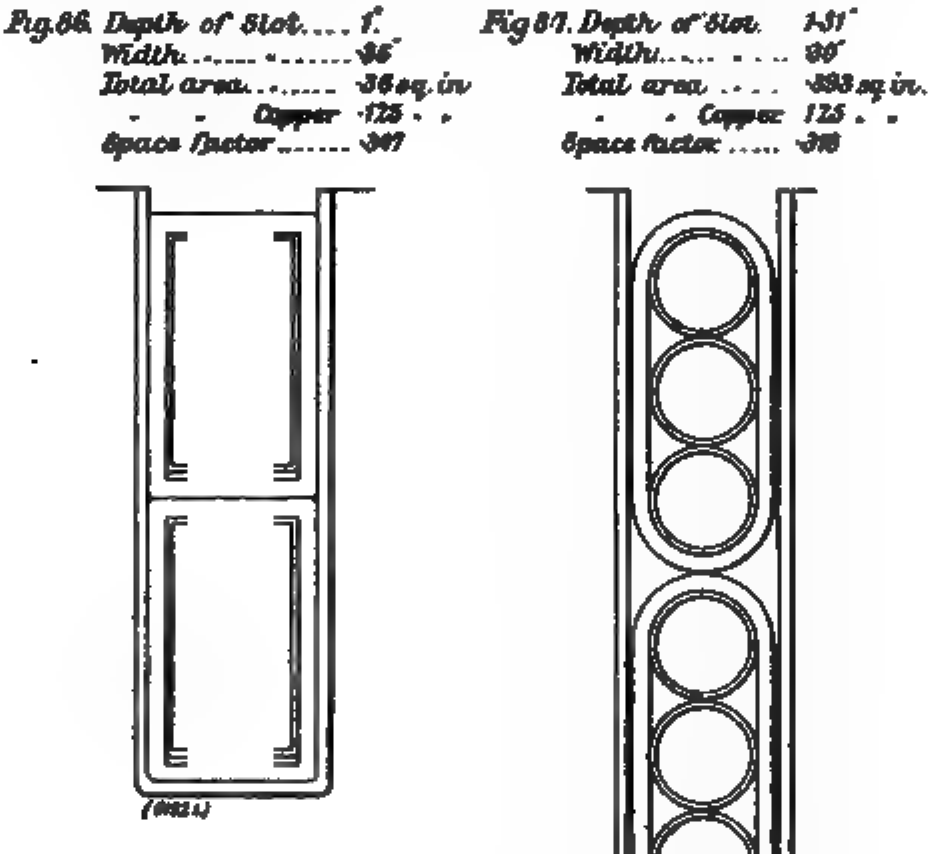
[*Plate* 2.

gle Winding of the formula $C = ny \pm 2m$ $(162 \cdot 4 \cdot 41 - 2 \cdot 1)$

2nd and 3rd segment omitted

page 89).

preferable to arrange the winding as shown in Fig. 85, Plate 2. The total number of conductors, and the number of commutator segments, remain unchanged, but the dimensions of the slot, and of the conductors, and their arrangement in the slot, are as shown in Fig. 85, whereas the coil winding would be arranged as shown in Fig. 86. The coil winding requires at the front (commutator) end of the armature, not only space for the end connections of the individual coils, but also further space for the conductors interconnecting these coils. The other arrangement is very much better, where each conductor is sufficiently large to consist of a



FIGS. 86 and 87.—Methods of Winding.

rod of rectangular cross section, and especially where the dimensions preferred for the slot require a conductor of such a cross section, since such conductors are less readily made up into coils. The superiority in orderliness should often make the winding preferable, even when the conductor is fairly small, and—other considerations aside — more readily wound into a coil. For the case chosen for the illustration the "space factor" is 8 per cent. lower for the bar winding.

The diagram in Fig. 85 represents the case of a two-circuit four-pole winding, with 162 face conductors and 27 segments, the

equivalent of a three-turn coil winding. The pitch y equals 41, and the winding is a two-circuit single winding, conforming to the formula $C=n\,y+2$ $(162=4\times 41-2)$.

There could be numerous other forms of this winding; thus every fifth turn could be connected to a segment, corresponding to a five-turn coil winding, or the winding could have six poles and still have every third, fifth, or seventh turn connected to commutator segments. In general for this winding, every $(n+1)$th pair of conductors,—n being any even number—has a corresponding commutator segment.

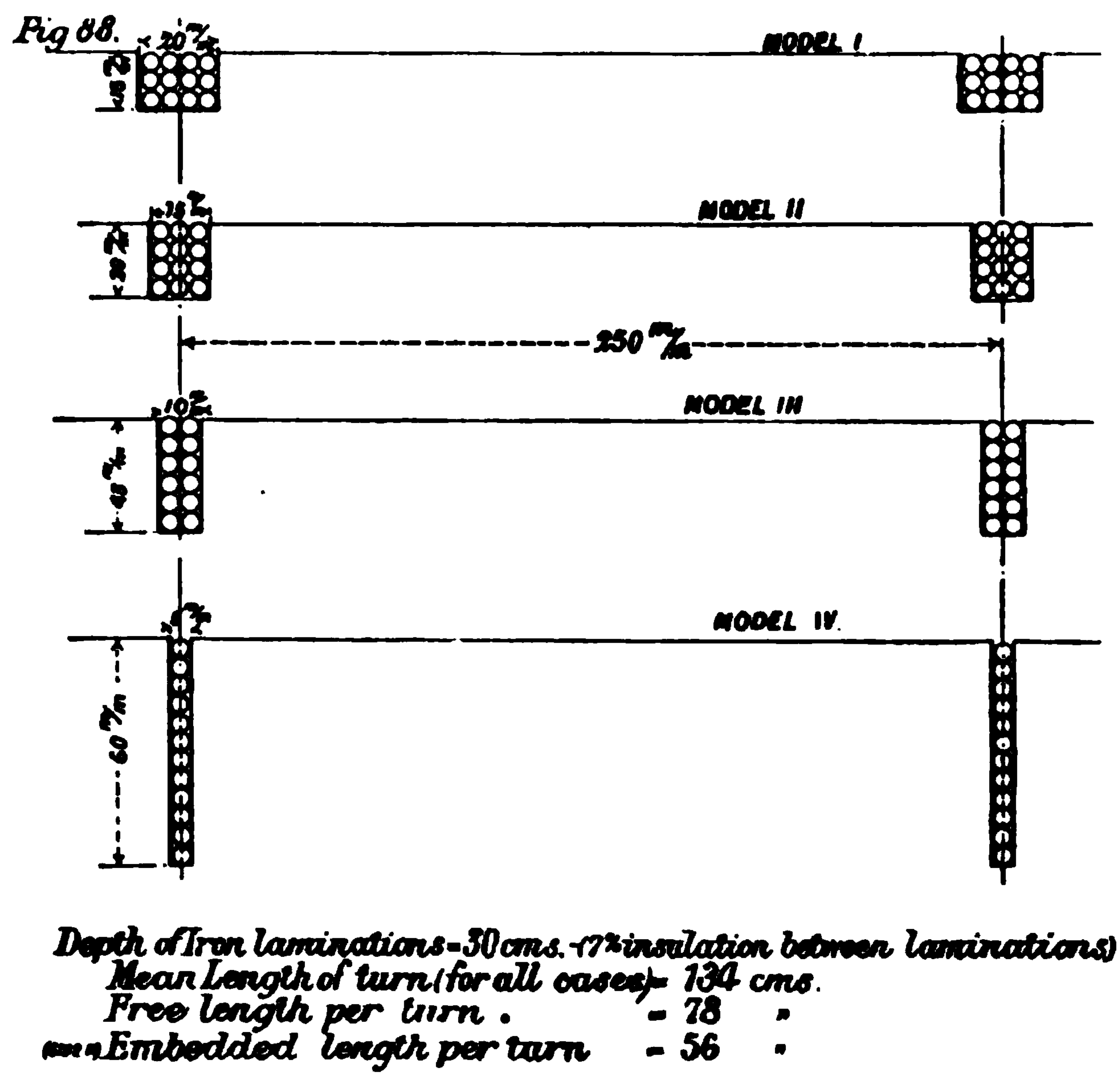


FIG. 88.—Models of Slot Dimensions.

§ 9. **Width and Depth of Slot.**—The chief disadvantage of wide slots is that they lead to eddy currents in the pole faces, if the latter be solid, due to the rapid local changes in the magnetic flux distribution caused by the successive passage of tooth and slot by a given point in the pole face. This has led in many cases to the adoption of laminated pole pieces, although, unless the slot is very wide and the air-gap short, high resistance, cast-iron pole faces suffice to reduce the pole face loss to very moderate proportions. Wide slots, when there are but very few slots per pole, also sometimes lead to noisy running, which can be best remedied by lengthening the air-gap, or chamfering, or otherwise shading

the pole pieces. These are troublesome and uncertain remedies, and it is desirable on this account not to choose too low a number of slots per pole. In the case of tramway, or crane, or other motors, operating in the midst of greater noises, this is not a point requiring to be taken into account. There is some slight advantage in wide and shallow slots from the standpoint of decreased inductance of the embedded portion of the winding, best expressed in c.g.s. lines per centimetre of length. The extent of the influence upon this value, of the width and depth of the slot, may be seen from Table XI., where the values experimentally observed on models with the slot dimensions shown in Fig. 88, page 90, are given.

TABLE XI.

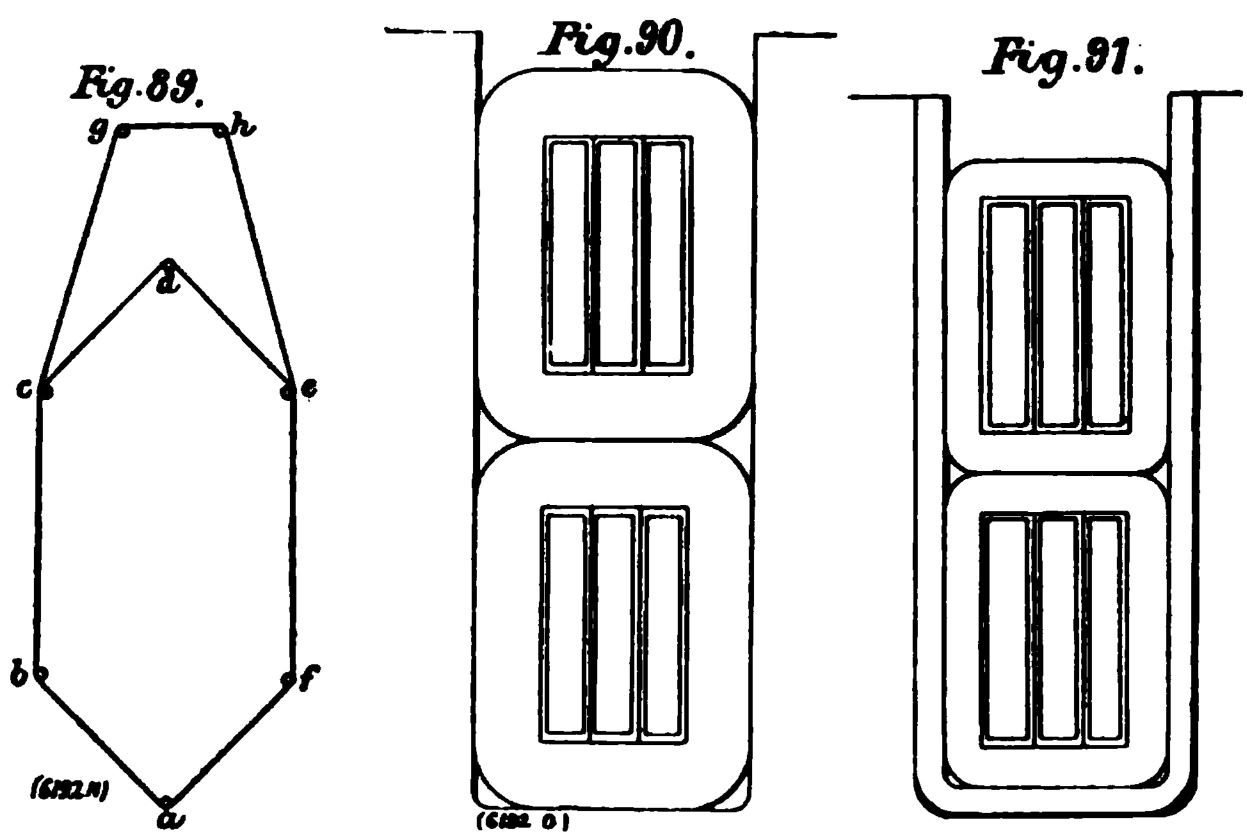
Model.	Lines per Centimetre of "Embedded Length."
I.	2·8
II.	3·2
III.	4·2
IV.	7·5

There is, however, the consideration to remember, that the deeper and narrower the slot, the shorter may be the end connections and the less their total inductance. The armature I^2R loss and the dimensions of the armature are also decreased by deep slots, so that they cannot at once be set aside as unsuitable, but must generally be taken into consideration in the case of the preparation of each design, and the relative advantages compared. Mechanically and thermally, it is much better to have wide and shallow slots, and several commutator segments per slot.

§ 10. **On Methods of Winding Armature Coils.**—When several coils are thus to be grouped in one slot, it is generally more economical to wind them at one operation from several reels of wire. Another and very interesting way has been described by Rothert, in the article already referred to (*Elektrotechnische Zeitschrift* for April 11th, 1901, also Rothert's U.S.A. Patent 660659). He describes it by the aid of a diagram reproduced in Fig. 89. The wire is wound in the winding form about the points *a b c d e f* as many times as there are to be turns per segment. The wire is then continued out about the points *g h*, and the turns corresponding to the next spool are wound again about the points *a b c d e f*, and a loop is again brought out about *g h*, and this is repeated till all the wires of the combined spool, whose two sides are to lie in a given pair of slots, have been wound. Then, either before or after assembling the winding on the armature core, the

loops are cut between g and h, and the six ends (supposing this to have been a case of winding with three segments per slot) are connected to the six corresponding segments.

§ 11. **Slot Insulation.**—Two methods of arranging the slot insulation are shown in Figs. 90 and 91. In the former (Fig. 90) the general plan of procedure is to have sufficient insulation in wrappings of braid impregnated with varnish, and to dispense with an additional insulation at the sides and bottom of the slot. In the latter (Fig. 91) there is less insulation upon the coils, the space thus saved being taken up by a U-shaped lining to the slot, which serves more to protect the formed coil from abrasion when being put in place, than as insulation. This latter method has the

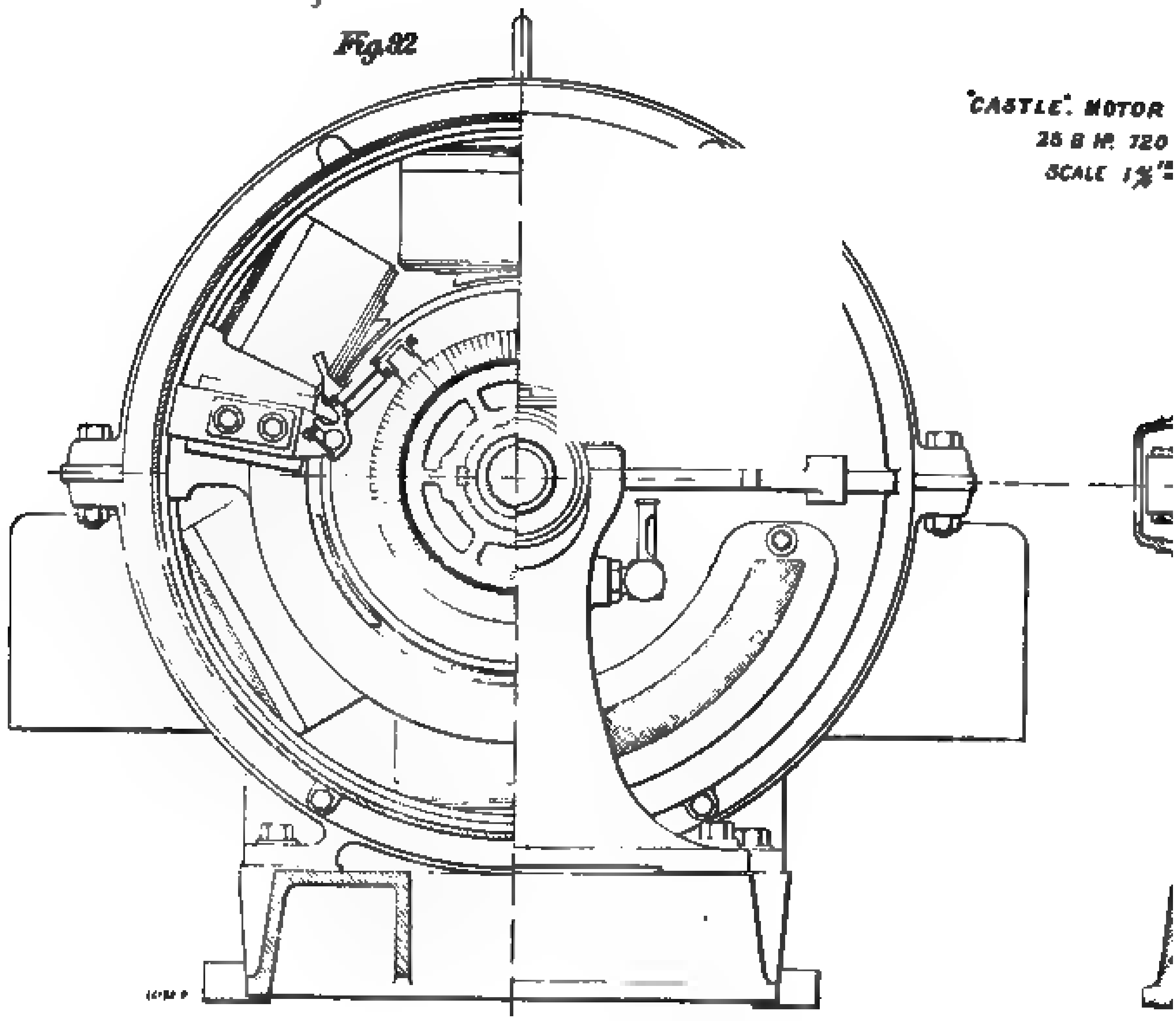


FIGS. 89, 90 and 91.—Methods of Winding (Rothert).

additional advantage that the end connections may be insulated practically to the same specification as the slot portion, and still not employ any more space at the ends of the armature than is required for their suitable protection and insulation.

§ 12. **"Castle" 25 B.H.P. Motor.**—A six-pole, semi-enclosed, compound-wound "Castle" motor, for 25 brake horse-power, at 720 revolutions per minute, is illustrated in Figs. 92 and 93, Plate 3. The writer is indebted to Messrs. J. H. Holmes & Co., of Newcastle, for permission to publish a description of this motor, which is one they have manufactured extensively, often for operating printing presses. The drawings are of a 220-volt motor, which differs from the 460-volt motor of the specification in Table XII. chiefly in number of slots, winding, length of commutator, and in being plain shunt wound.

Fig. 82



[*Plate* 3.

TABLE XII.—DESCRIPTION OF "CASTLE" MOTOR.

Number of poles	6
Speed	720 r.p.m.
Terminal voltage	460 volts
Full load current	45 amperes
Armature diameter	15 ins.
Gross length armature laminations between flanges	7 ins.
Width ventilating duct in laminations	$\frac{7}{16}$ in.
Effective length of armature laminations ...	6 ins.
Inside diameter of laminations	7·75 ins.
Number of slots	91
Depth of slot	$1\frac{1}{8}$ ins.
Width of slot	0·27 in.
Type of armature winding	2-circuit single
Pitch	61
Turns per commutator segment	2
Commutator segments per slot	2
Total number of face conductors	728
Total number of commutator segments	182
Commutator diameter	11·25 ins.
Active length commutator segment	3·5 ins.
Commutator segment made from strip of 1·125 ins. × ·163 in. × ·1242 in.	
Brushes of carbon $1\frac{1}{2}$ ins. wide and $\frac{3}{8}$ in. thick ...	2 brushes per set
MAGNETIC CIRCUIT—magnet core cross section ...	21·64 sq. ins.
Pole pieces are of steel and in one casting, with yoke.	
Cross section of yoke	13·25 sq. ins.
Length pole face	6·75 ins.
Length pole arc	5·13 ins.
Cross section of air-gap at pole face	34·5 sq. ins.
Diameter of bore of pole faces	$15\frac{5}{16}$ ins.
Depth of gap	$\frac{5}{32}$ in.
Full load armature ampere turns per pole ...	1370

After four hours' run at full rated load, the temperature increase above surrounding air by thermometer was as follows :—

Magnets	32 deg. Cent.
Armature core	24 "
End connections	24 "
Commutator	30 "

Temperature of surrounding air = 18 degs. Cent.

The arrangement of the conductors in the slot is shown in Fig. 94, drawn to a scale of 4 to 1.

There are eight conductors per slot, and each conductor measures (bare) ·12 in. × ·08 in.

The "space factor" is therefore 0·316.

The shunt winding has 3100 turns per limb, and the resistance of the six spools in series equals 380 ohms when warm.

An open-type six-pole motor of the same firm, and of about the same capacity as the motor just described, is shown in the engraving, Fig. 95, Plate 3. The set is for running a printing press, and the small auxiliary totally-enclosed motor at the left is employed at starting in order to obtain a small current. It is

Fig. 94.

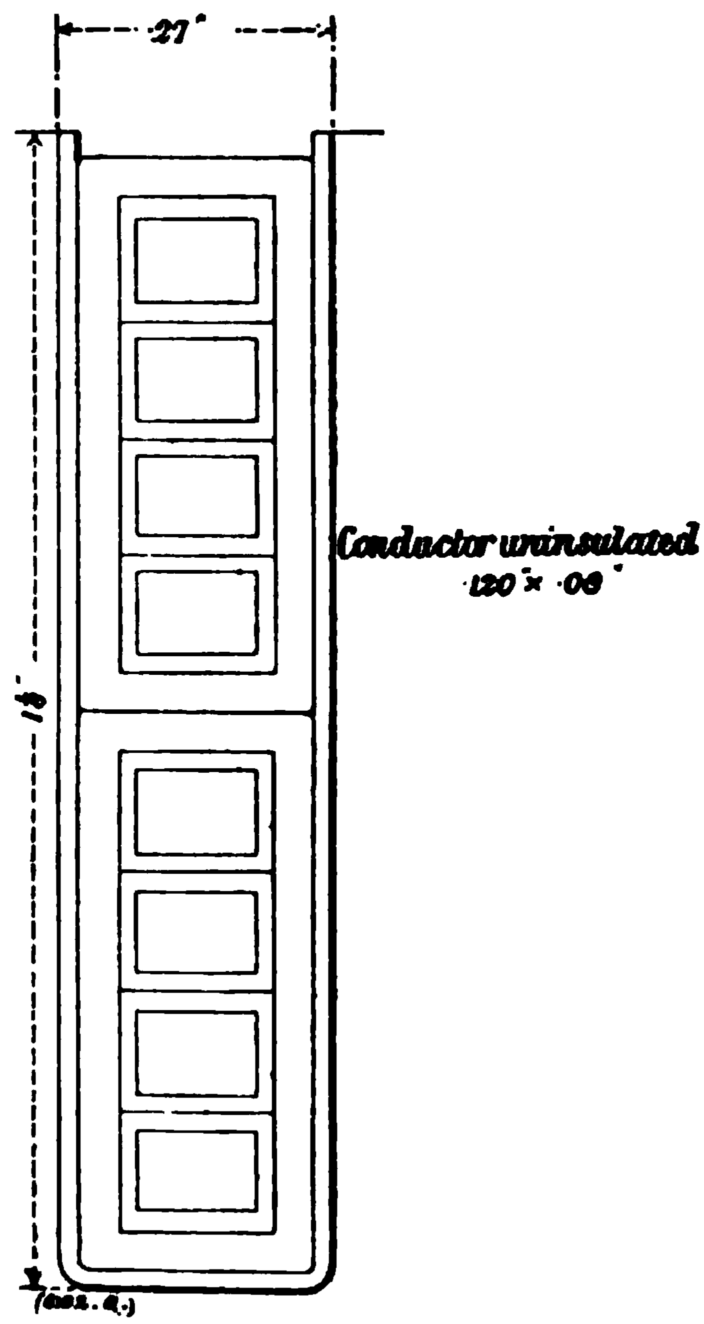
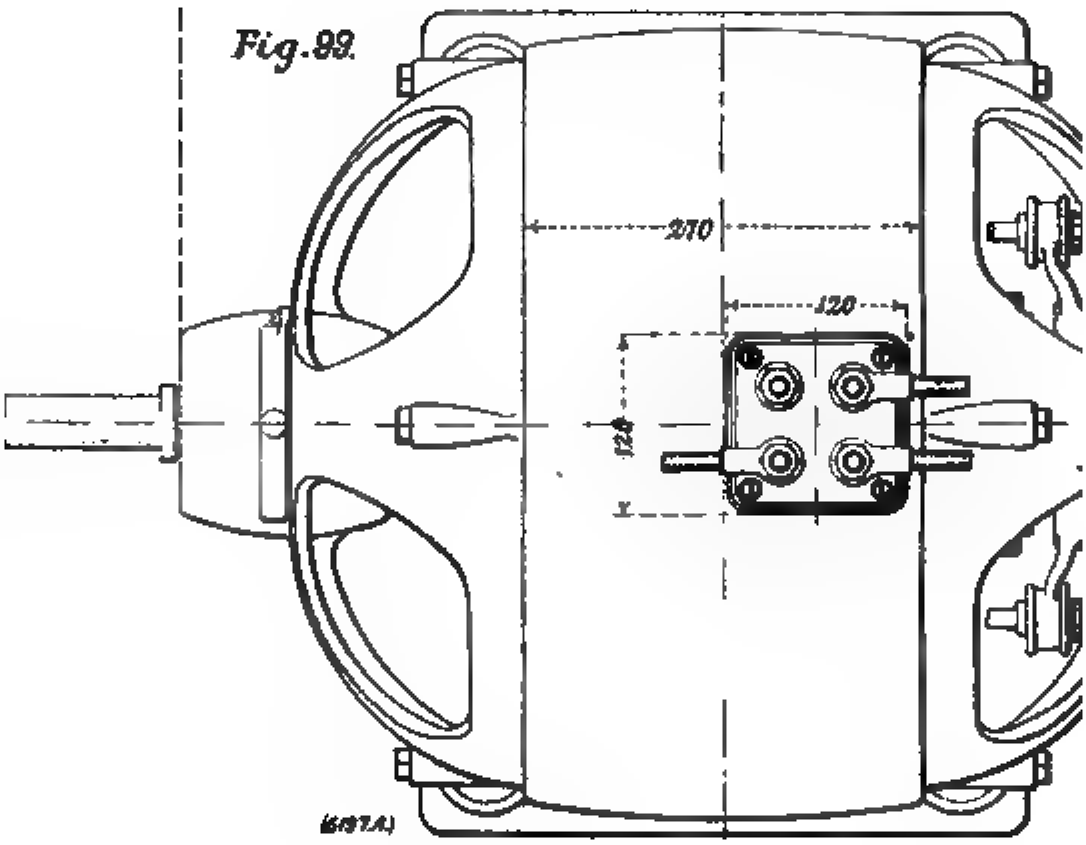


FIG. 94.—Arrangement of Conductors, "Castle" Motor.

subsequently, at the desired speed, automatically disconnected from the shaft by a toothed clutch, the work being taken over by the main motor.

§ 13. **Union Elektricitäts Gesellschaft 10 H.P. Motor.**—The Union Elektricitäts Gesellschaft of Berlin has kindly furnished the writer with full particulars of its latest 10 horse-power, semi-enclosed shunt-wound motor for 500 volts at 950 revolutions per minute. In Figs. 96 to 107 are given drawings of this motor, and Fig. 108, Plate 5, is a view of the next larger size of the



FIGS. 96, 97, 98, 9

10 Horse-power Motor by the Union

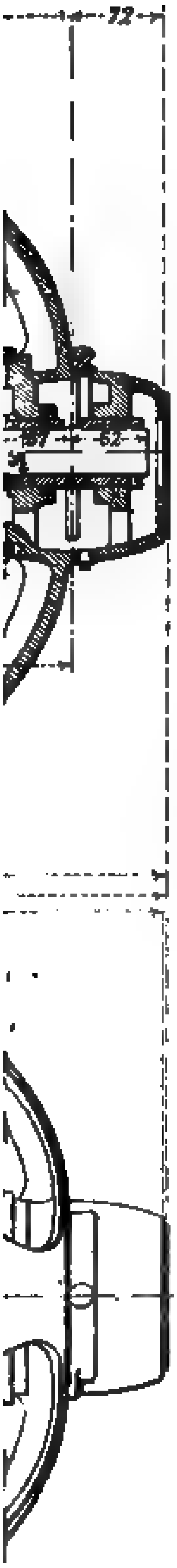


Fig. 97.

Fig. 98

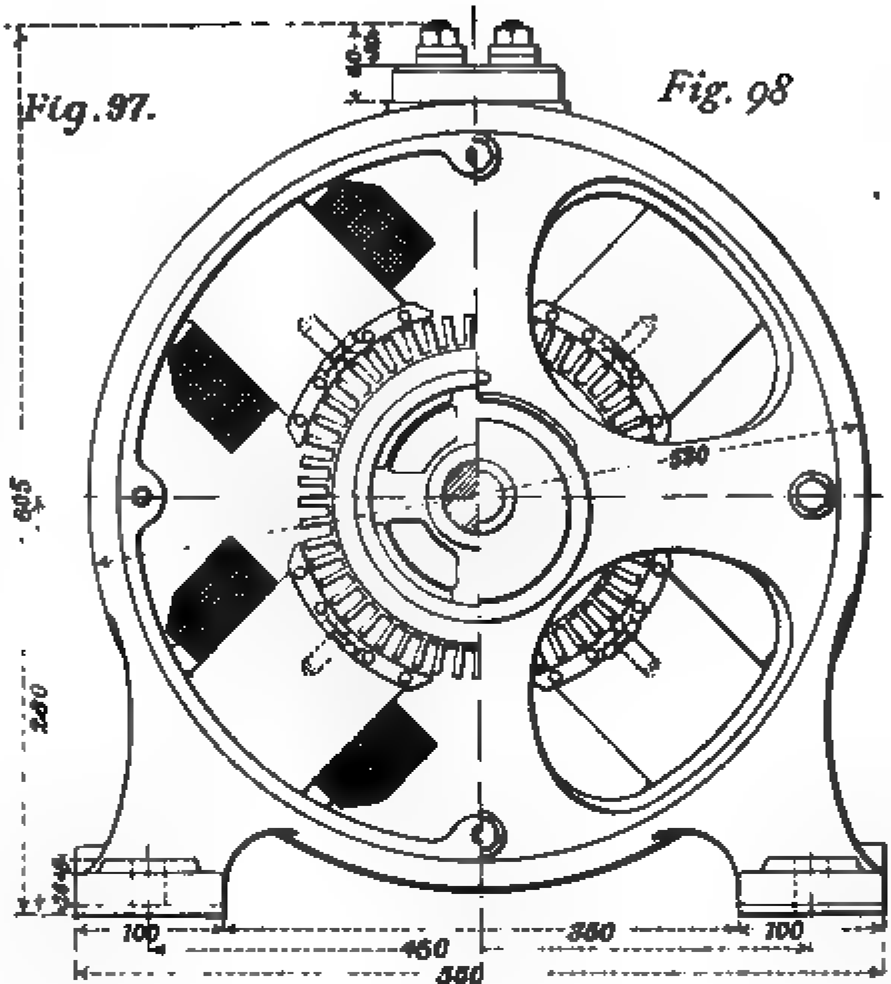
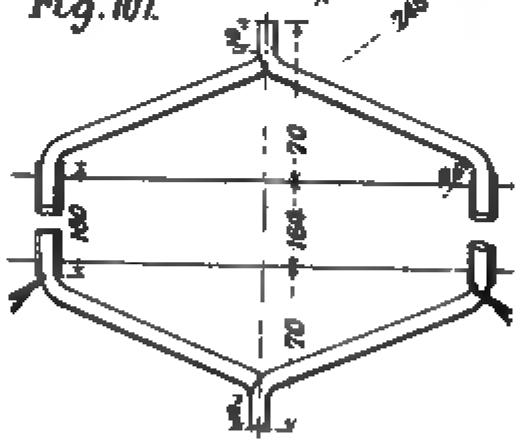


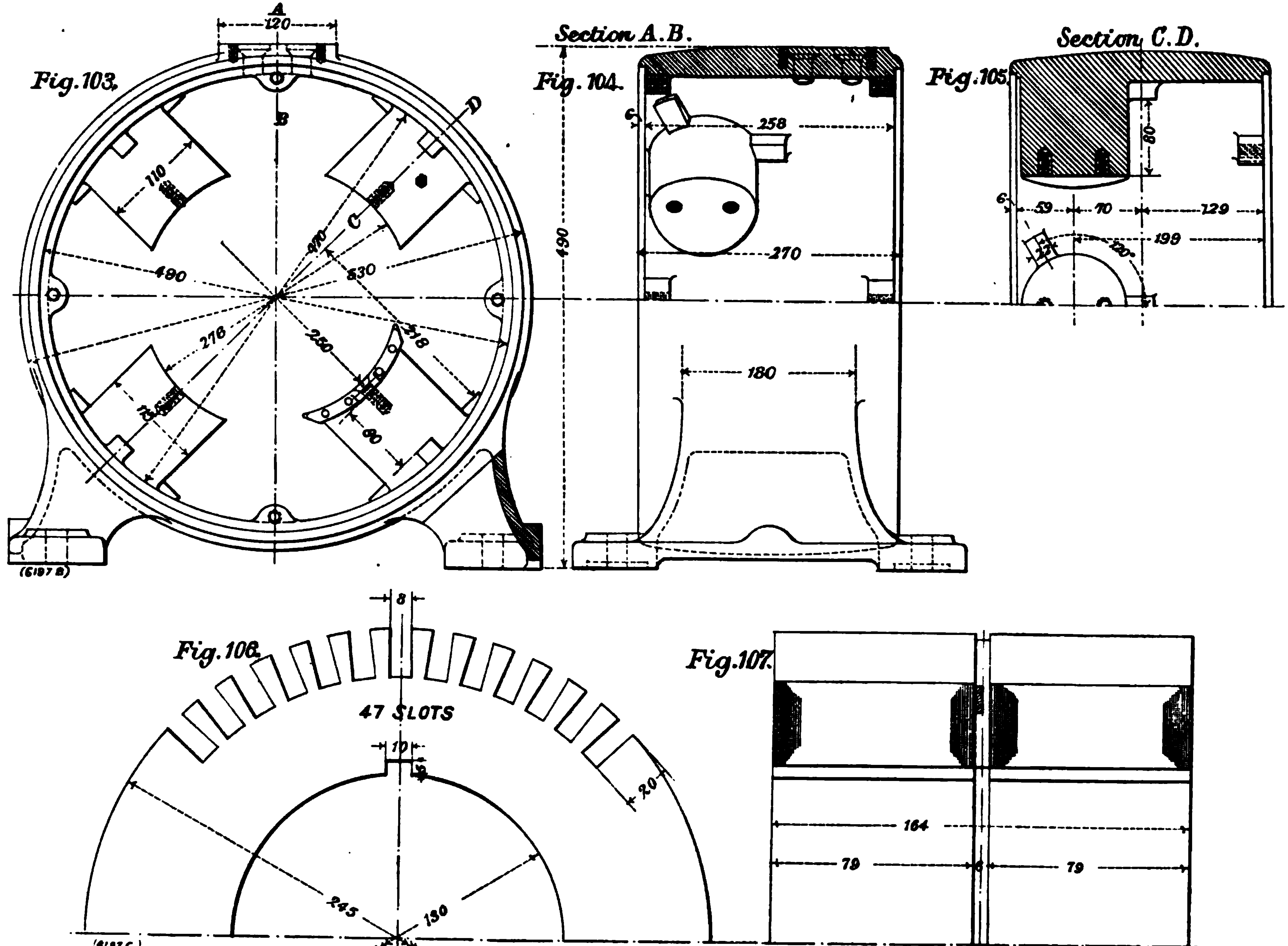
Fig. 100

Fig. 101.



1,46
1,71
1,85
3

9, 100, 101 and 102.
Elektricitäts Gesellschaft (see page 94).



FIGS. 103, 104, 105, 106 and 107.—10 Horse-power Motor by the Union Elektricitäts Gesellschaft.

same line of motors. All these small motors have the same number of slots, and the same commutator segments for all voltages, this corresponding to the minimum number considered to be required to give satisfactory results for the 500-volt motor, the length of the segment being determined by the permissible heating for the lowest rated voltage.

TABLE XIII.—DESCRIPTION OF THE UNION ELEKTRICITÄTS GESELLSCHAFT'S SEMI-ENCLOSED MOTOR.

Number of poles	4
Rated output	10 B.H.P.[1]
Rated voltage	500 volts
Rated speed	950 revs. per min.
Amperes input at full load	17·0 amperes
Amperes input at no load	1·5 „
Watts input at no load	740 watts
Armature:—	
External diameter	245 millimetres
Axial length of winding	304 „
Internal diameter of laminations	130 „
Effective length of laminations between flanges	142 „
Length occupied by ventilating duct	6 „
Axial length occupied by thickness of insulating varnish on laminations	16 „
Gross axial length of core between flanges ...	164 „
Thickness of each lamination	0·5 „
Number of slots	47
Depth of slot	20 „
Width of slot, as punched	8·3 „
Width of slot, as assembled	8·0 „
Width of tooth at root, as stamped	5·4 „
Average width of tooth, as stamped	6·7 „
Width of tooth at armature face, as stamped	8·1 „
Magnet Core:—	
Effective length of pole face parallel to shaft ...	155 „
Effective length of pole arc	147 „
Thickness of pole shoe at centre of pole arc ...	13 „
Radial length of the magnet core	80 „
Diameter of cross section of magnet core ...	115 „
Radial depth of air-gap	3 „
Ratio of effective pole arc to pitch	0·75 „

[1] The continental horse-power=736 watts (75 kilogrammetres per second).

Magnet Yoke:—

External diameter of yoke	530 millimetres
Internal diameter	470 "
Thickness	30 "
Axial width	270 "
Radial thickness of pole seat	17 "

Commutator:—

Diameter	175 millimetres
Number of segments	141
Thickness of a segment at periphery	3·2 "
Thickness of the insulation between segments	0·7 "
Effective length of segment	70 "

Armature Winding:—

Conductors per slot	24
Type of winding	2-circuit single
Size of wire	1·45 mm. diam.
Size of insulated conductor	1·71 "
Current per conductor	8·5 amperes
Current density in armature conductor ...	510 amperes per sq. centimetre
Armature slot "space factor"	0·25
Number of turns in series between brushes ...	282
Mean length per turn	77 centimetres
Resistance of winding from + to – at 60° Cent.	1·30 ohms
IR drop in armature winding at full load ...	22·0 volts
IR drop at brush contact surfaces	1·4 "
Total induced voltage at full load	476·6 "

Calculation of Reactance Voltage:—

Peripheral speed of commutator	8·7 m. per sec.
Length of arc of brush contact	14 millimetres
Frequency of commutation $=\dfrac{1000\times 8{\cdot}7}{2\times 14}=$ ...	313 cycles per sec.
Width of segment at periphery (including insulation)	3·9 millimetres
Maximum number of coils short-circuited under a brush	4
Turns per coil (q)	4
Maximum number of simultaneously commutated conductors per group (r) ...	32
"Free length" per turn (s)	49 centimetres
"Embedded length" per turn (t)	28 "
Lines per ampere turn per centimetre of "free length" (u)	0·8
Lines per ampere turn per centimetre of "embedded length" (v)	4·0
Lines per ampere turn for "free length" ($u\times s$)	39·2

Calculation of Reactance Voltage—continued.

Lines per ampere turn for "embedded length" $(v \times t)$	112
Lines per ampere for "free length" $\left(\frac{r}{2} \times u \times s = o\right)$	625
Lines per ampere for "embedded length" $(r \times v \times t = p)$	3580
Total lines linked with short-circuited coil per ampere $(o+p)$	4205
Inductance per segment $\left(\frac{q \times (o+p)}{10^8} = l\right)$ ...	·000168 henrys
Reactance per segment $(2\pi\, n\, l)$	0·33 ohm
Current per conductor at full load	8·5 amperes
Reactance voltage at full load	2·8 volts
Average voltage per segment	14·2 ,,
Armature ampere turns per pole at full load ...	1220

Magnetic Circuit Calculations:—

Full load internal voltage (E)	477
Turns in series between brushes (T)	282
Periodicity in cycles per second (N)	31·8
Flux entering armature per pole at full load (M expressed in lines) ... ... $(E = 4\, T\, N\, M \times 10^{-8})$	1·33 megalines
Leakage factor (assumed)	1·20
Flux generated per pole full load	1·60 ,,

	Cross Section (Square Centimetres).	Kilolines per Square Centimetre.	Magnetic Length (Centimetres).	Magneto-motive Force in Ampere Turns per Centimetre.	Magneto-motive Force in Total Ampere Turns.
Armature core below slots ...	106	12·6	12	8	100
Flux-carrying teeth	71	18·3	2	130	260
Pole face	228	5·8	0·3	4650	1400
Magnet core	104	15·4	10	35	350
Yoke	150	10·7	20	10	200
Total ampere turns per spool	...	...	...	...	2310

The motor at 17 amperes and two segments backward lead runs at 980 revolutions per minute.

Each field spool is wound with 2400 turns of single cotton-covered wire of 0·60 millimetre diameter (bare), and 3500 turns of 0·55 millimetre diameter (bare), a total of 5900 turns per spool,

FIG. 108.—Semi-enclosed Shunt Wound Motor by the Union Elektricitats Gesellscha
(see page 94).

FIG. 129.—20 Horse-power Shunt Motor, designed by A. V. Clayton, Ludvika, Sweden (see page 119).

FIG. 147.—M

[*Plate* 5.

FIG. 130.—35 Horse-power Four-pole Motor, designed by A. V. Clayton (see page 125).

vor & Coulson Enclosed Motor Geared to Horizontal Mining Pumps (see page 140).

and at 60° Cent. the shunt current is 0·45 ampere, the resistance being 1120 ohms for the four spools in series, as determined by resistance measurements.

Ampere turns per spool = 0·45 × 5900 = 2660 as against the calculated value of 2310.

Fig. 109.

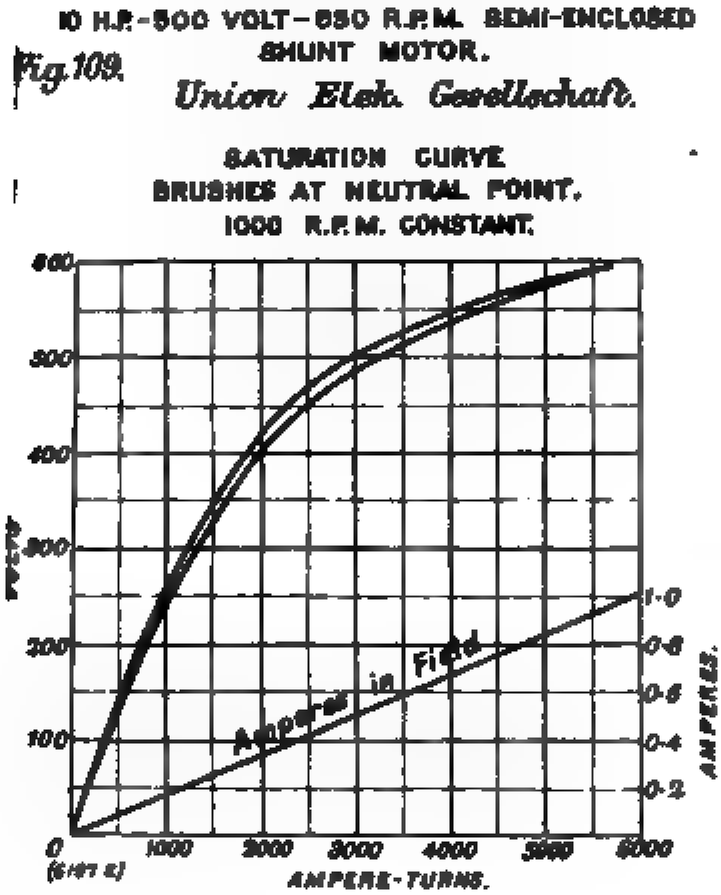


Fig. 110.

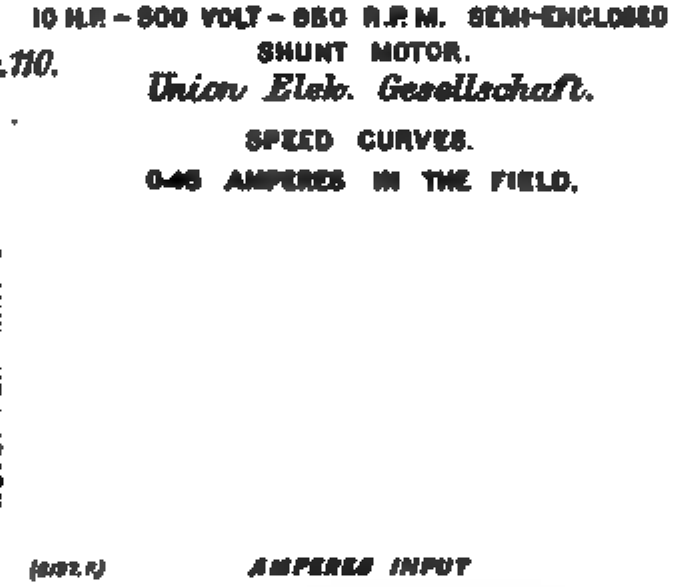


Fig. 111.

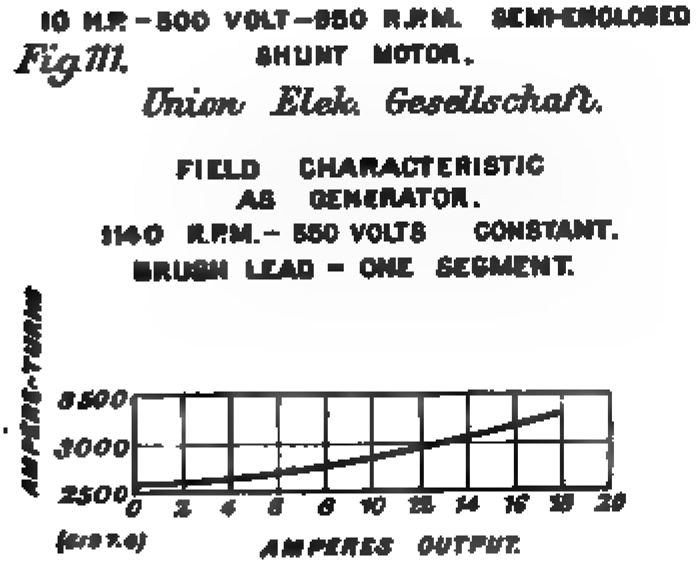
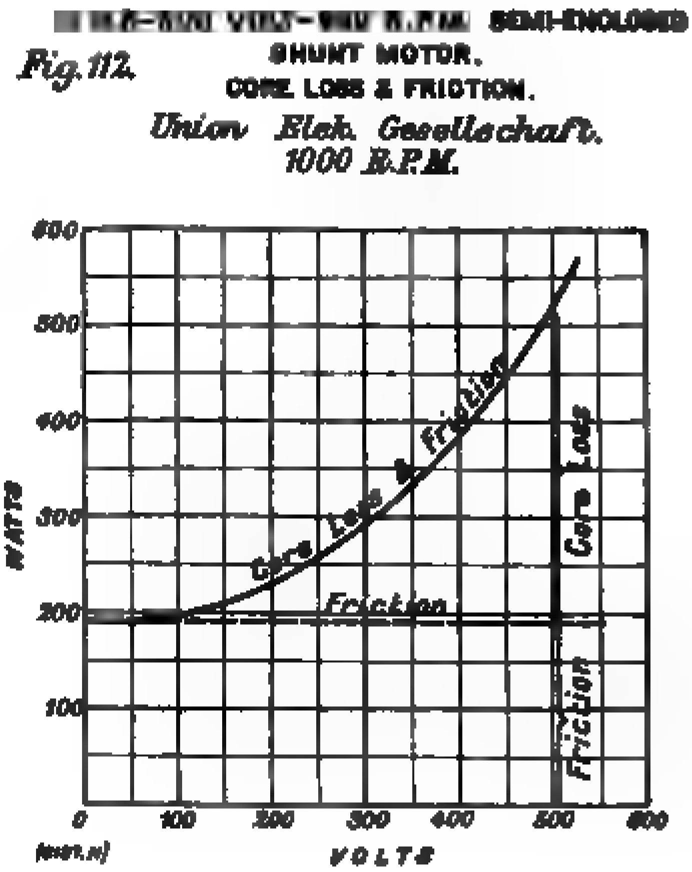


Fig. 112.



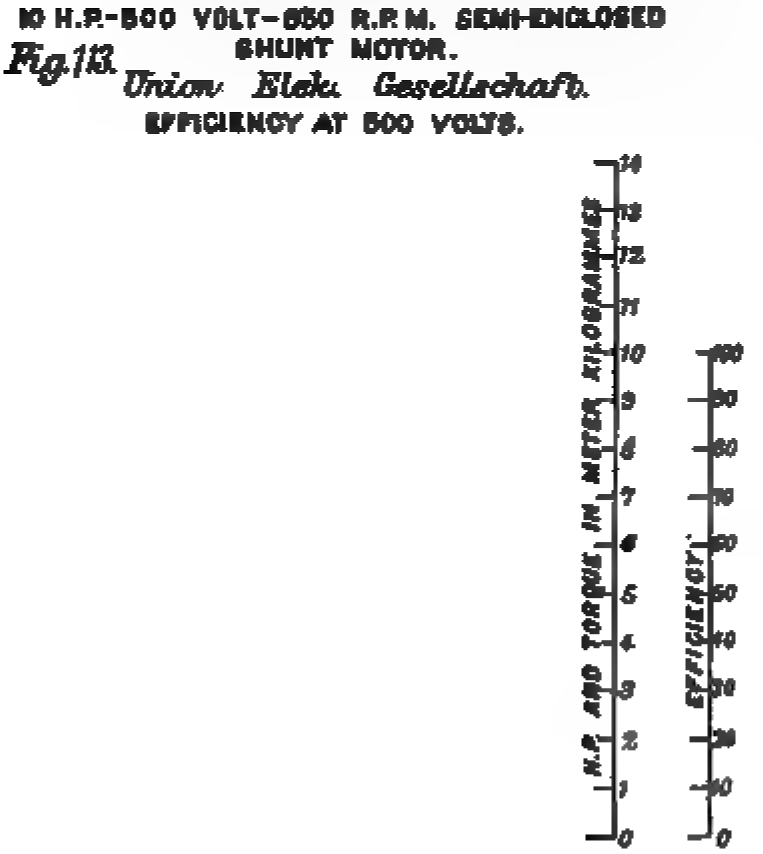
Figs. 109, 110, 111 and 112.

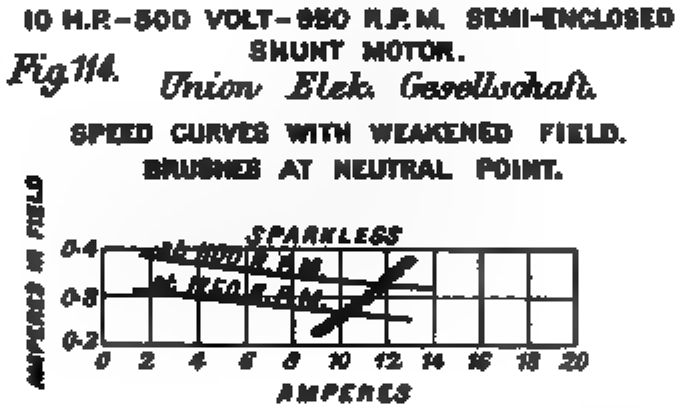
Energy for excitation = ·45 × 500 = 225 watts, or 56 watts per spool.

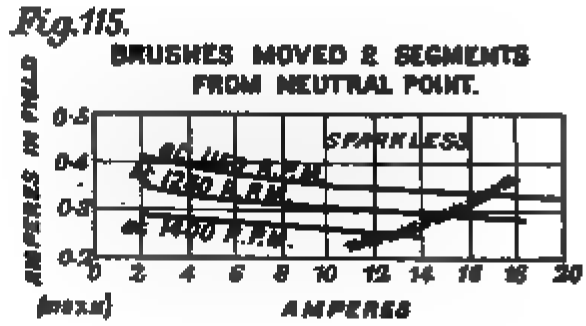
Taking the average depth of the spool winding as 5 centimetres, the external circumference is 6·75 decimetres, and the equivalent external cylindrical radiating surface may be taken as

5·5 square decimetres, thus giving about 10 watts per square decimetre.

The temperature increase, as determined by *resistance* measurements, is 40° Cent. at the end of a 4·5 hours' run at full load, or 4° Cent. per watt per square decimetre.







Figs. 113, 114 and 115.

Table XIV.—Calculation of Armature Losses and Thermal Characteristics.

Armature resistance at 60° Cent. = 1·30 ohms.

Armature I^2R loss = $17^2 \times 1{\cdot}30$ = 380 watts.

The core loss, as measured, was 325 watts, which, by chance, agrees exactly with the core loss as calculated by means of the curve of fig. 21 on page 29.

Total armature loss = 705 watts.

Cylindrical radiating surface, 23 square decimetres ∴ 31 watts per square decimetre.

Temperature increase at end of four and a half hours' run at full load, as determined from resistance measurements = 41° Cent.

Temperature increase per watt per square decimetre = 1·32° Cent.

Table XV.—Calculation of Commutator Losses and Thermal Characteristics.

Length of brush contact arc	14 millimetres
Width of brush	19 „
Contact surface per brush	2·67 sq. cm.
Number of brushes per pole	1

Total number of positive brushes	2
Contact surface of all positive brushes ...	5·34 sq. cm.
Current density at contact surface	3·2 amperes per sq. centimetre
I^2R loss in watts per ampere (see Table XIV.)	1·4
Total I^2R loss	24 watts
Peripheral speed of commutator	8·7 m. per second
Friction loss in watts per ampere	1·7
Total friction loss	29 watts
Total commutator loss	53 „
Cylindrical radiating surface	3·9 sq. decimetres
Watts per square decimetre cylindrical radiating surface	13·6
Observed temperature increase	33° Cent.
Temperature increase per watt per square decimetre	2·4° „

TABLE XVI.—EFFICIENCY AT 60° CENT.

Core loss	330 watts
Armature copper loss	380 „
Brush contact I^2R loss	20 „
Brush friction loss	30 „
Bearing and air friction loss	160 „
Shunt winding I^2R loss	220 „
Total of constant losses ...	740 „
Total of variable losses	400 „
Total of all losses	1140 „
Output at full load	7350 „
Input at full load	8490 „
Commercial efficiency at full load	86·5 per cent.
„ „ $1\frac{1}{4}$ „	87·0 „
„ „ $\frac{3}{4}$ „	85·0 „
„ „ $\frac{1}{2}$ „	81·2 „
„ „ $\frac{1}{4}$ „	70·0 „

TABLE XVII.—WEIGHTS AND COSTS OF NET EFFECTIVE MATERIALS.

	Weight in Kilogrammes.	Assumed cost in Pence per Kilogramme.	Total Cost in Shillings.
Armature copper	6·2	24	12·4
Commutator copper	9·5	24	19·0
Field copper	31·0	24	62·0
Armature laminations	29	3·6	8·7
Magnetic circuit portion of cast steel frame	115	4·5	43·0
Pole shoe laminations	10	3·6	3·0
Commutator mica	1·8	24	3·6

Total cost of net effective material	... ...	151·7 shillings
Total cost per horse-power rated output	...	15·2 „
Weight of motor, complete, without pulley or belt-tightener	... ...	310 kilogrammes

Figs. 109 to 115 (pages 99, 100) are a series of curves plotted from test results of this machine.

§ 14. **Commutator Design.**—The commutator must be proportioned with regard both to heating and to the avoidance of sparking. It may be assumed that in a well-proportioned motor, current in the short-circuited coils, and sparking, are so minimised that the heating may be considered as due entirely to the ohmic resistance of the brush contacts, and to the brush friction. Very complete investigations have been carried out by Arnold and others[1] to determine the value of the brush contact resistance, and its dependence upon the peripheral speed, the current density at the brush surface, the brush pressure, and the material of the brush. While the brush contact resistance has been shown by these investigations to vary considerably with variations in each of these factors, by far the greatest influence is exerted by the quality of the brushes, and the current density at the surfaces of contact. Copper brushes, now rarely employed, have an extremely low contact resistance (less than one-tenth that of carbon brushes), and were their use practicable, the length of the commutator could be very greatly reduced, because of the greatly decreased losses due to brush contact resistance and to brush friction. The employment of copper brushes for motors was almost entirely abandoned in favour of carbon brushes many years ago, because it was found that carbon brushes permitted of sparkless running in motors which would have sparked undesirably with copper brushes. It is probable that with the general improvement in design as regards commutation, it would now be much more practicable to employ copper brushes in many cases, but no such brush has yet been produced which compares at all favourably with the carbon brush as regards the small amount of attention required, and the great durability. The carbon brush is also superior with respect to operation where the direction of rotation is frequently reversed. While the specific resistance of a carbon brush is some four thousand times that of copper, the *contact* resistance is only ten to fifteen times greater. Different grades

[1] *Die Gleichstrommaschine*, vol. i., 1902, by E. Arnold, page 476, gives a useful summary of much experimental work on this subject. Also Parshall and Hobart's *Electric Generators*, pages 271 to 280.

of carbon and graphite brushes vary widely in specific resistance (resistance between opposite faces of a cubic centimetre of the material), but the contact resistance is much less, and may, for practical calculations, be taken at 0·2 ohm per square centimetre of bearing surface,[1] at a current density of 4 amperes per square centimetre, and decreasing at values for higher current densities rather more slowly than in proportion to the increase in current density. Graphite and carbon brushes should, in the interests of good commutation, preferably not be run at higher current densities than from 4 to 7 amperes per square centimetre, although it is claimed for some qualities that densities much higher than these values may be employed. Better satisfaction will be obtained with the lower values, although at the expense of increased loss due to mechanical friction.

Measurements of the specific resistance *of the material* of the brush are misleading. The chief consideration is that the brush *contact* resistance shall be low, and that the material and method of manufacture of the brush shall be such as to ensure smooth, quiet running, and to maintain a hard, clean, glazed commutator surface.

Table XVIII. shows at a glance that the general order of magnitude of these two components of the total loss is the same, hence it is not very necessary in the interests of a minimum total loss[2] to make any departure from the values found from other considerations to be the most desirable. These other considerations relate to commutation.

[1] Arnold, on page 481 of *Die Gleichstrommaschine* (1902), gives the following table of values for different qualities of carbon brush, which are based on the assumption that the brush contact resistance is inversely proportional to the current density :—

Le Carbone (quality X), softest quality	0·45 to 0·6 volt.
Ordinary soft quality	0·7 to 1·0 „
Harder quality	1·0 to 1·2 „
Very hard quality	1·2 to 1.5 „

At a current density of 4 amperes per square centimetre, this would give the following contact resistances, expressed in ohms per square centimetre of bearing surface :—

Softest quality, about	0·13 ohm	per sq.	centimetre.
Ordinary soft quality, about	0·21	„	„
Harder quality, about	0·28	„	„
Very hard quality, about	0·34	„	„

[2] The total loss is a minimum at that load at which the two component losses are equal.

As we have already seen, commutation will, in general, be better the greater the number of segments per pole, and the less the average voltage and the reactance voltage per segment. From this point of view the commutator should be permitted, where

TABLE XVIII.—ESTIMATION OF COMMUTATOR LOSSES WITH CARBON BRUSHES.

Current Density in Amperes per Square Centimetre.	Sum of Volts Drop at Positive and Negative Brushes, or Loss in I^2R at Brush Contact Surface expressed in Watts per Ampere.			Friction Loss at Positive and Negative Brushes, expressed in Watts per Ampere at following Peripheral Speeds in Metres per Second (Brush Pressure=0·1 Kilogramme per Square Centimetre, and Friction Coefficient = 0·3).									
				Metres per Second.									
	A	B	C	8	10	12	14	16	18	20	22	24	26
3	1·6	1·4	1·2	1·6	2·0	2·3	2·7	3·1	3·5	3·9	4·3	4·7	5·1
4	1·6	1·6	1·6	1·2	1·5	1·8	2·1	2·4	2·6	2·9	3·2	3·5	3·8
5	1·6	1·8	2·0	0·94	1·2	1·4	1·6	1·9	2·3	2·4	2·6	2·8	3·1
6	1·6	2·0	2·4	0·78	0·98	1·2	1·4	1·6	1·8	2·0	2·2	2·4	2·6
7	1·6	2·2	2·8	0·67	0·84	1·0	1·2	1·3	1·5	1·7	1·9	2·0	2·2

For A the brush contact resistance is taken as inversely proportional to the current density.

For B the brush contact resistance is taken as decreasing less rapidly than inverse proportion to the current density.

For C the brush contact resistance is taken as having for all current densities the value of 0·2 ohm per square centimetre.

B would generally be a safe value to use, A giving in practice rather too low, and C too high results for the customary working densities of from 4 to 7 amperes per square centimetre.

A brush pressure of the low value of 0·1 kilogramme per square centimetre can only be obtained with the best types of brush holders, and in stationary motors. For tramway motors, the specific pressure must be at least 50 per cent. greater.

desirable, to have rather high peripheral speed, thus being proportioned of large diameter and short. It is, however, well to decrease the friction loss to the extent permissible by the choice of fairly high current densities—say, in small motors, up to 6 or 7 amperes per square centimetre—for the brush friction loss is a constant loss at all loads, and in the interests of high efficiency at light loads it should be made as low as is consistent with good commutation. The commutator should be no longer than is necessary for obtaining the required radiating surface, and to

obtain the necessary brush bearing surface the arc of brush contact should be increased rather than the length of the commutator. When, however, with fairly high current density at the brush contact, the arc of brush bearing surface is large, additional care must be taken to so fit the brush to the commutator surface that there shall be good contact at all parts. These are the best lines on which to construct the commutator. If, with the intention of obtaining a reduced total loss by low peripheral speed, by a long commutator of small diameter and relatively few segments, the friction loss is materially reduced, there will in all probability be introduced an additional loss much greater than the saving sought, and this loss will be in sparking, not necessarily so severe as to be destructive to the commutator, or even very noticeable, but

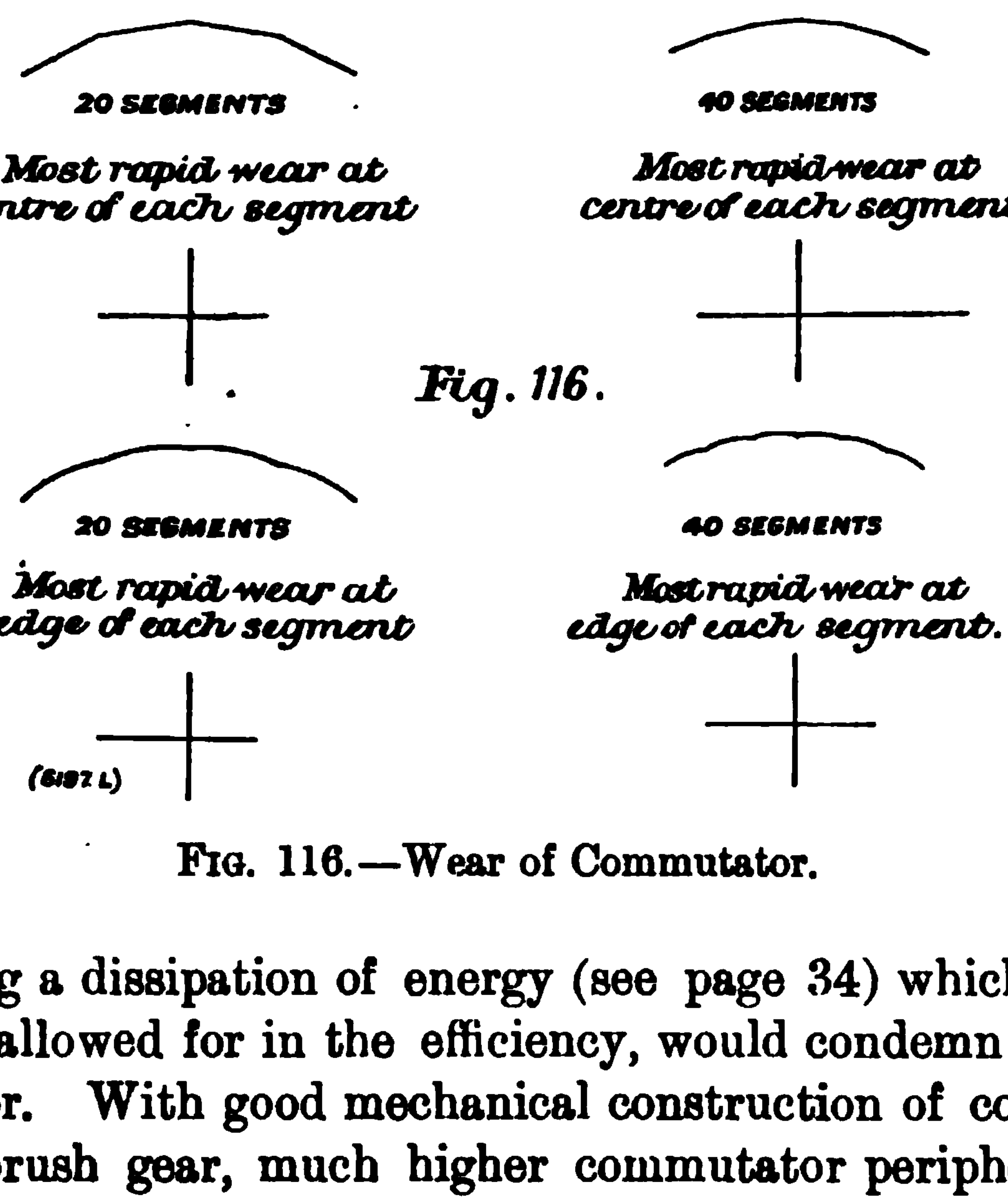


FIG. 116.—Wear of Commutator.

representing a dissipation of energy (see page 34) which, if determined and allowed for in the efficiency, would condemn the design of the motor. With good mechanical construction of commutator, shaft, and brush gear, much higher commutator peripheral speeds than are at present customary may be used with entire success. Indeed, shortness of commutator is an essential element to rigidity, and the larger the diameter the shorter need it be for a given radiating surface. The slightly increased labour involved in building up a commutator of many segments is, in view of the great gains thereby obtained, altogether negligible, and, with suitable design, the rigidity is in no wise impaired. The surface wears and remains more truly cylindrical, the greater the subdivision. This may be illustrated by Fig. 116, which shows

portions of the surfaces of two commutators with twenty and forty segments respectively, which are assumed to have a tendency, due to sparking or to the quality of the mica or the copper, to wear, in the one case more rapidly at the centre, in the other case at the edge of the segment.

Care should, of course, be exercised in selecting a soft and uniform quality of mica for the insulation between the segments. The quality of the copper segments should also be uniform, but there is no sufficient reason why its *conductivity* should be specified, as is often done, for it is the *contact* resistance which is of importance, and not the resistance of the copper itself. Copper brushes have given way to carbon brushes with four thousand times as great resistance, which carry the current continually, but the commutator segments, which carry the current intermittently, and, but for an exceedingly small fraction of the time, are often specified to be of the highest electrical conductivity. It has, in fact, never yet been demonstrated that copper is the most suitable material of which to construct commutator segments.

§ 15. **Commutator Loss.**—The use of Table XVIII. for determining the commutator loss may be illustrated by the case of a 110-volt 30 horse-power motor, whose commutator runs at a peripheral speed of 14 metres per second, and with brushes proportioned for a current density of 5 amperes per square centimetre.

$$\text{Ampere input} = \frac{30 \times 746}{110 \times \cdot 89} = 230$$

Commutator I^2R loss in watts per ampere	1·8
Commutator friction loss in watts per ampere	1·6
Total I^2R loss at commutator $= 230 \times 1{\cdot}8$	$= 413$ watts
Total friction loss at commutator $= 230 \times 1{\cdot}6$	$= 367$ watts
Total commutator loss	$= 780$ watts

Had the motor been of the same capacity, but for 440 volts, the watts per ampere would still have remained 3·4 (1·8 + 1·6), but the amperes would have been but one quarter as great (57·5 amperes) and the total commutator loss but $\frac{780}{4} = 195$ watts. Hence, while for the customary temperature increase of 40 degs. Cent., the 110-volt commutator would have required a total cylindrical radiating surface of $\frac{780}{50} = 15{\cdot}6$ square decimetres (based on a temperature increase of 0·8 deg. Cent. per watt per

square decimetre), the 440-volt motor would require but $\frac{15 \cdot 6}{4} = 3 \cdot 9$ square decimetres for an equal temperature rise.

Now, with a limited number of segments, it is relatively easier to design a 110-volt motor for few segments without having sparking, than a 440-volt motor. Hence the latter, to find room for a sufficient number of segments, should have a relatively large diameter. But as it requires but small radiating surface, it should be very short. The 110-volt motor, however, in order to have the four times greater radiating surface for its commutator without the latter being too long, should also have a large commutator diameter. Such a 30 horse-power motor works out most satisfactorily with the same diameter of commutator for all voltages, but of lengths more or less proportionate to the greater current, *i.e.*, inversely as the rated voltage.

Motors of 10 horse-power capacity and less will have, even for the lowest voltages, when designed on these lines, such short commutators that it is hardly worth while varying them in length with the voltages. It *is*, however, a great advantage, and in the interests of giving a group of machines a high rating to which those of all voltages will comply equally satisfactorily (*i.e.*, in using a minimum of material most effectively), to vary the number of segments with the voltage, even in these small motors,[1] and for motors from 15 horse-power or 20 horse-power upwards it is decidedly economical to employ active lengths of segments inversely proportional to the rated voltage, widening the armature core, as in the case of the motors of Fig. 73, p. 79, for the higher voltages, and keeping the same overall length for all voltages.

Segments are often made of considerably greater radial depth than was required for mechanical stability, the additional depth

[1] An exception is to be made in the case where manufacturers find it expedient to employ in 500-volt motors such a large number of segments as to ensure thoroughly satisfactory commutation at that voltage, *and to employ this same number*—then, except for 500 volts, unnecessarily high—for the lower voltages. On this basis it is not difficult to arrange, for the lower voltages, windings corresponding to this large number of segments. The only trouble is that, on this plan, the manufacturer will be inclined to adopt rather less than the most satisfactory number of segments for the 500-volt motor. It is highly probable that even in small motors the greatest economy is, in the end, attained by using for each voltage the number of segments and the winding best adapted to that voltage. All attempts in the other direction involve sacrifices in quality in many ways only fully appreciated by the designer.

being for the purpose of allowing for wear and for occasional turning down of the commutator. Any considerable allowance for wear should not be necessary in a well-designed motor. Radial depths of from 2 to 4 centimetres, according to the capacity of the motors, are nevertheless necessary from considerations of the mechanical design, and, for segments of any considerable axial length, even greater radial depth is sometimes required in order to avoid any outward bending due to centrifugal force.[1]

In motors of from 10 to 100 horse-power rated output, present constructions require about 3 kilogrammes of copper per square decimetre of external cylindrical radiating surface, measured from the joint of the connection of the armature winding with the commutator segment, to the outer end of the commutator segment. With really good ventilation through the inside of the commutator spider, and light brush pressure, the temperature increase need not exceed 0·8° Cent. per watt per square decimetre, hence 50 watts per square decimetre may be permitted.

§ 16. **Cost of Commutator Segments.**—At 1s. 8d. per kilogramme the cost of commutator copper is $\frac{1{\cdot}7 \times 3}{50} = 0{\cdot}10$ shilling per watt dissipated at the commutator. By means of this value, and of the constants of Table XVIII., using column B for the I^2R losses, there has been calculated, for Table XIX., the cost of the commutator copper per ampere for various peripheral speeds.

TABLE XIX.—COST OF COPPER COMMUTATOR SEGMENTS (AT 1s. 8d. PER KILOGRAMME) IN SHILLINGS PER AMPERE, AT FOLLOWING PERIPHERAL SPEEDS.

Current Density at Brush Contact in Amperes per Square Centimetre.	Metres per Second.									
	8	10	12	14	16	18	20	22	24	26
3	·30	·34	·37	·41	·45	·49	.52	·57	·60	·64
4	·26	·29	·32	·35	·38	·40	·43	·46	·49	·52
5	·23	·26	·28	·30	·33	·35	·38	·40	·42	·45
6	·21	·23	·26	·28	·30	·32	·34	·36	·38	·40
7	·20	·22	·24	·26	·27	·29	·31	·33	·34	·36

[1] The centrifugal force at the periphery is conveniently calculated by the following formula:—

Centrifugal force $= 0{\cdot}00000559\ D\ S^2$ kilogrammes per kilogramme, in which—

D = diameter in centimetres.

S = revolutions per minute.

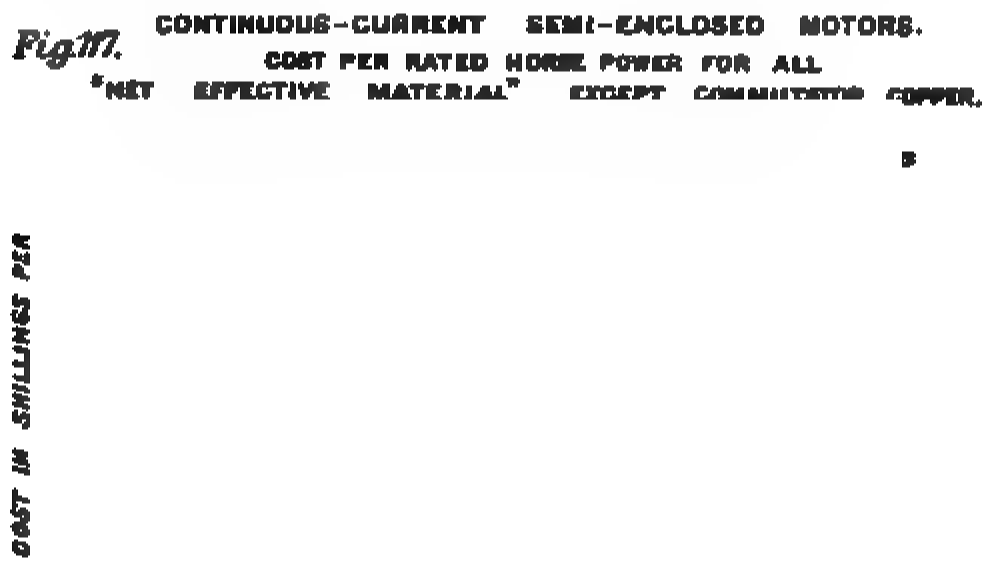
This table is very instructive, and is useful in preliminary estimates. While the cost of all the other conducting and magnetic material in the motor is a function of the horse-power rated output and speed, the commutator segments fall into an entirely different category. The commutator serves to collect the current, and its cost is practically independent of the voltage. For this reason the cost of the commutator segments constitutes a greater percentage of the total cost of effective material in the motor the lower the rated voltage. For similar reasons it forms also a greater percentage of the total cost, the higher the speed of the motor in revolutions per minute, since the current to be collected remains unchanged. In fact the commutator tends then to be more expensive, since rather higher peripheral speeds (and, consequently, higher friction losses) must be employed in order to find room for the required number of segments; the mechanical construction must also be more solid, and the larger segments must be of somewhat greater radial depth to resist bending outward under the great centrifugal force.

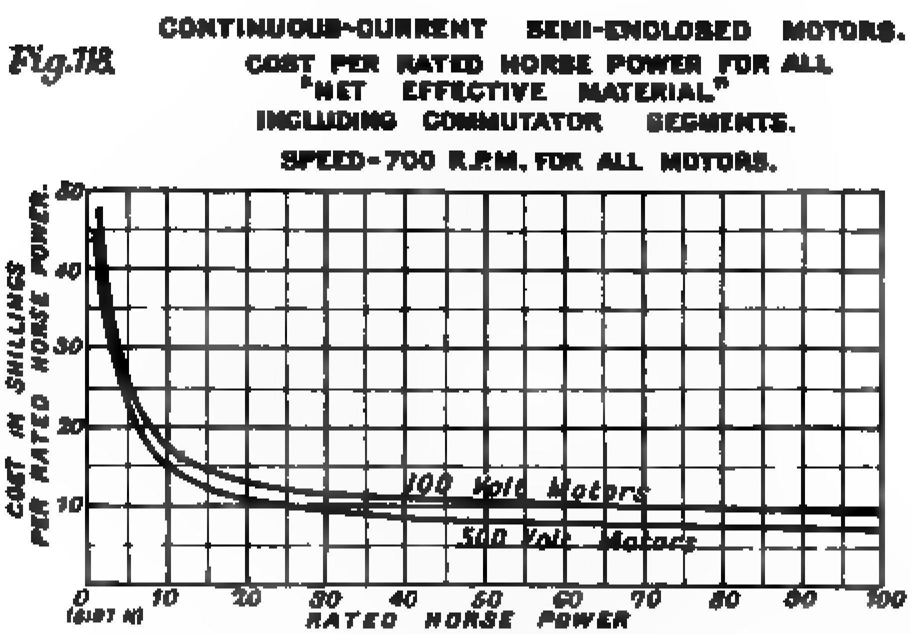
§ 17. **Cost of Field and Armature Copper.**—The component cost of armature copper, field copper, and of magnetic material, cannot be tabulated in this manner, since, for thoroughly satisfactory designs, these components may be very greatly varied. Thus, one designer may prefer an outlay of 50 for field and armature copper, and 50 for magnetic material, where another would devote but 25 to copper and 75 to magnetic material. The writer's own preference is based on the consideration that the percentage of cost of field and armature copper to cost of total "effective material," exclusive of commutator copper, should be higher the lower the rated voltage, because the amount of insulation associated with the use of each kilogramme of copper is very much less the lower the rated voltage. These considerations, in their full application, lead to such proportions for different voltages as have been shown in Fig. 73, page 79, and Table IV., page 78.

The total of these costs, *i.e.*, field copper, armature copper, and effective material in the magnetic circuit, may, for ordinary semi-enclosed shunt motors, be taken roughly, as shown in the curve of Fig. 117. For the two curves of Fig. 118, for 500-volt and 100-volt motors, the cost of the commutator segments is included. These curves show very clearly that while there would be scarcely any economy in using different lengths of segments for the different voltages for motors of less than, say, 7½ horse-power, the difference in cost of net effective material is at 30 horse-power

already some 15 per cent., and rapidly increases with higher capacities. This margin amply provides for the greater cost for insulation and for the labour of winding the many turns of fine wire in the higher voltage motors, and for motors of more than 15 horse-power capacity the higher voltage motors should cost the least.

All such figures must necessarily be only very rough estimates,





Figs. 117 and 118.

but the important point to be understood is the influence of the cost of the commutator segments upon the basis of design.

The curve of Fig. 119 gives, for 700 revolutions per minute, semi-enclosed shunt motors, the approximate weight per rated horse-power. The weights vary considerably with different manufacturers, and, of course, the manufacturing costs vary to a still greater extent, but the values given in Fig. 120 give a rough

idea of the range of variation in the ratio of the total cost of labour and material to the cost of net effective material for 250-volt, 700 revolutions per minute, semi-enclosed motors.

The amount by which such motors must be rated down, in order not to exceed permissible temperature limits on continuous running as totally enclosed motors, depends mainly upon the ratio

Fig.119.

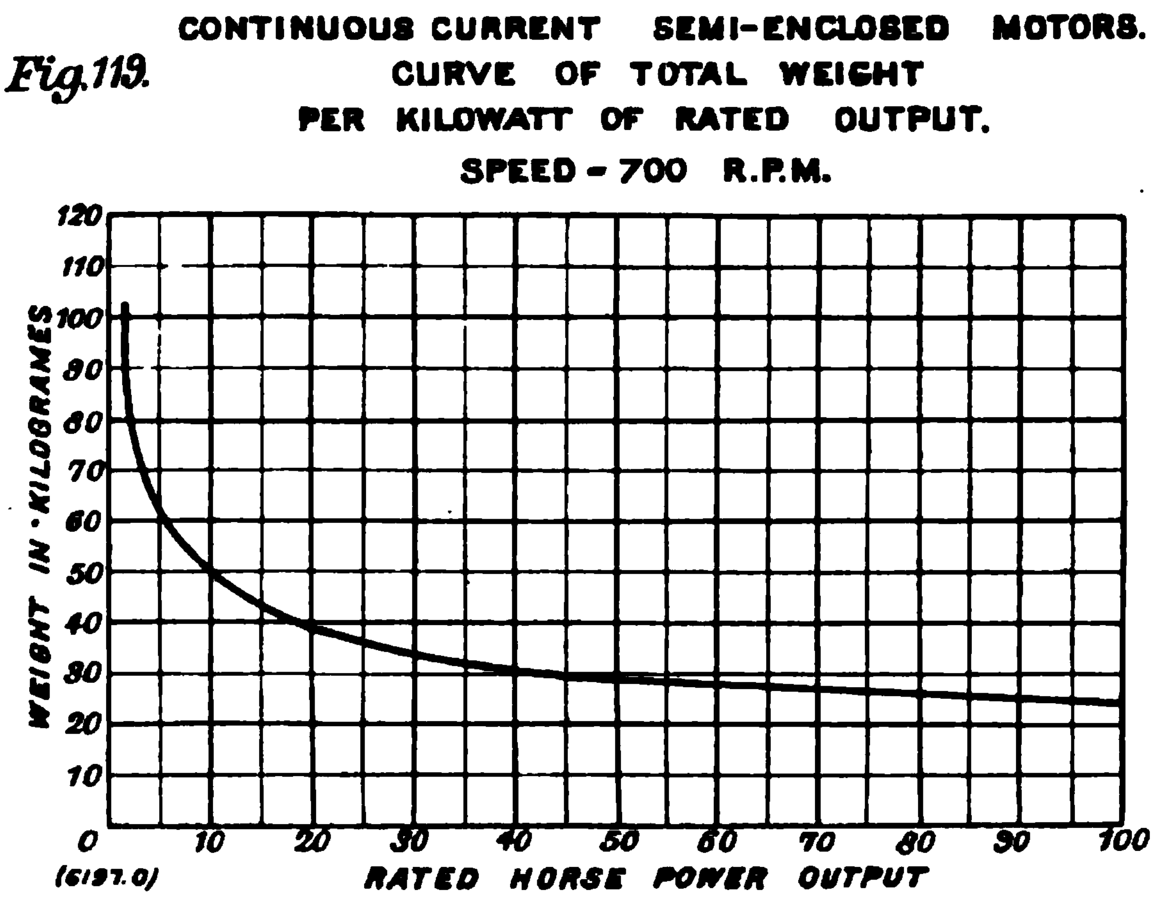
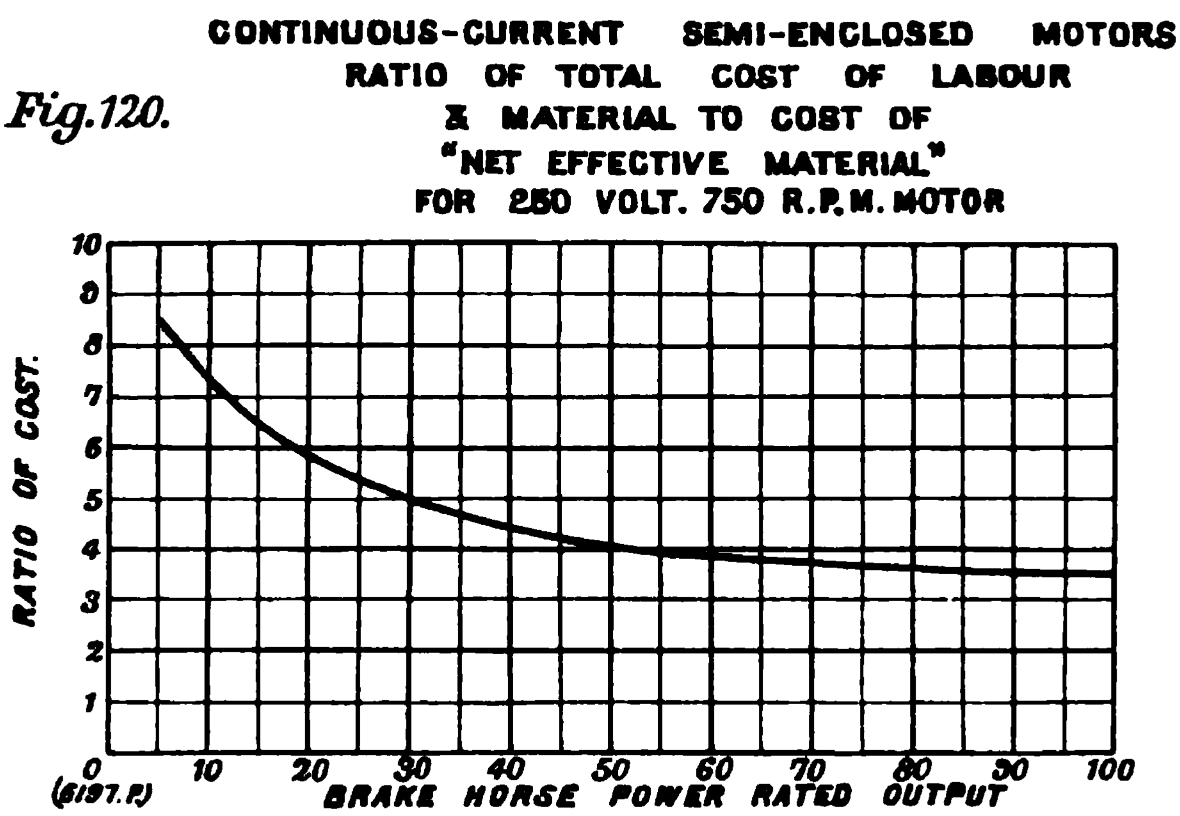


Fig.120.



FIGS. 119 and 120.

of the "constant" and "variable" losses, as already explained on pages 13 to 15. Motors of different manufacturers at present vary greatly in this respect, although all employ undesirably and needlessly high values for this ratio, and hence such totally enclosed motors are, for a given output, either needlessly expensive, or else they run undesirably hot.

CHAPTER VII

STANDARDISATION OF ELECTRIC MOTORS

§ 1. **The Verband Deutscher Elektrotechniker Rules.**—The *Verband Deutscher Elektrotechniker* in 1902 issued a set of standardising rules for electrical machinery. While these may be taken as setting forth the limits within which manufacturers in Germany, and probably also fairly generally in Austria, Switzerland, Belgium, and Sweden, will undertake to guarantee their apparatus, it does not necessarily follow that they will deem it expedient to take full advantage of the rather high temperature limits permitted, the extremely low prescribed insulation tests, and the rather vague requirements in regard to commutation. In fact, the writer's observations have shown him that, while this machinery is rated higher than conforms to the more strict requirements of English and American practice, the ratings are generally considerably more conservative than would be indicated by the rules adopted by the *Verband Deutscher Elektrotechniker.* So far as they relate to continuous current motors, the standardising rules are in substance as follows:—

Motors are divided into three classes—

A.—For intermittent work, where the periods of work and rest alternate every few minutes, as for cranes, lifts, tramways, etc.

B.—For work lasting for such a short time that the final temperature corresponding to continuous running at full load is never reached, and where the pauses are sufficiently long to permit the temperature to fall to approximately that of the surrounding air.

C.—For continuous running, where the working period is sufficiently long to cause the motors to reach a constant temperature.

For Class A the rating of the motors should be that output

which, maintained for one hour, will not cause a greater temperature increase than that specified in the clause relating thereto.

For Class B this same temperature increase must not be exceeded when the motor is operated continuously for the specified time at the rated load.

For Class C the rating must not exceed that giving the permissible temperature increase when the motor is operated at its rated load until the attainment of constant temperature or until the expiration of ten hours.[1]

Temperature Increase.—When the temperature of the surrounding air does not exceed 35° Cent., the increase in temperature above the air shall not exceed the following amounts:—

For windings on moving parts—

	Degs. Cent.
Where cotton insulation is employed	50
Where paper insulation is employed	60
Where mica or asbestos insulation is used	80

For windings on stationary parts the temperature increase may be permitted to be 10° Cent. higher than the above values. For the iron in which windings are embedded, the same temperature limits as for the windings themselves are permitted.

For commutators the temperature increase must not exceed 60° Cent.

For field spool windings the temperature increase is to be determined by resistance measurements on the assumption of an increase of resistance of 0·4 per cent. per degree Centigrade.

For all other parts the temperature increase is to be determined by thermometric measurements.

Where insulations are made up from more than one of the above described materials, the lower limiting temperature shall not be exceeded.

For tramway motors the temperature increase shall not exceed the values specified in the following:—

The windings and iron of both moving and stationary parts:—

	Degs. Cent.
Where cotton insulation is employed	70
Where paper insulation is employed	80
Where mica or asbestos insulation is used	100
Commutator	80

Commutation.—With the brushes well fitted and set in the

[1] The temperature may still be increasing at the end of ten hours.

most favourable position, the motor must run at all loads with sufficient freedom from sparking not to require treating the commutator with sandpaper or its equivalent more than once per twenty-four hours.

Overloads.—In practical operation motors shall only be required to carry overloads for so short a time, or when at such temperatures that the permissible temperature increase shall not be exceeded. With this limitation they shall be capable of sustaining the following overloads:—

25 per cent. overload for one half-hour.
40 per cent. overload for three minutes.

The commutation at these overloads shall be such as not to require any departure from the conditions specified in the preceding general clause relating to commutation.

Motors required to run at various speeds by field regulation shall not be required to withstand overloads.

Insulation.—Continuous current motors are to be tested for one half-hour, when warm from windings to frame, with twice normal voltage. When these tests are made with an alternating current, the R.M.S. voltage need be but 1·4 times the normal continuous current voltage of the motor.[1]

§ 2. **The American Institute of Electrical Engineers' Rules.**—The recommendations of the American Institute of Electrical Engineers are, in most particulars, rather more exacting. So far as they relate to continuous current motors, they are, in substance, to the following effect:—

Rise of Temperature.—The temperature of field and armature,

[1] In the *Elecktrotechnik Zeitschrift* for October 3, 1902, page 839, Dettmar explains that these rules of the *Verband Deutscher Electrotechniker* correspond only to the highest permissible rating, and that the quality of electrical machinery manufactured in Germany is not surpassed by that of other countries. He states that "all good German firms give much higher guarantees than those prescribed by the *Verband Deutscher Elektrotechniker*, and the following specification is almost universally complied with :—

"25 to 30 per cent. overload for three hours without harmful sparking or heating.

"Thermometrically determined temperature increase of not over 35° to 40° Cent.

"Ability to withstand momentary overloads of 100 per cent. without harmful sparking or heating.

"Constant brush position for all these conditions.

"In many cases still higher guarantees are undertaken, and the machines are, as a rule, considerably better than required by the guarantees."

as determined by resistance measurements, should, with rated full load, not exceed an increase of 50° Cent. above that of the surrounding air; that of commutator, by thermometer, 50° Cent.; and that of bearings and other parts of motor, by thermometer, 40° Cent.

When a thermometer applied to a coil or winding indicates a higher temperature elevation than that shown by resistance measurement, the thermometer indication should be accepted.

In the case of apparatus intended for intermittent service, the temperature elevation which is attained at the end of the period corresponding to the term of full load shall not exceed 50° Cent. by resistance in electric circuits.

Insulation.—Insulation resistance tests should, if possible, be made at the pressure for which the apparatus is designed. The insulation resistance of the complete machine must be such that the rated voltage of the apparatus will not send more than $\frac{1}{1,000,000}$ of the full load current, at the rated terminal voltage, through the insulation. Where the value found in this way exceeds one megohm, one megohm is sufficient.

Dielectric Strength.—The dielectric strength or resistance to rupture should be determined by a continued application of an alternating electromotive force for one minute, and should be made from windings to frame at the following R.M.S. voltages:—

Rated Terminal Voltage.	Rated Output. B.H.P.	Testing Voltage (R.M.S. Volts).
Not exceeding 400	Under 15	1000
Not exceeding 400	15 and over	1500
From 400 to 800	Under 15	1500
From 400 to 800	15 and over	2000

Overload Capacities.—Motors should be able to carry a reasonable overload without self-destruction by heating, sparking, mechanical weakness, etc., and with an increase of temperatures not exceeding 15° Cent. above those specified for full loads.

The following overload capacities are recommended:—

25 per cent. for one half-hour,
50 per cent. for one minute,

except in railway motors and other apparatus intended for intermittent service.

More severe requirements than those recommended by either

of these two associations are often exacted both in England and America, and serve a good purpose in keeping up the standard.

The writer—in the interests of sound construction—considers the following insulation tests to be desirable for continuous current motors:—

Rated Voltage.	Guaranteed Insulation Test from Copper to Iron at 60° Cent. for One Minute.
100	2000 R.M.S. volts.
200	2400 " "
400	3000 " "
600	3600 " "

As to temperature rise and sparking, these points may, in the writer's opinion, be briefly and yet satisfactorily covered by the following guarantee:—

Twenty-five per cent. overload during one half-hour, without harmful sparking or heating. Thermometrically measured temperature increase not over 50° Cent. above surrounding atmosphere during continuous operation at rated load. No harmful sparking or heating with momentary overloads of 50 per cent. Fixed brush position for all these conditions.

As already stated, the basis of rating of railway motors—independently of the recommendations of associations—has become quite generally that the determined thermometric temperature rise of the hottest accessible part, as tested on a testing stand for one hour at the rated nominal capacity, shall not exceed 75° Cent. above the surrounding air.[1]

It is necessary to keep in mind these methods prevailing in different countries, in order to draw any intelligible conclusions in comparing the weights and costs of different motors.

§ 3. **Weights of Shunt Motors.**—In Fig. 121 are reproduced curves of weights of continuous current motors as given by others. Curve A is taken from the *Zeitschrift fur Elektrotechnik*, vol. xix. (1901), page 246, from an article by Seefehlner. Curve B is from page 910 of vol. xviii. (1901), of the *Transactions of the American Institute of Electrical Engineers*, and was communicated by Golds-

[1] On standardising rules, see also *Proceedings of the American Institute of Electrical Engineers*, vol. xv. (1898), pages 3 to 32; vol. xvi. (1899), pages 255 to 268; *Transaction and Transmission*, vol. i., No. 1 (1900)—article by Mr Parshall on "Standardisation of Electrical Apparatus"; *Elektrotechnik Zeitschrift* (1900), page 727—Dettmar on "Standardisation"; *Elektrotechnik Zeitschrift* (1900), page 1058—Oelschlager on the subject of the rating of intermittently loaded machinery; *Elecktroteclnik Zeitschrift* (1901), page 499—Dettmar on "Standard Tests for Electrical Machinery."

borough. In both cases the authors were comparing the weights with those of corresponding induction motors, and consequently

Fig. 121. CURVES OF WEIGHTS OF CONTINUOUS-CURRENT MOTORS.
Curve A.–Zeit. für Elek.–Vol. XIX. (1901) p. 246,– Article by Seefehlner: Represents average of many Continuous Current Shunt motors from 15 manufacturers in several different countries. Speeds not stated.
Curve B.– Transac. Am. Inst. Elec. Eng.– Vol. XVIII (1901) p. 910, Communication by Goldsborough & relates to moderate Speed Shunt Motors.

to

(6191.Q) HORSE POWER, RATED OUTPUT.

Fig. 121.

were justified in not stating the speeds corresponding to the motors in question. This, nevertheless, detracts from the usefulness of

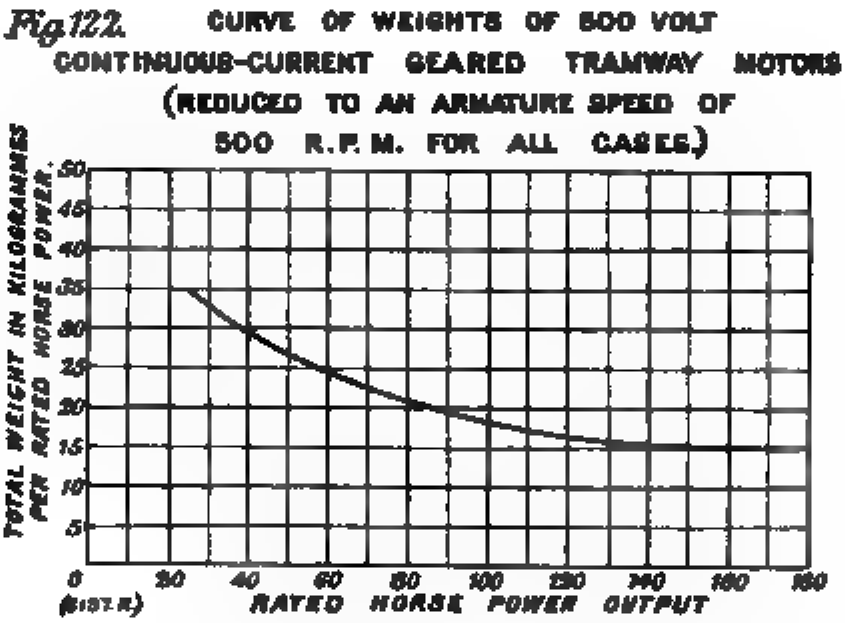


Fig. 122.

the curves. Curve C, also from Seefehlner's article, gives similar data for tramway motors.

§ 4. **Weights of Series Motors.**—In Fig. 122 will be found

a curve giving the weights of continuous current 500-volt tramway motors in kilogrammes per rated horse-power, reduced to the uniform basis of 500 revolutions per minute of the armature, which is about the speed generally employed for geared tramway motors at their full load rating. The basis of the rating is that given above.

CHAPTER VIII

EXAMPLES OF MOTOR DESIGNS BY DIFFERENT MANUFACTURERS

§ 1. Data and Tests of a Ludvika 20 H.P. 500-volt Shunt Motor.—The illustrations, Figs. 123 to 128 (pp. 121, 123, and 125), and Fig. 129 (see Plate 5), are of a four-pole 20 horse-power shunt motor, for 700 revolutions per minute and 500 volts, built by the Elektriska Aktiebolaget Magnet of Ludvika, Sweden, from the designs of Mr. Aubrey V. Clayton, who has kindly permitted the writer to publish the particulars of the machine, as arranged by him in the form set forth in Table XX.

TABLE XX.—SPECIFICATION AND CALCULATION FOR FOUR-POLE, 20 HORSE-POWER, 700 REVOLUTIONS PER MINUTE, 500-VOLT DIRECT CURRENT SHUNT-WOUND MOTOR.

Core, diameter (outer)	340 millimetres
Core, diameter (inner)	155 "
Number of slots	83 "
Conductors per slot	12 "
Style of winding	2 circuit single
Turns in series	$\frac{6 \times 83}{2} = 249$
Flux (480 internal volts) (M)	2·08 megalines
	$(480 = 4 \times 249 \times \frac{700 \times 2}{60} \times M \times 10^{-8})$
Full load current input	33 amperes
Armature ampere turns per pole	$\frac{248}{2} \times \frac{33}{2} = 2050$
Size of conductor, bare	2·2 millimetres diameter
Size of conductor, D.C.C.	2·52 " "
Density in conductor...	$\frac{33}{2(2{\cdot}2^2 \times {\cdot}7854)} = 430$ amperes per sq. cm.
Size of slot	7·3 mm. × 20 mm.
Ratio of width to depth	·365
Width of tooth at face	5·6 millimetres
Width of tooth at root	4·0 "

Mean width of tooth	4·8 millimetres
Ratio of mean tooth width to slot width ...	·66
Average length of pole arc	187 „
Effective length of armature laminations ...	145 „
Average density in teeth	$\frac{2\cdot08}{16(4\cdot8\times145)}=$ 18·7 kilolines per sq. cm.
Length of arm. between flanges	$145\times1\cdot10$ (insulation) plus 2 air-ducts each 6 mm. wide = 170 millimetres
Density at pole face	$\frac{2\cdot08}{187\times170}=$ 6·5 kilolines per sq. cm.
Density at magnet core (cast steel)	$\frac{2\cdot8\times1\cdot12}{147^2\times\cdot7854}=$ 13·8 kilolines per sq. cm.
Density at magnet yoke (cast steel) ...	$\frac{2\cdot08\times1\cdot2}{2(110\text{sq.cm})}=$ 10·6 kilolines per sq. cm.
Density at armature core	$\frac{2\cdot08}{2(145\times72\cdot5)}=$ 10·0 kilolines per sq. cm.

Calculation of Field Ampere Turns:—

	Length.	Density in Kilolines per Sq. Cm.	Ampere Turns
Armature core	9 cm.	10·0	55
Armature teeth	2 cm.	17·8 (corrected value)	320
Air-gap in centre	3·5 mm.	6·5	2050
Air-gap at pole tip	7·0 mm. Aver. 4 mm.		
Magnet core	15 cm.	13·8	240
Magnet yoke	25 cm.	10·6	200
Sum	...	...	2865
Further allowance	...	...	300
Calculated total ampere turns ...	...	...	3165

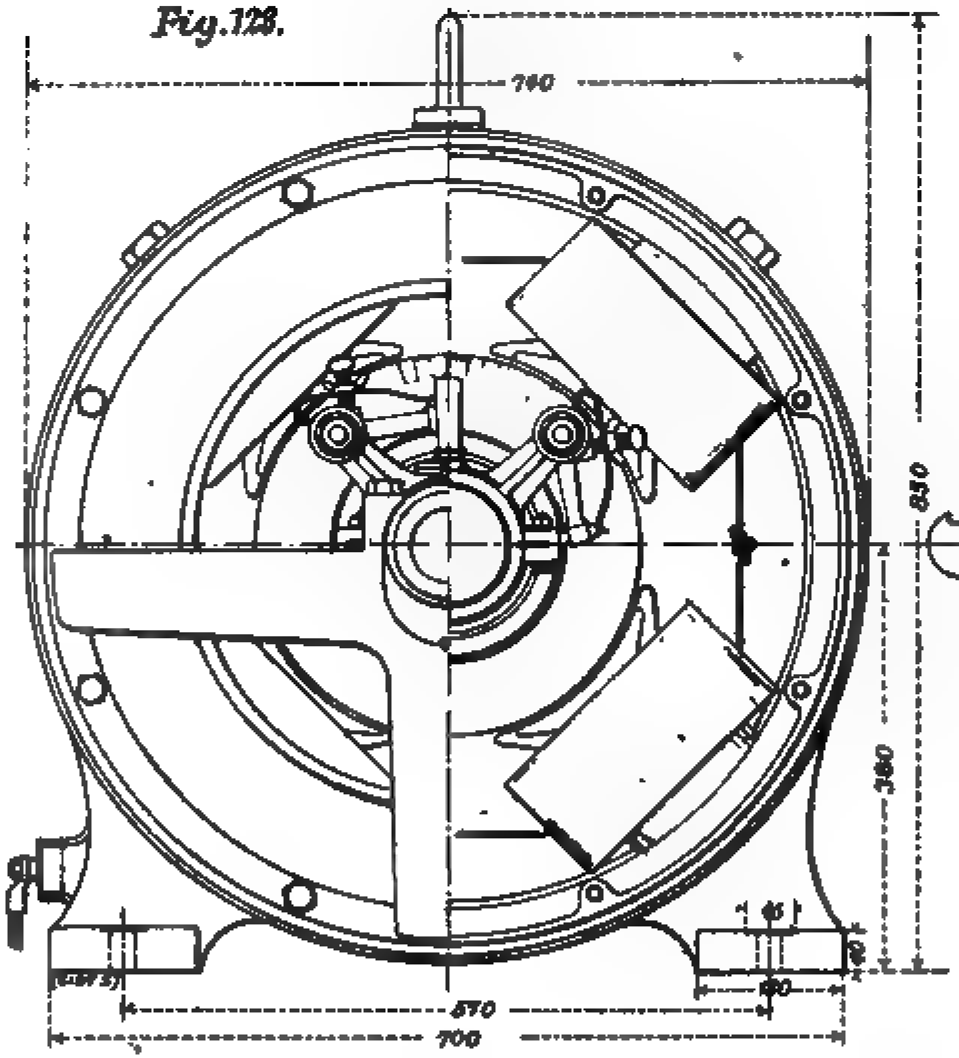
Field Spool Calculation:—

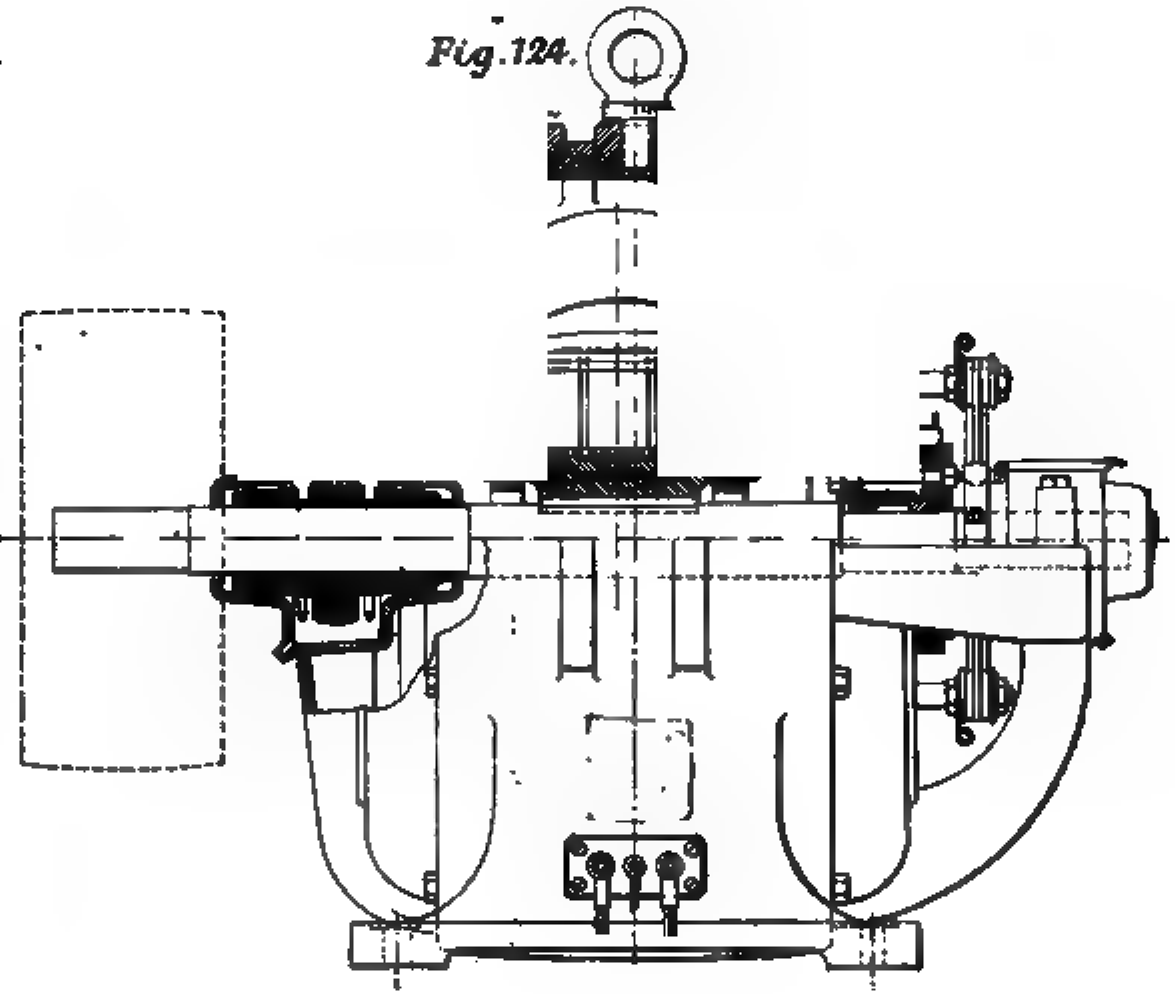
Permissible loss for excitation = 1·7 per cent. of output = 250 watts, or $\frac{250}{4}=62\cdot5$ watts per spool.

Therefore amperes $=\frac{250}{500}=0\cdot5$; and turns per spool $\frac{3165}{0\cdot5}=6330$.

Resistance per spool must be $\frac{500}{0\cdot5}\div4=250$ ohms when warm, or, allowing for 40° Cent. rise, 215 ohms at 20° Cent.

On the assumption that 8·5 watts can be radiated per square decimetre of the outer cylindrical surface of the spools, $\frac{62\cdot5}{8\cdot5}=7\cdot4$ square decimetres are required. The magnet core diameter is 147





FIGS. 123 and 124.—120 Horse-power Shunt Motor, designed by A. V. Clayton, Ludvika, Sweden.

millimetres, and adding to this 5 millimetres for insulation, and 70 millimetres for winding, gives an outer circumference of

$$\pi\,(147+5+70)=700 \text{ millimetres},$$

hence the winding space must be $\dfrac{74000}{700}=105$ millimetres long.

As 1 cubic centimetre Cu has, at 20° Cent., a resistance of ·00000174 ohm, area of copper in winding will be

$$\frac{\cdot 00000174\times 6330\times\pi(147+5+35)}{215}=0\cdot 03 \text{ sq. millimetre.}$$

Wire 0·65 millimetre diameter, with an area of 0·033 square millimetre, may be taken. This has a diameter of 0·77 millimetre single cotton covered, and can be wound in forty-six layers of 142 turns each, the space required being 115 × 38 millimetres. The measured resistance of spool is 208 ohms at 20° Cent.

Commutator and Commutation Calculation:—

Diameter	200 millimetres
Number of segments	165

Average volts per segment $500\div\dfrac{165}{4}=12\cdot 2$

Breadth of segment at face $\dfrac{\pi\times 200}{165}-0\cdot 7$ (insulation) $=3\cdot 1$ mm.

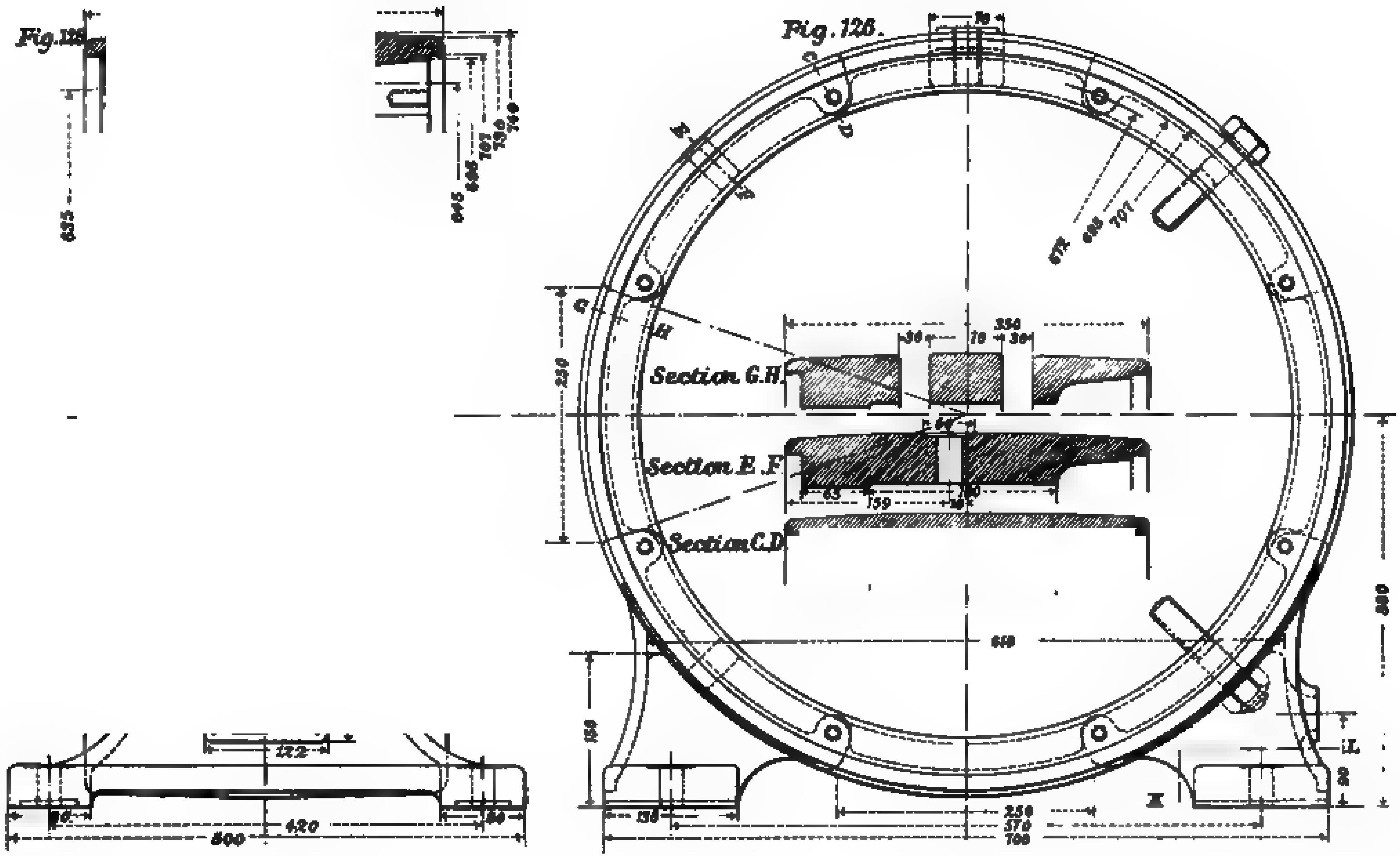
Breadth of brush face on arc 11 mm. (brush is 10 mm. thick)

In one of the slots there is only one coil connected, the ends of the other being insulated, and the coil used only as a dummy or filler.

To explain the basis of the calculation of the frequency it is necessary to mention that the brushes short circuit three coils. Therefore the time required for complete reversal of the current is that taken by any segment to move across the brush face until it leaves the opposite brush corner; or the time required for a given point on the commutator to travel the distance of the brush are, plus the thickness of one commutator segment (11 + 3·1 = 14·1 mm.). Hence, frequency in cycles per second is,

$$\frac{200\pi\times 700}{2\times 14\cdot 1=60}=258$$

Coils short circuited per brush	3
Turns per coil	3
Conductors in each group simultaneously undergoing commutation	18
Flux set up per ampere turn, per centimetre of gross length	9 c.g.s. lines
Gross length	17 centimetres



FIGS. 125 and 126 —20 Horse-power Shunt Motor, designed by A. V. Clayton, Ludvika, Sweden.

Flux set up per ampere turn	153 lines
Flux per ampere	$18 \times 153 = 2750$ lines
Linkage per ampere	$3 \times 2750 = 8250$ lines
Inductance	·0000825 henry
Reactance	$\cdot 0000825 \times 2\pi \times 258 = \cdot 134$ ohm

But this is the reactance of the short-circuited coils when four sets of brushes are employed. With this motor but two sets of brushes were used, and the reactance was hence that of two coils in series, or ·268 ohm.

Amperes per conductor	16·5
Reactance Voltage	$16 \cdot 5 \times \cdot 268 = 4 \cdot 4$ volts
Brushes per spindle	2 of 25 mm. × 10 mm.
Average density at brush face	6 amps. per sq. cm.
Resistance at positive and negative brushes	·074 ohm
I^2R loss at positive and negative brushes	$33^2 \times \cdot 074 = 80$ watts
Peripheral speed of commutator	7·3 metres per sec.
Watts lost in friction...	22
Assumed stray watts...	10
Total loss in commutator	112 watts
Effective length, commutator	75 millimetres
Watts per square decimetre of commutator surface	23

The machine with three segments backward lead of brushes stands 25 per cent. overload without sparking, no movement of the brushes being required from no load up to 25 per cent. overload.

After six hours' run as generator, with 30 amperes output, at 740 revolutions per minute, the following final temperatures were observed by thermometer :—

Heating :—

	Ultimate Temperature in Degs. Cent.	Degs. Cent. Rise.
Field spool	39	23
Magnet yoke	32	16
Pole face	40	24
Armature	42	26
Commutator	37·5	21·5
Bearing (pulley side)	39	23
Bearing (commutator side)	36	20
Room temperature	16	...

These low values were in large measure due to the ventilation afforded by the ducts through the magnet yoke, which secured a very thorough circulation of air.

Efficiency:—

	Watts.
Measured no load losses (core loss, brush and bearing friction, and windage)	700
Armature I^2R (resistance measured 0·58 ohm at 20° Cent.) $33^2 \times 0{\cdot}65$	710
Field losses	245
Commutator I^2R and stray losses (calculated)	90
Total losses	1,745
Total output, $20 \times 736 =$	14,720
(Using contintental H.P. = 736 watts)	
Total input	16,465

Commercial efficiency at full load, 89·5 per cent.

Fig. 128.

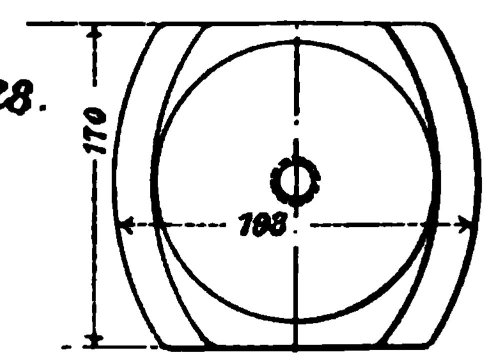
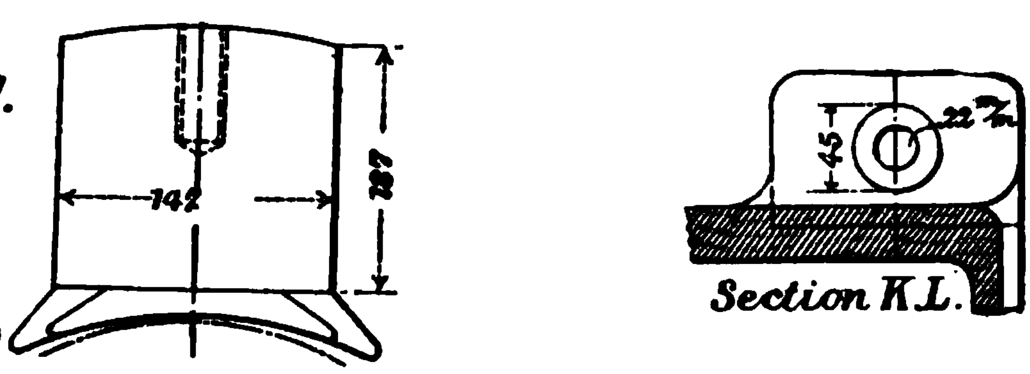


Fig. 127.



FIGS. 127 and 128.—20 Horse-power Shunt Motor, designed by A. V. Clayton, Ludvika, Sweden.

The four-pole, 35 horse-power, 950 revolutions per minute, 440-volt shunt motor shown in the engraving, Fig. 130, Plate 5, is developed from the 20 horse-power motor just described, the same yoke, press flanges, bearings, and shaft being used. The bearings and brackets are also the same as for the firm's induction motors. The heating constants are higher, but this is offset by the better ventilation afforded by the higher speed.

§ 2. **Data on 35 H.P. 220-volt Shunt Motor.**—The 25 horse-power, 950 revolutions per minute motors, described in Table XXI., differs from that shown in Fig. 130, Plate 5, only in the commutator and windings, which are for 220 volts. The same bearing brackets were used, the bar winding for the 220-volt

armature taking up less space than the 440-volt coil winding, and hence permitting the extra room required for the longer commutator.

TABLE XXI.—SPECIFICATION AND CALCULATION FOR FOUR-POLE, 35 HORSE-POWER, 950 REVOLUTIONS PER MINUTE, 220-VOLT DIRECT CURRENT SHUNT-WOUND MOTOR.

Armature:

Core diameter (outer)	343 millimetres
Core diameter (inner)	155 „
Number of slots	71
Conductors per slot	8
Style winding...	Multiple circuit single
Turns in series	71
Flux (210 internal volts)	2·35 megalines
Full load current input	129 amperes
Armature ampere turns per pole	$\frac{71 \times 129}{4} = 2280$
Size of conductor, bare	$1{\cdot}3 \times 9$ millimetres
Density in conductor...	275 amperes per sq. cm.[1]
Resistance of armature winding at 20° Cent. (measured)	·0255 ohm
Size of slot	$8{\cdot}5 \times 23$ millimetres
Ratio width slot to depth of slot	·37
Width tooth at face	6·6 millimetres
Width tooth at root	4·7 „
Mean width tooth	5·65 „
Ratio tooth width to slot width	·66
Average length pole arc	187 „
Effective length of armature laminations ...	150 „
Density in teeth	$\frac{2{\cdot}35}{14(5{\cdot}65 \times 150)} = 19{\cdot}8$ kilolines
Length of armature between flanges ...	$150 \times 1{\cdot}09 + 1$ duct, 6 millimetres wide = 170 mm.
Density at pole face	$\frac{2{\cdot}35}{187 \times 170} = 7{\cdot}4$ kilolines
Density at magnet core	$\frac{2{\cdot}35 \times 1{\cdot}11}{155^2 \times {\cdot}785} = 14{\cdot}4$ „
Density at magnet yoke	$\frac{2{\cdot}35 \times 1{\cdot}11}{2 \times 110} = 11{\cdot}9$ „
Density at armature core	$\frac{2{\cdot}35}{2 \times (150 \times 72)} = 10{\cdot}8$ „

[1] The 440 and 500-volt motors of this size have two-circuit wire-wound armatures with current densities of about 450 amperes per square centimetre, the extra loss thereby entailed being offset by the much lower losses at the commutator.

Magnetic Circulation:—

	Length.	Density in Kilolines per Sq. Cm.	Ampere Turns.
Armature core	9·0 cm.	10·8	60
Armature teeth	2·3 cm.	19·0 (corrected)	600
Air-gap in centre	3·5 mm.		
Air-gap at pole tip	7·0 mm.	7·4	2220
Average air-gap	3·8 mm.		
Magnet core	17·0 cm.	14·4	420
Magnet yoke	28·0 cm.	11·9	320
			3620
Further allowance			380
Calculated total ampere turns			4000

Field Spool:—

Number of turns 4100

Size of conductor { bare diameter ... ... 1·10 millimetres; single cotton covered diameter ... ... 1·23 „

Commutator:

Diameter	200 millimetres
Number of segments	142
Reactance volts	2·7
I^2R loss	200 }
Friction	160 } =400 watts total loss
Stray losses	40 }
Effective length of commutator	152 millimetres
Watts per square decimetre of cylindrical surface of commutator	$\frac{400}{\pi \times 2{\cdot}0 \times 1{\cdot}5} = 43$

Brushes:—

Number of spindles	4
Brushes per spindle	4
Dimensions of brushes	16 × 25 millimetres

Efficiency:—

Measured core loss, brush and bearing friction, and windage (average of several machines)	1,260
Armature I^2R, $129^2 \times 0{\cdot}0295 =$	490
Field losses from saturation curve	220
Commutator I^2R and stray losses (calculated)	240
Total losses (watts)	2,210
Watts output (35 × 736) = ...	25,750
Watts input	27,960
Commercial efficiency = ...	92·2 per cent.

Tests.—Machine runs from no load to 25 per cent. overload with fixed brushes at two segments backward lead, sparklessly. No complete heating tests were carried out, the only observed temperatures at end of two and a half hours being: Pole-shoe, 30° Cent. rise; commutator, 35° Cent. rise; armature, 32° Cent. rise.

Weight of machine complete = 764 kilogrammes.

§ 3. **Test Results of Ludvika 20 H.P. and 35 H.P. Motors.**—In Figs. 131 and 132 are given saturation curves, and in Figs. 133 and 134 curves of efficiencies and losses for the two motors.

§ 4. **Enclosed Motors of 5 H.P. and 30 H.P.**—As examples of enclosed motors, two designs by Mr Henry A. Mavor (of the firm of Mavor & Coulson, Glasgow) will be described. They are shunt-wound motors of 5 brake horse-power and 30 brake horse-power normal rated capacity respectively, and have cast-steel frames. Drawings of the 5 brake horse-power motor are given in Figs. 135 and 136, page 130, and of the 30 brake horse-power in Figs. 137 to 142, Plate 6. Fig. 143, Plate 7, is from a photograph of the 30 brake horse-power motor.

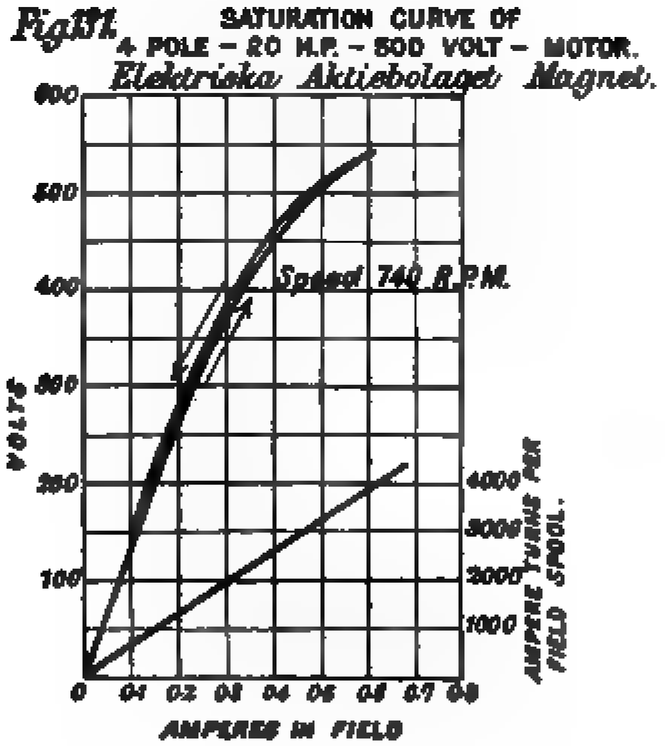
TABLE XXII.—SPECIFICATION OF ENCLOSED MOTORS.

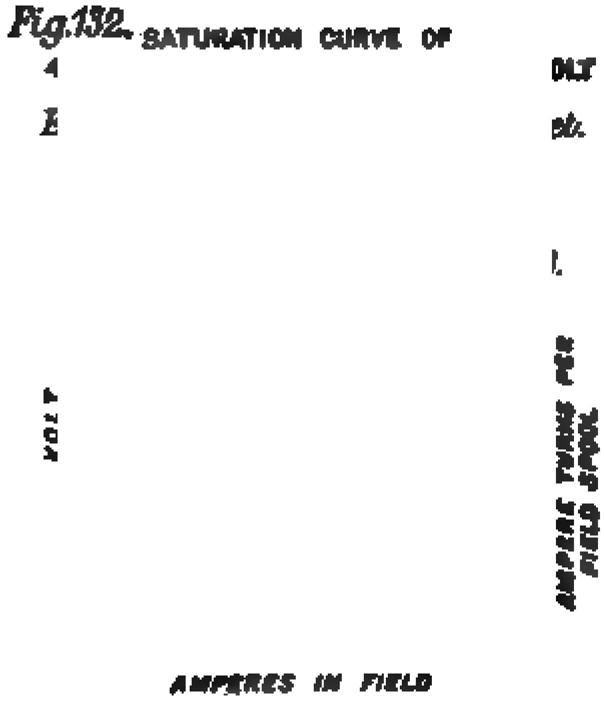
	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Powe .
Speed in revolutions per minute	1000	600
Periodicity in cycles per second	33·3	20
Voltage	250	220
Amperes input at full load	18·8	113
Amperes input at no load	3	9·5
Watts imput at no load	750	2090

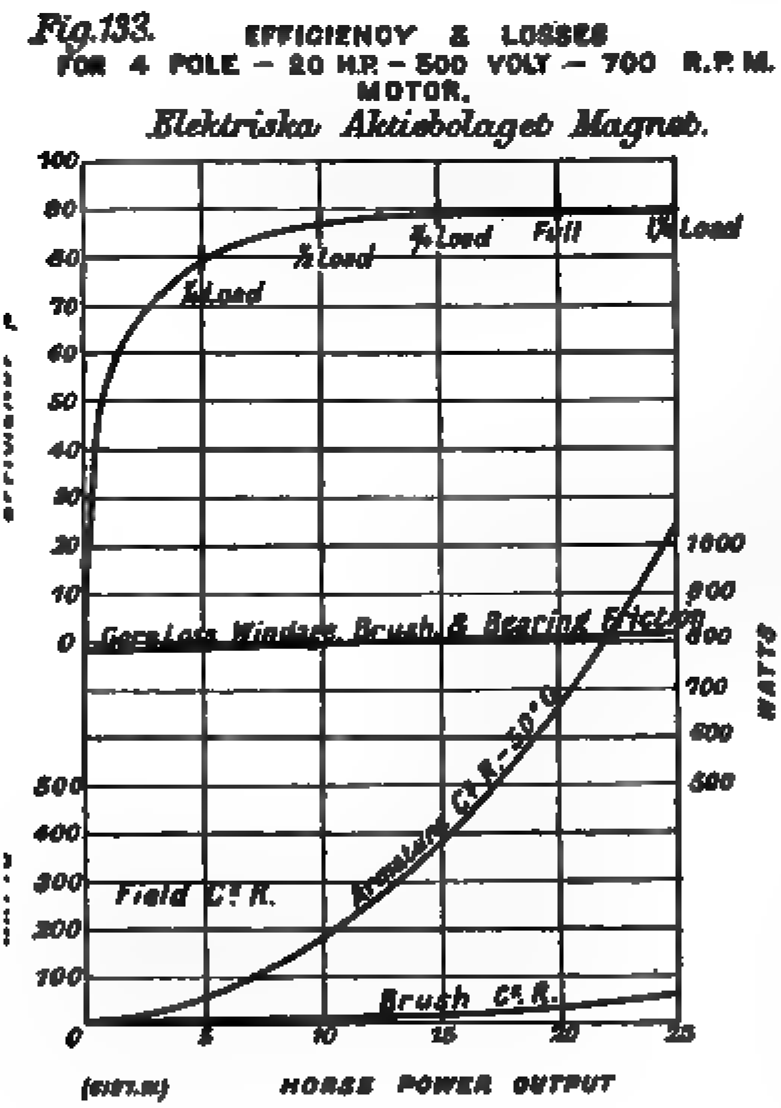
(Dimensions in millimetres.)

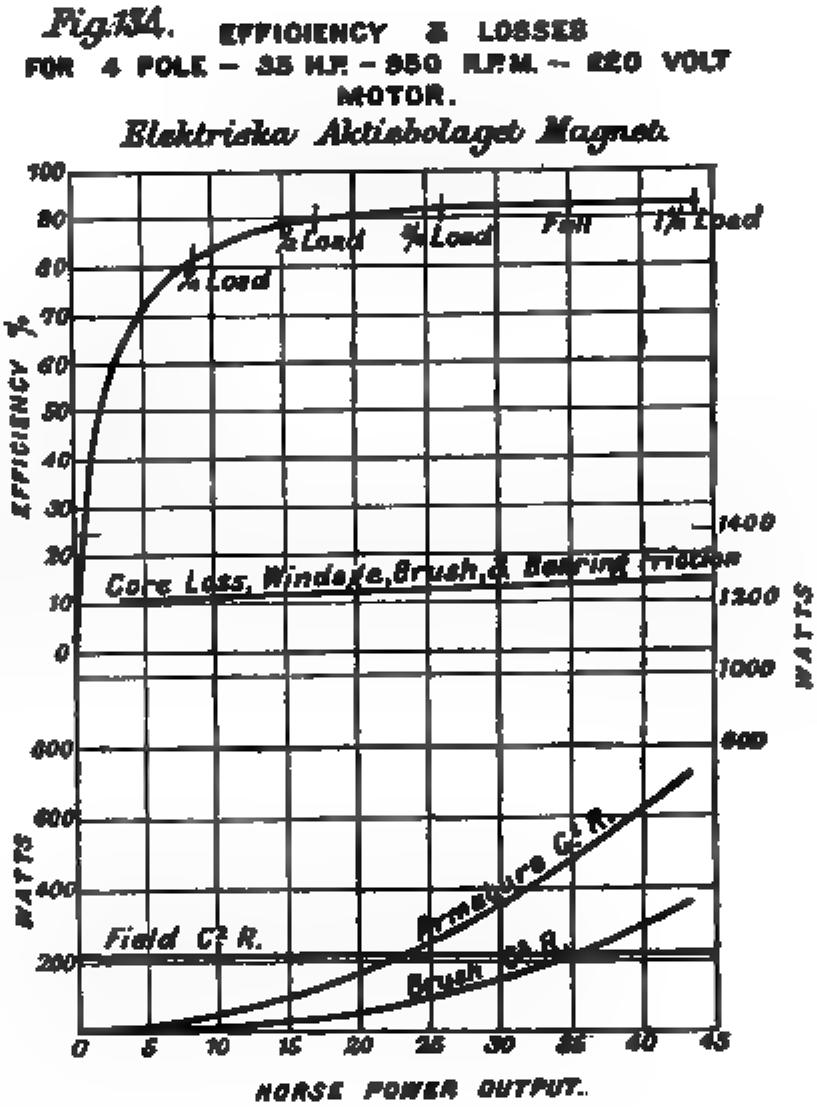
Armature:—

External diameter	254	546
Axial length of the winding	241	456
Internal diameter of the laminations ...	57	191
Axial length of core between flanges ...	102	228
Effective length of core (magnetic iron) ...	91	205
Depth of the slot	15·9	31·7
Width of the slot	6·35	11·5
Number of slots	64	70
Width of tooth at periphery, as stamped ...	6·12	9·1
Minimum width of tooth, as stamped ...	4·57	6·2









FIGS. 131, 132, 133 and 134.—Saturation and Efficiency Curves of 20 and 35 Horse-power Motors.

	Rated Output 5 Brake Horse-Power.	Rated Output 20 Brake Horse-Power.
Magnet Core:—		
Length of the pole face parallel to the shaft	102	228
Diameter of the bore of the pole face ...	262	465·5
Pitch at the bore of the pole face	206	367
Length of pole arc	127	267

Fig. 135.—5 Brake Horse-power Enclosed Motor, by Henry A. Mavor.

Ratio of the pole arc to pitch	·61	·73
Thickness of the pole shoe at the centre of the arc	9·5	30
Radial length of the magnet core	89	95
Width of the magnet core parallel to the shaft	76	228
Width of the magnet core at right angles to the shaft	121	165
Radial depth of the air gap	3·81	4·75

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Yoke:—		
External diameter	520	811
Internal diameter	476	709
Thickness of yoke for magnetic purposes ...	22	51
Axial width (effective magnetic portion) ...	222	470

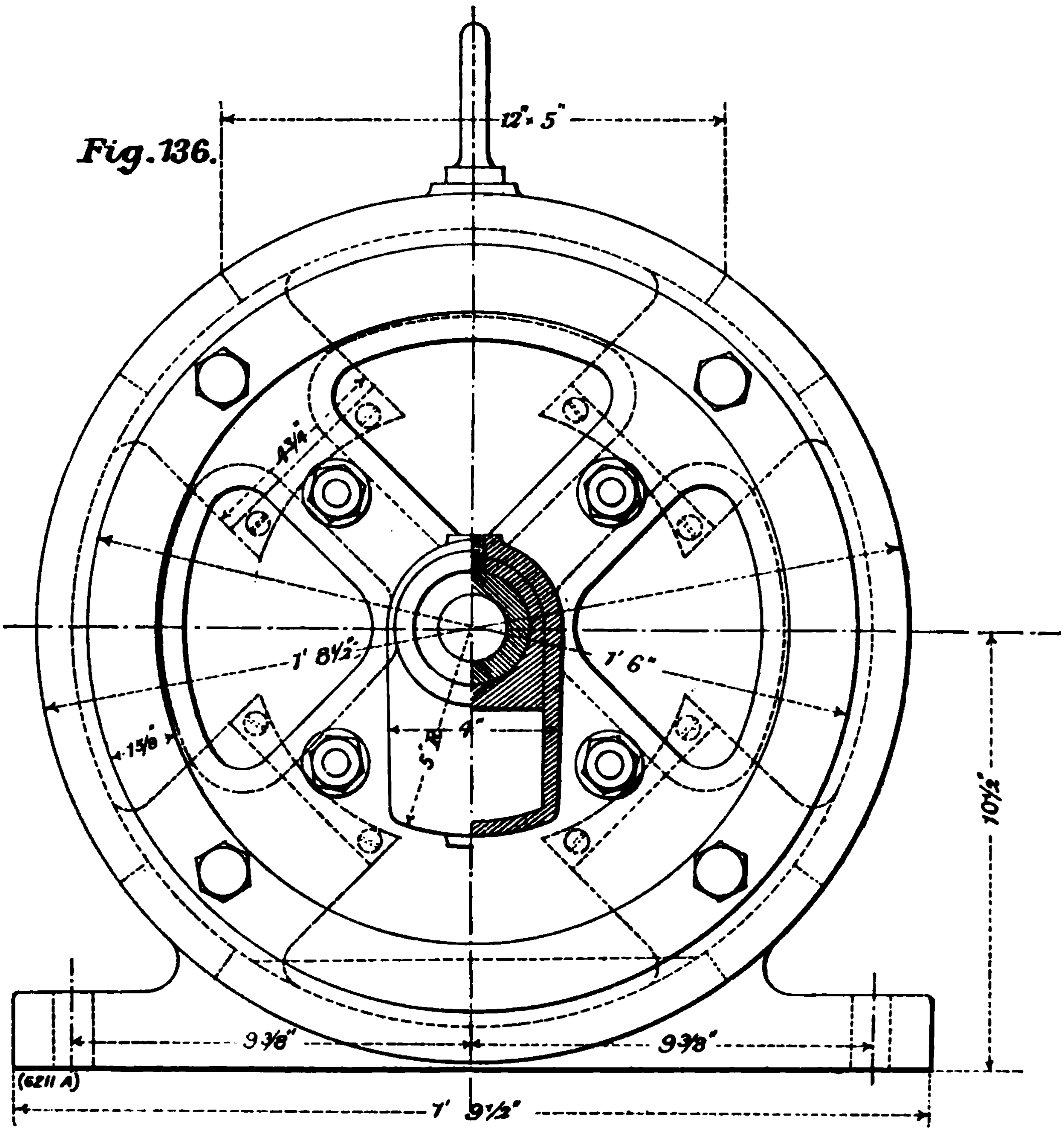


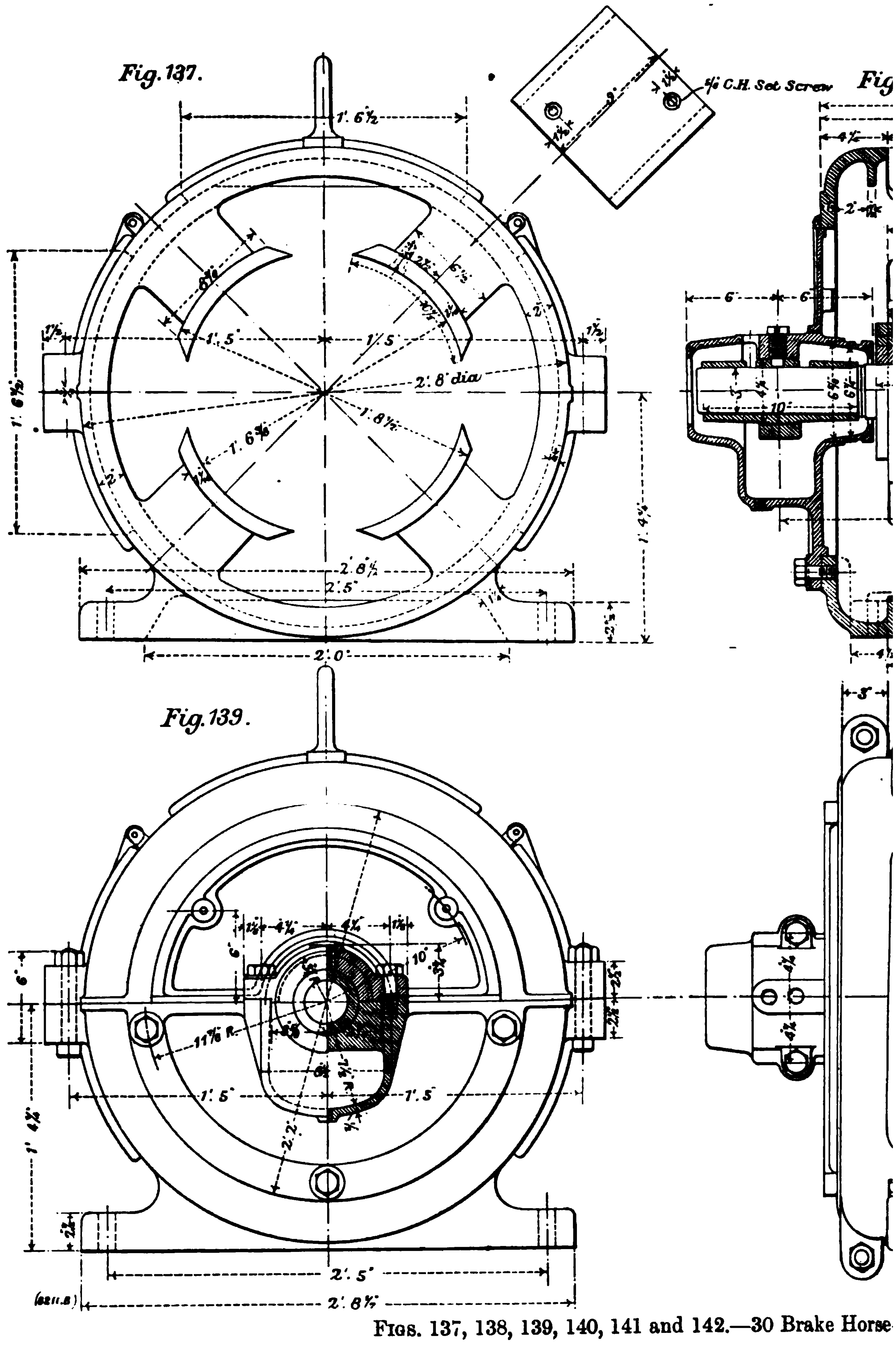
Fig. 136.—5 Brake Horse-power Enclosed Motor, by Henry A Mavor.

Commutator:—		
Diameter	178	305
Number of segments...	127	139
Thickness of segments + insulation at periphery	4·4	6·9
Thickness of segment at the periphery ...	3·6	6·1
Length from external end to commutator connection	51	114

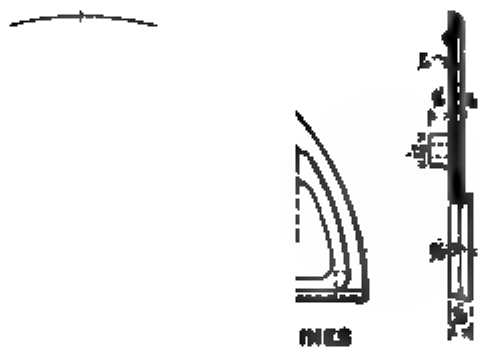
	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Armature:—		
Terminal voltage	250	220
Number of face conductors	762	278
(One "dummy" coil in each armature not counted)		
Number of conductors per slot	12	4
Arrangement of the conductors in the slot	2×6	2×2
Style of winding	2-circuit	2-circuit
Total amperes to commutator	17·6	110
Amperes per conductor	8·8	55
Mean length of a single turn, centimetres...	72	136
Number of turns in series between brushes	191	69·5
Total length of conducting path between brushes, centimetres	13700	9450
Dimensions of Bare Armature Conductors ...	1·63 (diameter)	12·7×3·8
Dimensions of Insulated Armature Conductor	1·93 (diameter)	13·5×4·56
Cross section of one conductor, square centimetre	·0209	·482
Cross section of all parallel conductors ...	·0418	·964
Specific resistance at 60° Cent., ohm ...	·00000200	
Resistance of winding from + to − at 60° Cent., ohm	0·65	0·0196
IR loss in armature at 60° Cent., volts ...	12·2	2·25
IR loss in brush contact surfaces, volts ...	1·5	1·4
Total internal IR loss, volts	13·7	3·65
Total induced voltage, full load	236·3	216·7
Total copper cross section per slot, square centimetres...	·250	1·93
Width × depth of slot, square centimetres...	1·01	3·64
"Space factor" of slot	·248	·53
Amperes per square centimetre in armature conductor	420	115
Calculation of Reactance Voltage:—		
Periphery of the commutator, metres ...	·560	·959
Revolutions per second	16·7	10
Peripheral speed in metres per second (=A)	9·35	9·59
Length of the arc of contact (=B), millimetres	13·5	20
Frequency of commutation, cycles per second $\left(=\frac{1000\text{ A}}{2\text{ B}}=n\right)$	347	240
Width of a segment at the periphery, including insulation	4·4	6·9
Maximum number of coils short circuited under a brush	4	3

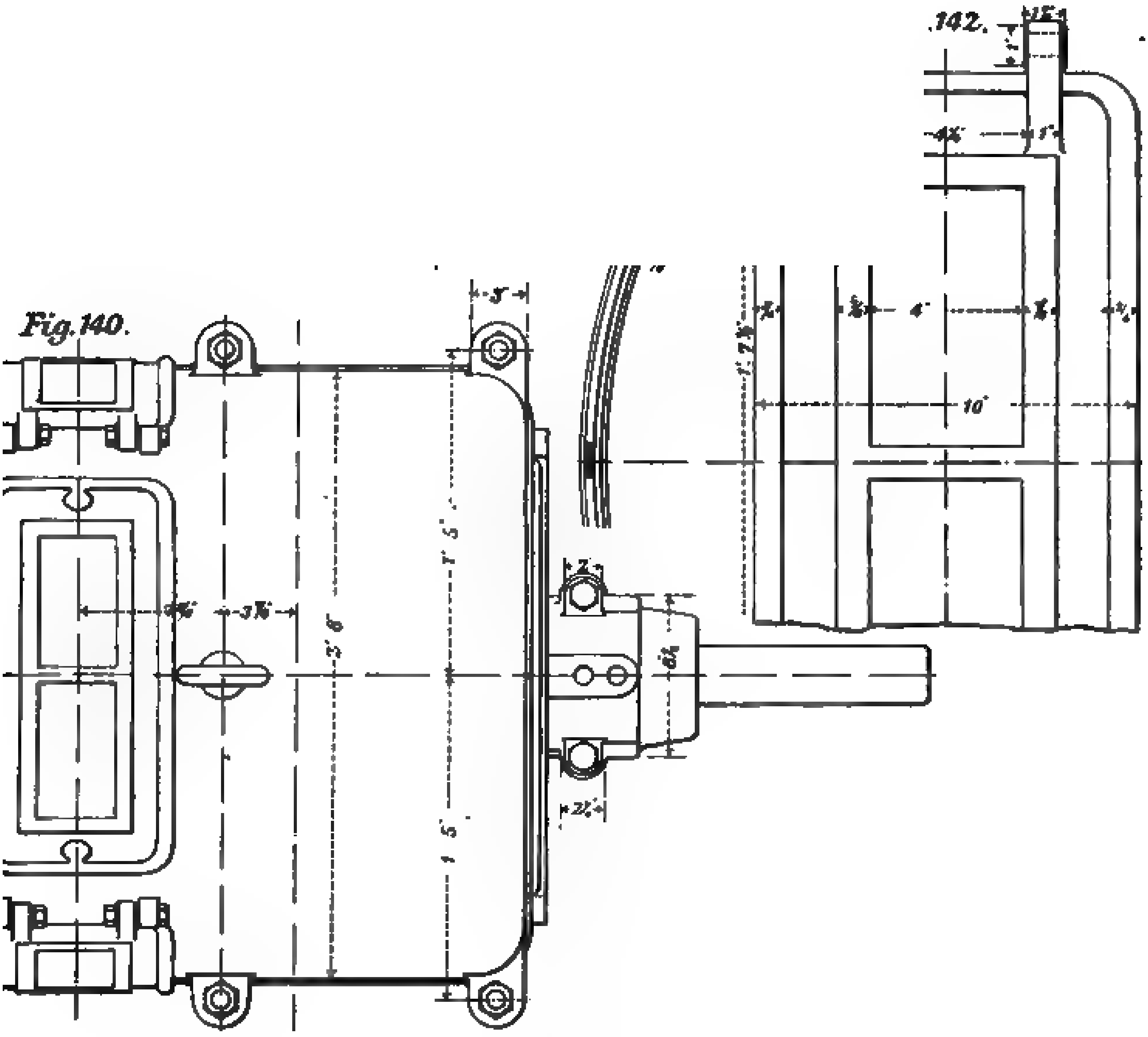
Fig. 137.

Fig. 139.



Figs. 137, 138, 139, 140, 141 and 142.—30 Brake Horse-





·wer Enclosed Motor, by Henry A. Mavor (see page 128).

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Calculation of Reactance Voltage—continued.		
Turns per coil (q)	3	1
Maximum number of simultaneously commutated conductors per group (r) ...	24	6
Mean length of one turn, centimetres ...	72	136
Effective length of core, centimetres ...	9·2	20·5
"Free length" per turn (s)	54	95
"Embedded length" per turn (t)	18	41
Lines per ampere turn per centimetre of "free length" (u)	0·8	0·8
Lines per ampere turn per centimetre of "embedded length" (v)	4·0	4·0
Lines per ampere turn for "free length" ($u \times s$)	43	76
Lines per ampere turn for "embedded length" ($v \times t$)	72	164
Lines per ampere for "free length" $\left(\frac{r}{2} \times u \times s\right) = o$	515	228
Lines per ampere for "embedded length" $(r \times v \times t) = p$	1730	982
Total lines linked with short-circuited coil per ampere $(o+p)$	2245	1210
Inductance per segment $\frac{q \times (o+p)}{10^8} = l$ (henry)	·0000672	·0000121
Reactance per segment, ohm ($2\pi\, n\, l$) ...	·147	·018
Number of sets of brushes employed ...	2	4
Minimum series reactance of short-circuit conductors	·294	·018
Amperes per conductor	8·8	55
Reactance voltage	2·58	0·99
Magnetic Circuit Calculations :—		
Flux entering armature per pole, full load, megalines	0·94	3·90
Corresponding internal voltage	217	236
Corresponding terminal voltage	220	250
Leakage factor	1·2	1·2
Flux generated per pole, full load, megalines	1·12	4·68
Armature :—		
Cross section of the core, square centimetre	151	415
Density, full load, c.g.s. lines	6200	9300
Ampere turns per centimetre, full load ...	1·0	1·2
Magnetic length per pole, centimetres ...	7	13
Ampere turns, full load	7	16

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Teeth:—		
Number of teeth per pole	16	17·5
Number of teeth directly below a mean pole arc	9·8	12·8
Percentage increase allowed for spread ...	10 per cent.	10 per cent.
Total number of flux carrying teeth per pole	10·8	14·1
Cross section of one tooth at root, square centimetres	4·2	12·8
Total cross section at the bottom of these teeth, square centimetres	45	180
Apparent density, full load, c.g.s. lines ...	21,000	21,600
Mean width of tooth ÷ width of slot ..	·85	·67
Corrected density, full load, c.g.s. lines ...	20,100	19,900
Ampere turns per centimetre, full load ...	340	320
Length, centimetres	1·59	3·17
Ampere turns, full load	540	1020
Air Gap:—		
Cross section at pole face, square centimetres	129	610
Density at pole face, full load, c.g.s. lines...	7300	6400
Length of air gap, iron to iron, centimetres	·381	·475
Ampere turns, full load	2230	2440
Magnet Core:—		
Cross section, square centimetres	92	377
Density, full load, c.g.s. lines	12,200	12,450
Ampere turns per centimetre, full load ...	12	13
Magnetic length, centimetres	11	12
Ampere turns, full load	130	160
Yoke:—		
Cross section, square centimetres	99	477
Density, full load, c.g.s. lines	11,400	10,000
Ampere turns per centimetre, full load ...	10	8
Magnetic length, per pole, centimetres ...	15	25
Ampere turns, full load	160	200
Ampere Turns per Spool:—		
Armature core	10	20
Armature teeth	540	1020
Air gap	2230	2440
Magnet core	130	160
Yoke	160	200
Estimate total number of ampere turns per spool	3070	3840
Actually required	2600	4100

The value actually required varies according to the brush position chosen.

	Rated Output 5 Brake Horse-power.	Rated Output 30 Brake Horse-power.
Shunt Spool Winding Calculations:—		
Voltage per shunt spool at 60° Cent. ...	62·5	55·0
Radial depth of winding, centimetres ...	5·5	9·0
Internal periphery of spool, centimetres ...	39·4	79
External periphery of spool, centimetres ...	83·8	153
Mean length of one shunt turn, metres (a)	·616	1·16
Ampere turns per shunt spool (b)	2610	4100
a b	1610	5000
$.000176 \times a^2b^2$	455	4400
Axial length of shunt spool, centimetres ...	7·5	9
Cross section of shunt spool winding, square centimetres (r)	41·2	81
"Space factor" of shunt spool (s)	·28	·32
Cross section of copper in shunt spool ($t = r \times s$)	11·6	26
Cubic centimetres copper in shunt spool (100 a t)	715	3020
Kilogrammes copper per shunt spool (1 cubic centimetre copper = ·0089 kilogramme)	6·3	26·8
Watts per shunt spool (watts = $\frac{\cdot000176 \times a^2b^2}{\text{Weight in kilogs.}}$)	72	164
External cylindrical surface per spool, square decimetres	6·3	13·8
Watts per square decimetre of external cylindrical spool surface	11·5	11·9
Amperes per shunt spool (watts ÷ volts per spool)	1·17	2·97
Turns per shunt spool	2232	1384
Cross section copper per turn, square centimetre	·0052	·0188
Current density in amperes per square centimetre	225	158
Diameter of bare copper conductor... ...	·81	1·55
Insulation employed on conductor... ...	D.C.C.	D.C.C.
Watts in all shunt spools at 60° Cent. ...	290	657
Weight total shunt copper in kilogrammes, all spools	25·2	107
Resistance four shunt spools at 60° Cent., ohms	212	74
Observed total temperature increase by thermometer	38° C.	28° C.
Ditto per watt per square decimetre external cylindrical surface	3·3° C.	2·4° C.
Observed total temperature increase by resistance	47° C.	44° C.
Ditto per watt per square decimetre ...	4·1° C.	3·7° C.
Corresponding number of hours run at full load	4·5	3·0

(Calculations of Armature Losses and Temperature Increase.)

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Armature Copper Loss:—		
Resistance of the winding from + to – at 60° Cent., ohm ...	·65	·0196
Total amperes to commutator ...	17·6	110
Watts lost in armature copper at 60° Cent.	200	235
Core Loss:—		
Weight of the armature teeth, kilogrammes	4	27
Weight of the armature core, kilogrammes	26	146
Total weight of armature laminations ...	30	173
Flux density in the core, kilolines (D) ...	6·2	10
Periodicity, cycles per second (N) ...	33·3	20
$D \times N \div 100$ ...	2·07	2·00
Watts lost in iron per kilogramme[1] ...	5·3	5·0
Total core loss (estimated) watts ...	159	865
Total core loss (observed) watts ...	136[2]	700
Armature Temperature Increase:—		
Armature copper loss, watts...	200	235
Armature iron loss, watts ...	136	700
Total armature loss, watts ...	336	935
Circumference, decimetres ...	8·0	14·4
Axial length of the winding, decimetres ...	2·4	4·6
Peripheral surface, square decimetres ...	19	66
Watts per square decimetre of peripheral surface ...	18	14
Observed total temperature increase by thermometer ...	42° C.[3]	24° C.
Total thermometrically determined temperature increase per watt per square decimetre	2·3° C.	1·7° C.
Corresponding number of hours run at full load ...	4·5	3·0
Commutator Losses:—		
Length of brush contact arc, millimetres ...	13·5	20
Width of brush, millimetres ...	38	50
Contact surface per brush, square centimetres	5·1	10
Number of sets of brushes ...	2	4
Number of brushes per set ...	1	2
Total number of positive brushes ...	1	4

[1] Obtained by curve of Fig. 21, page 30.
[2] Average result for two machines.
[3] 37° Cent. for winding, and 47° for core.

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Commutator Losses—continued.		
Contact surface of all positive brushes, square centimetres	5·1	40
Amperes to commutator	17·6	110
Amperes per square centimetre of brush contact surface	3·4	2·7
I^2R loss in watts per ampere [1]	1·5	1·4
Total I^2R loss at brush contacts, watts ...	26	154
Peripheral speed of commutator in metres per second	9·4	9·6
Brush friction loss in watts per ampere [1] ...	1·5	1·9
Brush friction loss in watts	26	208
Total commutator loss, watts	52	362
Commutator Temperature Increase:—		
Total commutator loss, watts	52	362
Circumference, decimetres	5·6	9·6
Length of commutator surface, decimetres	·51	1·14
Cylindrical surface of commutator, square decimetres	2·9	10·9
Watts per square decimetre of cylindrical surface	18·0	34·8
Observed total temperature increase at the peripheral surface	36° C.	30° C.
Total temperature increase per watt per square decimetre	2·0° C.	0·86° C.
Corresponding number of hours run at full load	4·5	3·0
Efficiency at 60° Cent.:—		
(*a*) Iron loss, watts	136	700
(*w*) Watts lost in armature copper	200	235
(*x*) Watts lost at the brush contact resistance at the commutator	26	154
(*b*) Brush friction loss at the commutator, watts	26	208
(*b*) Friction loss at bearings and air friction, watts	300	500
(*c*) Watts lost in shunt winding	290	656
Total of all losses	978	2543
Output at full load, watts	3730	22,400
Input at full load, watts	4708	24,853

[1] Values taken from Table XVIII., page 104.

	Rated Output 5 Brake Horse-Power.	Rated Output 30 Brake Horse-Power.
Efficiency at 60° Cent.—continued.		
Commercial efficiency at $1\frac{1}{4}$ load	80·7	91·1
„ „ full load	79·2	90
„ „ $\frac{3}{4}$ „	76·1	87·6
„ „ $\frac{1}{2}$ „	69·8	83·6
„ „ $\frac{1}{4}$ „	55·0	72·5
Characteristic Losses:—		
$(a+b+c)$ = constant losses, watts	752	2064
$(w+x)$ = variable losses, watts	226	389
External radiating surface of case, square decimetres	120	320
Watts per square decimetre external radiating surface of case	8·2	7·7

When totally enclosed, motors have excellent internal circulation of air, so that air has ready access to all parts, and transfers the heat promptly from those parts to the internal surfaces of the walls of the case; 7 watts per square decimetre of external radiating surface will generally ensure a temperature increase not exceeding 8° Cent. per watt per square decimetre of external surface.

When perforated covers, or other departures from absolutely total enclosure are employed, a considerably lower temperature rise is secured.

Weights of the Effective Materials in Kilogrammes:—		
Armature laminations, net	30	173
Armature copper	5	82
Commutator segments	9	50
Magnet cores	30	110
Pole shoes	—	50
Yoke	65	450
Shunt copper on magnet spools	25	107
Total effective material	164	1022
Effective material per horse-power	32·8	34
Weight of complete motor	310	1530
Ditto per horse-power	62	51

Total Cost of Effective Materials in Shillings:—		
Armature copper at 24 pence per kilogramme	10	164
Commutator copper at 24 pence per kilogramme	18	100
Spool copper at 24 pence per kilogramme ...	50	214
Armature laminations at 3·5 pence per kilogramme...	9	52
Cast steel at 5 pence per kilogramme ...	40	255
Total effective material, shillings	127	785
Effective material per horse-power, shillings	25·4	26·2

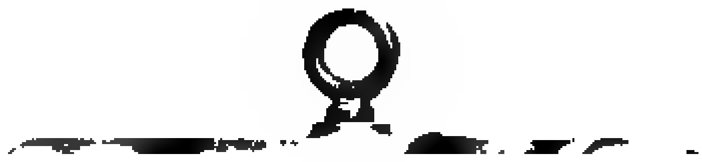


Fig. 143.—30 Brake Horse-power Enclosed Motor, by Henry A. Mavor (see page 129).

Fig. 144.—10 Brake Horse-power 110-volt Enclosed Series Wound Motor (see page 139).

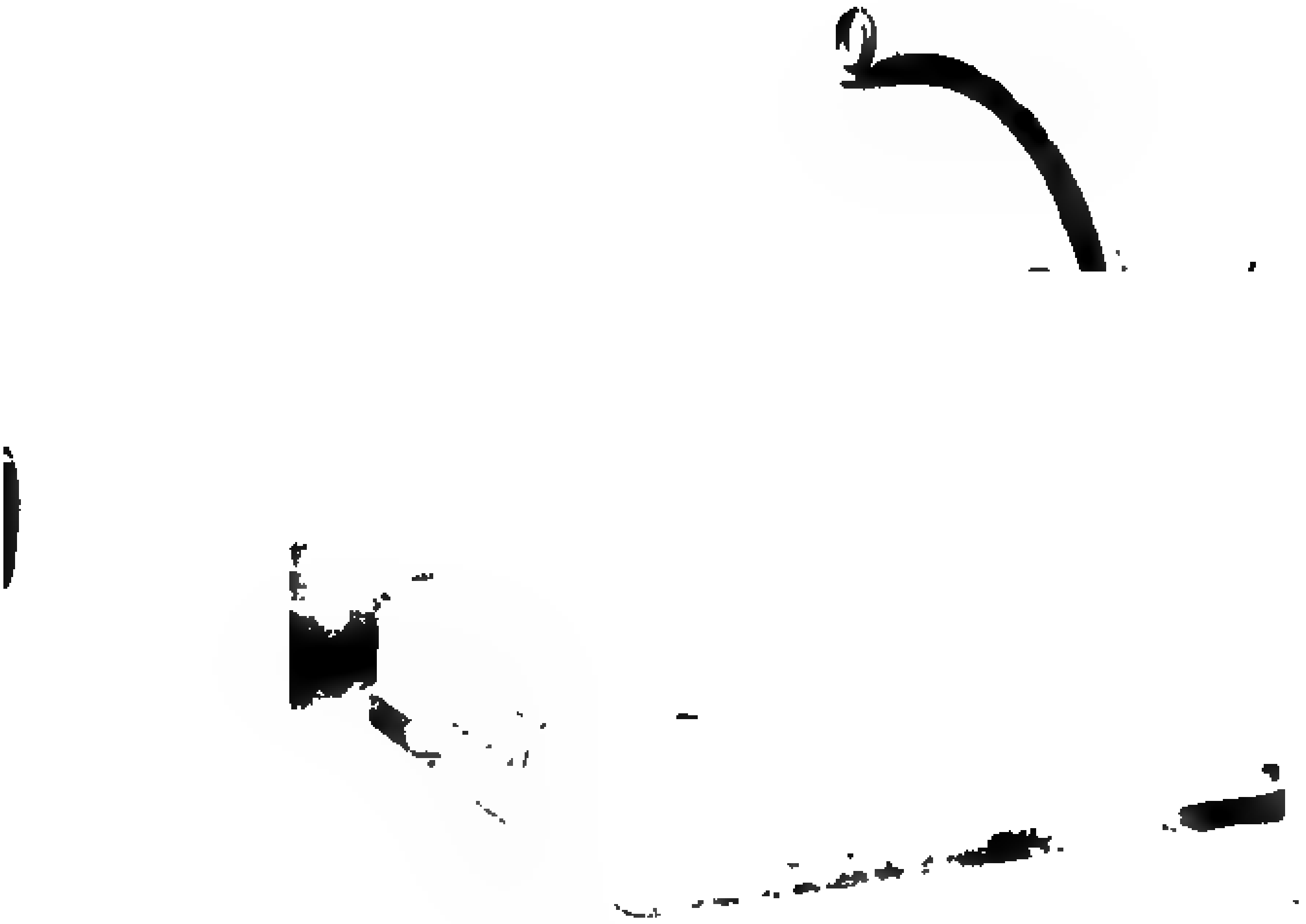


Fig. 145.—Small Open Type Mavor & Coulson Motor (see page 139).

Fig. 146.—Enclosed Motor Geared to Vertical Mining Pump (see page 140).

§ 5. **Enclosed Series Motor of 10 H.P. 110 volts.**—Fig. 144, Plate 7, illustrates a 10 horse-power, 110 volts, 700 revolutions per minute, series wound totally enclosed winch motor, derived from the 5 horse-power motor already described by lengthening the core between flanges from 102 millimetres to 152 millimetres, the other parts of the motor being, of course, also lengthened. The above rating is for intermittent working, and is about double the rating which would be given to the motor for a continuous run of six hours with the specified rise of 70° Cent.

Fig. 145, Plate 7, is an engraving of Messrs Mavor & Coulson's next larger diameter of armature, in an open frame. In all open type motors these makers employ longer and shallower magnet coils.

The 30 horse-power motor described in the preceding specification is, as an open type motor, rated at 42 horse-power (220 volts and 160 amperes) at 600 revolutions per minute for a temperature rise of 39° Cent. Including two bearings and bed-plate, its weight is then 2200 kilogrammes, or 52 kilogrammes per rated horse-power.

§ 6. **Comparative Tests of 3-turn and 4-turn Armatures.**—Mr Mavor has also kindly furnished the writer with the results of an interesting series of tests which he has made to compare the 5 horse-power motor described with a motor constructionally identical in all respects except the following:—

49 armature slots instead of 64.
Slots 23·8 millimetres deep instead of 15·9 millimetres.
Slots 7·6 millimetres wide instead of 6·4 millimetres.
Tooth 8·7 millimetres wide at top instead of 6·1 millimetres.
Tooth 5·6 millimetres wide at bottom instead of 4·6 millimetres.
776 armature conductors instead of 762.
16 conductors per slot instead of 12.
2·03 millimetres diameter for armature conductor instead of 1·63 millimetres diameter.
270 amperes per square centimetre in armature conductor instead of 420.
97 commutator segments instead of 127.
4 turns per segment instead of 3 turns.

The 3-turn per coil armature (described in column 1) is, of course, preferable from the commutation standpoint, as its reactance voltage is but 2·6 volts as against 3·5 volts for the 4-turn design.

The following results are the average for at least two machines of each design:—

	Armature turns per coil. 3.	4.
Armature I^2R loss at full load, watts ...	190	121
Armature I^2R loss at half load, „ ...	42	28
Armature iron loss, „ ...	130	164
Friction loss (estimated), „ ...	300	300
Commutator loss, full load, „ ...	61	61
Commutator loss, half load, „ ...	41	41
Spool I^2R loss, „ ...	293	293
Total loss, full load, „ ...	974	938
Total loss, half load, „ ...	806	826
Efficiency, full load, „ ...	79·2	79·7
Efficiency, half load, „ ...	70·0	69·5

The thermometrically determined temperature rise after full load runs for four and a half hours:—

	Armature turns per coil. 3.	4.
Temperature of air in Centigrade degrees ...	18	18
Commutator, above air „ ...	36	27
Armature core, above air „ ...	47	35
Armature coils, above air „ ...	38	30
Field spools, above air „ ...	38	36

Full load speed at 220 volts = 1000 revolutions per minute in both cases when hot.

Figs. 146 and 147, Plates 5 and 7, show two applications of small enclosed motors of these types to driving pumps.

§ 7. **Open Type 27 Horse-power Westeras Shunt Motor.**—Mr Ernest Danielson, Technical Director of the Allmanna Svenska Elektriska Aktiebolaget, of Westeras, Sweden, has sent the writer the data for the following description of his company's standard open type 110-volt 27 horse-power shunt motor for 800 revolutions per minute. Fig. 148, Plate 8, is taken from a photograph of this motor, and outline drawings are shown in Figs. 149 and 150. Efficiency, saturation, and compounding curves of this machine, taken at the practically equivalent generator rating of 20 kilowatts, are given in Figs. 151 and 152.

TABLE XXIII.—DESCRIPTION OF THE ALLMANNA SVENSKA ELEKTRISKA AKTIEBOLAGET OPEN TYPE 27 HORSE-POWER SHUNT MOTOR.

(Dimensions in millimetres.)

Number of poles ...	4
Rated output, brake horse-power ...	27
Rated voltage ...	125
Rated speed, revolutions per minute ...	800
Amperes output, full load ...	179
Amperes input, no load ...	12
Watts input at no load ...	1500

FIG. 148.—Open Type 27 Horse-power Shunt Motor, by E. Daniel

on, Allmanna Svenska Elektriska Aktiebolaget, Sweden (see page 140).

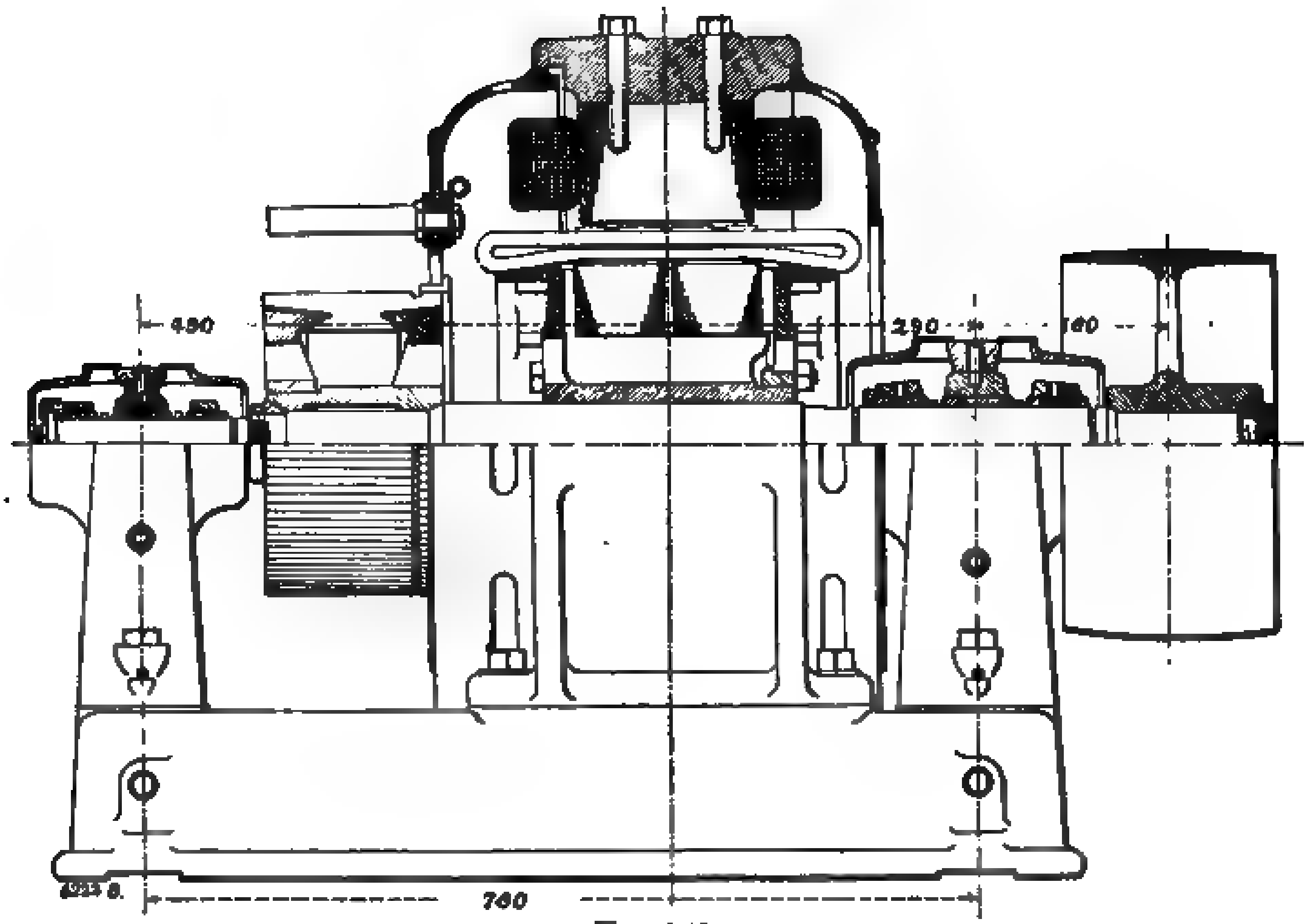


FIG. 149.

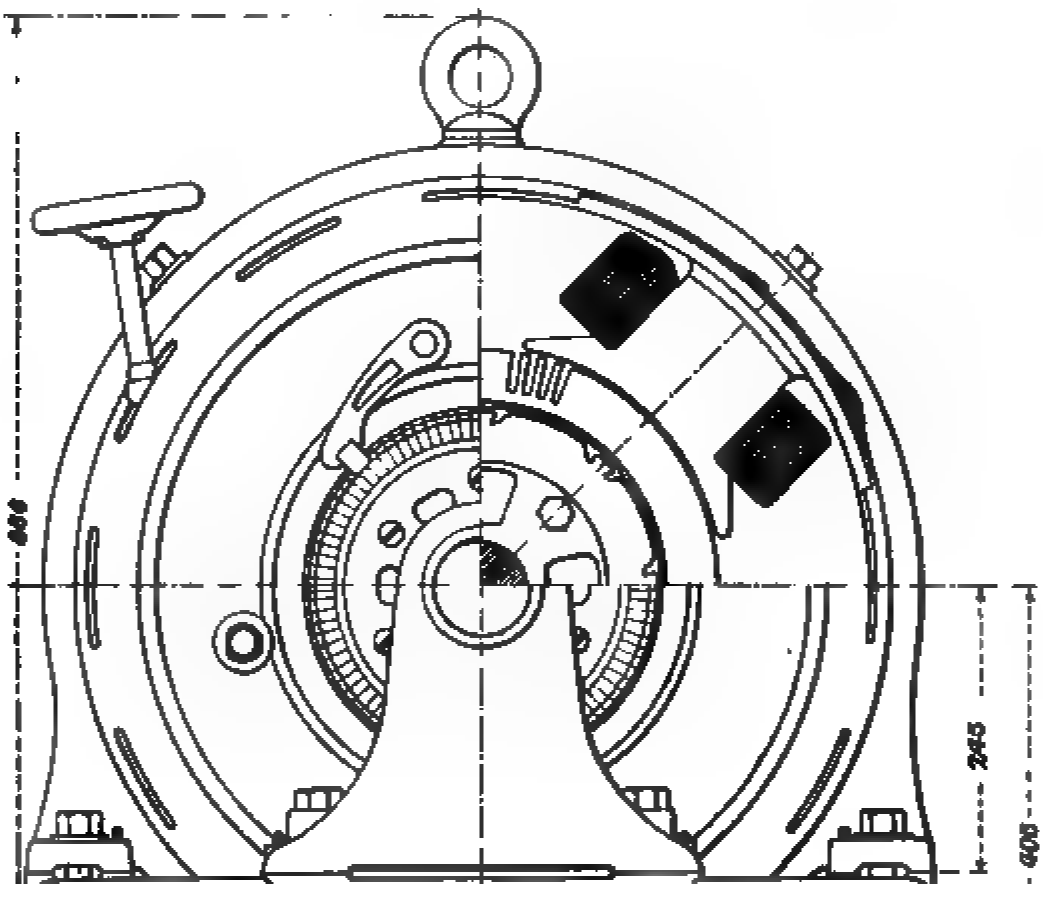


FIG. 150.

Armature:—

External diameter	400
Axial length of the winding	360
Diameter at the bottom of the slots	333
Internal diameter of the laminations	200
Number of intermediate ventilating ducts	2[1]
Width of each intermediate ventilating duct	12
Length occupied by intermediate ventilating ducts	24
Length occupied by insulating varnish	16

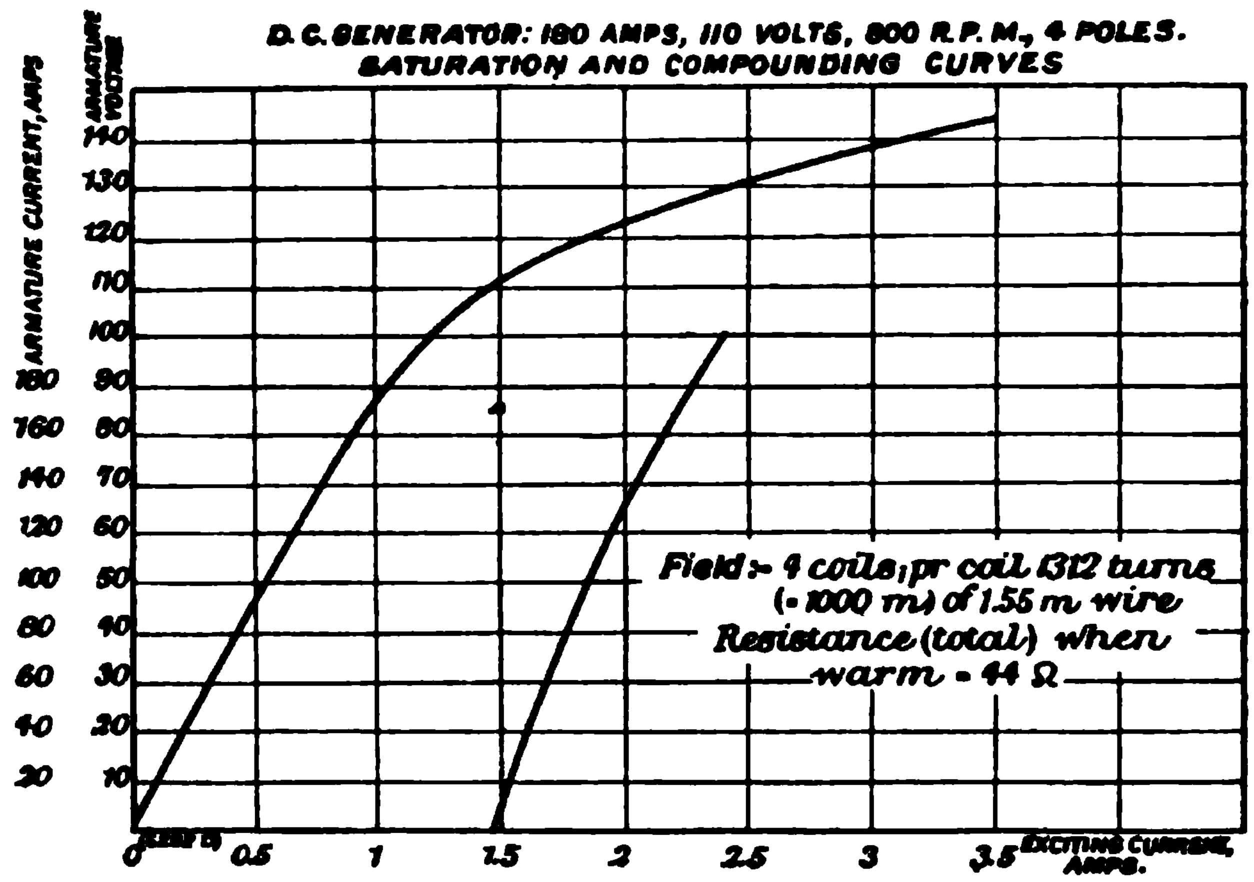


FIG. 151

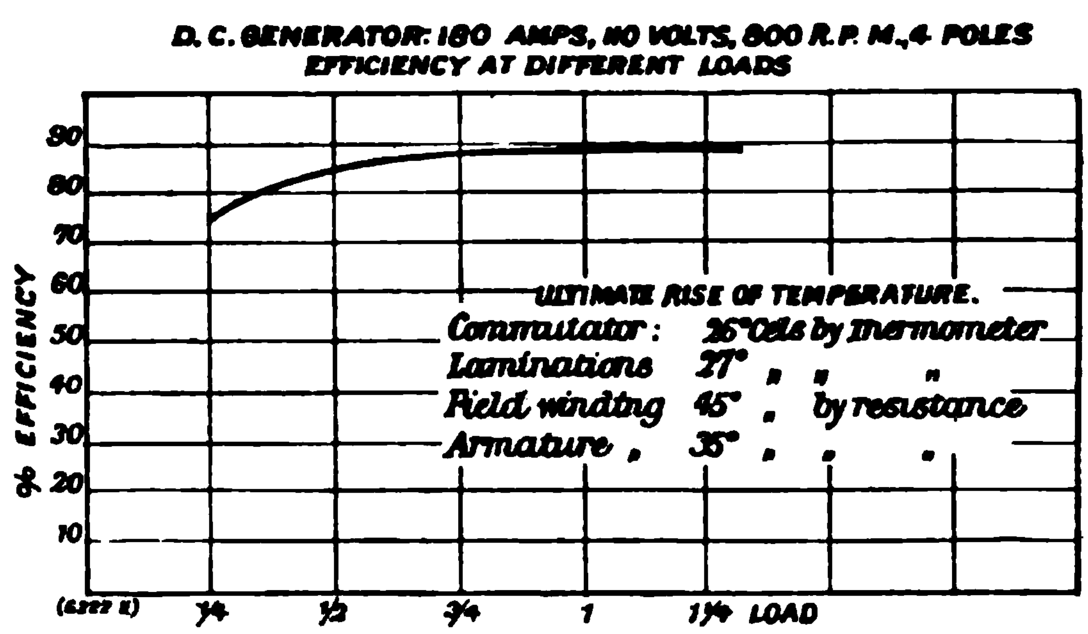


FIG. 152.

Length occupied by effective iron	144
Gross axial length of core between flanges	184
Depth of the slot	33·5
Width of the slot	7·0
Number of slots	89
Width of tooth at periphery, as stamped	7·2
Minimum width of tooth, as stamped	4·8
Average width of tooth, as stamped	6·0

[1] Also two end ducts, each 12 millimetres wide.

Magnet Core:—

Length of the pole face parallel to the shaft	162
Length of the pole arc	238
Radial length of magnet core to bore of pole face	112
Width of magnet core parallel to the shaft	162
Width of magnet core at right angles to shaft	125
Radial depth of the air gap	2·25

Magnet Yoke:—

External diameter	746
Internal diameter	661
Thickness of yoke	47·5
Axial width	250
Radial thickness of the pole seat	14

Commutator:—

Diameter	280
Number of segments	89
Thickness of segment + insulation at periphery	9·9
Thickness of insulation between segments	0·7
Thickness of a segment at the periphery	9·2
Thickness of a segment at the inner edge	6·6
Height of a segment, total	37
Length from external end to commutator connection	140

Armature Winding:—

Conductors per slot	4
Each conductor consists of two wires in parallel, each wire 3 millimetres diameter, ∴ eight wires per slot, arranged eight deep.	
Type of winding, 4 circuit single	
Current per conductor, amperes	44·8
Cross section one conductor, square centimetre	·142
Current density in amperes per square centimetre	316
Armature slot "space factor"	·24
Number of turns in series between brushes	44·5
Mean length of one turn, centimetres	128
Resistance of winding from + to − at 60° Cent., ohm	·020
IR drop in armature winding at full load, volts	3·6
IR drop at brush contact surfaces, volts	1·7
Total induced voltage at full load, volts	119·7

Calculation of Reactance Voltage

Peripheral speed of commutator in metres per second (= A)	11·8
Length of the arc of contact (= B)	16
Frequency of commutation (cycles per second) $\left(=\frac{1000A}{2B}=n\right)$	368
Width of a segment at the periphery (including insulation) ...	9·9
Maximum number of coils short circuited under a brush ...	2
Turns per coil (q)	2
Maximum number of simultaneously commutated conductors per group (r)	8

Mean length of one turn, centimetres	128
Effective length of core, ...	14·4
"Free length" per turn (s)	99
"Embedded length" per turn (t)	29
Lines per ampere turn per centimetre of "free length" (u) ...	0·8
Lines per ampere turn per centimetre of "embedded length" (v)	4·0
Lines per ampere turn for "free length" ($u \times s$)	79
Lines per ampere turn for "embedded length" ($v \times t$)	116
Lines per ampere for "free length" $\left(\frac{r}{2} \times u \times s\right) = o$	316
Lines per ampere for "embedded length" $(r \times v \times t) = p$ ...	926
Total lines linked with short-circuited coil per ampere $(o+p)$	1242
Inductance per segment $\left(\frac{q \times (o+p)}{10^8} = l\right)$, henry	·0000248
Reactance per segment, ohms ($2\pi\ n\ l$)	·0573
Current per conductor at full load	44·8
Reactance voltage at full load	2·6
Average voltage per segment	5·6
Armature ampere turns per pole at full load	2000

MAGNETIC CIRCUIT CALCULATIONS.

Full load internal voltage (E)	119·7
Turns in series between brushes (T)	44
Periodicity in cycles per second (N)	26·7
Flux entering armature pole $M = \left(\frac{E \times 10^8}{4\ T\ N}\right)$	2·50
Assumed leakage factor	1·20
Flux generated per pole	3·00

	Cross Section, Square Centimetres.	Kilolines per Square Centimetre.	Magnetic Length (Centimetres).	Magneto-motive Force in Ampere Turns per Centimetre.	Total Magneto-motive Force in Ampere Turns.
Armature core below slots	190	13·2	10	10	100
Flux carrying teeth	127	19·0	3·4	200	670
Pole face	385	6·6	0·225	5250	1180
Magnet core	200	15·0	11·2	20	220
Yoke	236	12·8	26·0	14	360
Total ampere turns per spool	...	...	...	...	2530

Shunt Spool Winding Calculations.

Voltage per shunt spool at 60° Cent.	31·2
Radial depth of winding, centimetres	6·3
Internal periphery of spool, centimetres...	57
External periphery of spool, centimetres	107
Mean length of one shunt turn, metres (a)	·82
Ampere turns per shunt spool (b)	2530
a b	2080
$\cdot000176 \times a^2b^2$	760
Axial length of shunt spool, centimetres	8·5
Cross section of shunt spool winding, square centimetres (r)	53·5
"Space factor" of shunt spool (s)	·49
Cross sections copper in shunt spool ($t = r \times s$)	26·3
Cubic centimetre copper in shunt spool (100 a t)	2160
Kilogrammes copper per shunt spool (1 cubic centimetre copper = ·0089 kilogramme)	19·2
Watts per shunt $\left(\text{watts} = \dfrac{\cdot000176 \times a^2b^2}{\text{weight in kgs.}}\right)$	39·6
External cylindrical surface per spool, square decimetres ...	9·1
Watts per square decimetre of external cylindrical spool surface	4·35
Amperes per shunt spool (watts ÷ volts per spool)	1·27
Turns per shunt spool	2000
Cross section copper per turn	·0132
Current density in amperes per square centimetre	96
Diameter of bare copper conductor	1·3
Watts in all shunt spools at 60° Cent.	158
Weight total shunt copper in kilogrammes, all spools... ...	77
Ultimate rise of temperature, by resistance, observed, degrees Cent.	45
Ditto per watt per square decimetre, degrees Cent.	10·3

Armature Losses and Thermal Constants.

Weight armature laminations, kilogrammes	83
Flux density in core, kilolines, D	13·2
Periodicity, N	26·7
$D \times N \div 100$	3·53
Watts lost per iron per kilogramme [1]	9·5
Total core loss (estimated), watts...	790
Armature I^2R loss at 60° Cent., watts	640
Total armature loss, watts	1430
Cylindrical radiating surface, square decimetres	12·6
Watts per square decimetre	45·3
Ultimate temperature rise as experimentally determined by resistance measurements, degrees Cent.	35
Ditto per watt per square decimentre, degrees Cent.	1·1

[1] Derived from curve of Fig. 21, page 30.

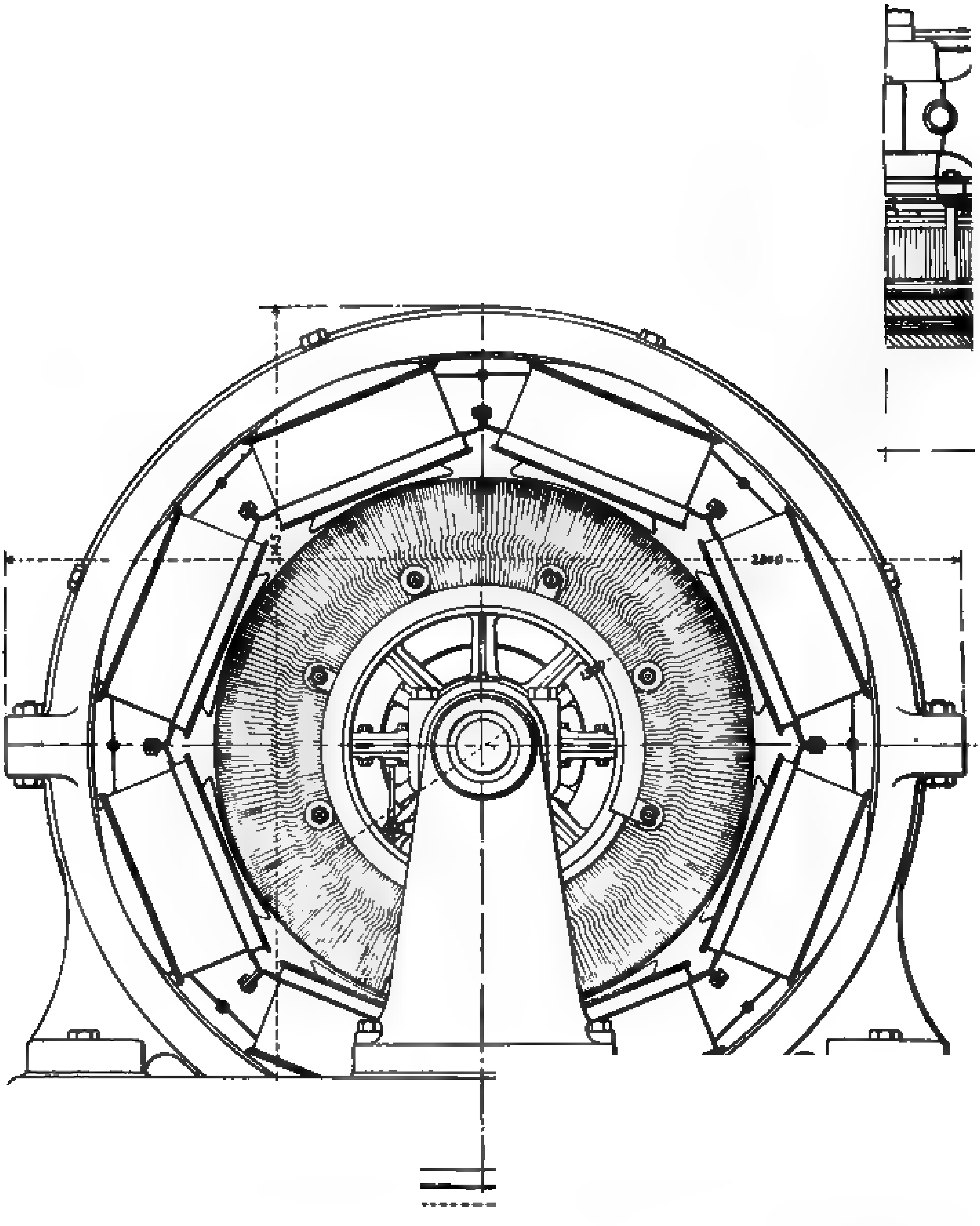
Calculation of Losses and Thermal Characteristics of Commutator.

Length of brush contact arc	1·6
Width of brush	2·9
Contact surface per brush, square centimetres	4·64
Number of brushes per pole	4
Total number of positive brushes	8
Contact surface of all positive brushes, square centimetres	37
Current strength of the machine, amperes	179
Amperes per square centimetre of brush contact surface	4·8
I^2R loss in watts per ampere [1]	1·8
Total I^2R loss at brush contacts, watts	320
Peripheral speed of commutator in metres per second... ...	11·8
Brush friction loss in watts per ampere [1]	1·4
Brush friction loss in watts	250
Total commutator loss, watts	570
Cylindrical surface of commutator, square decimetres ...	12·3
Watts per square decimetre of cylindrical surface	46·5
Observed total ultimate temperature increase at the peripheral surface, degs. Cent.	26
Ditto per watt per square decimetre	0·56

Efficiency at 60° Cent.

Iron loss, watts	790
Watts lost in armature copper	640
Watts lost at the brush contact resistance, at the commutator	320
Brush friction loss at the commutator	250
Friction loss at bearings + air friction	300
Watts lost in shunt winding	160
Total of constant losses	1500
Total of variable losses	960
Total of all losses	2460
Output at full load, watts	19,900
Input at full load, watts	22,360
Commercial efficiency at $1\frac{1}{4}$ load	89·2
" " full "	89·0
" " $\frac{3}{4}$ "	88·0
" " $\frac{1}{2}$ "	85·0
" " $\frac{1}{4}$ "	76·0

[1] These values are taken from Table XVIII., page 104.

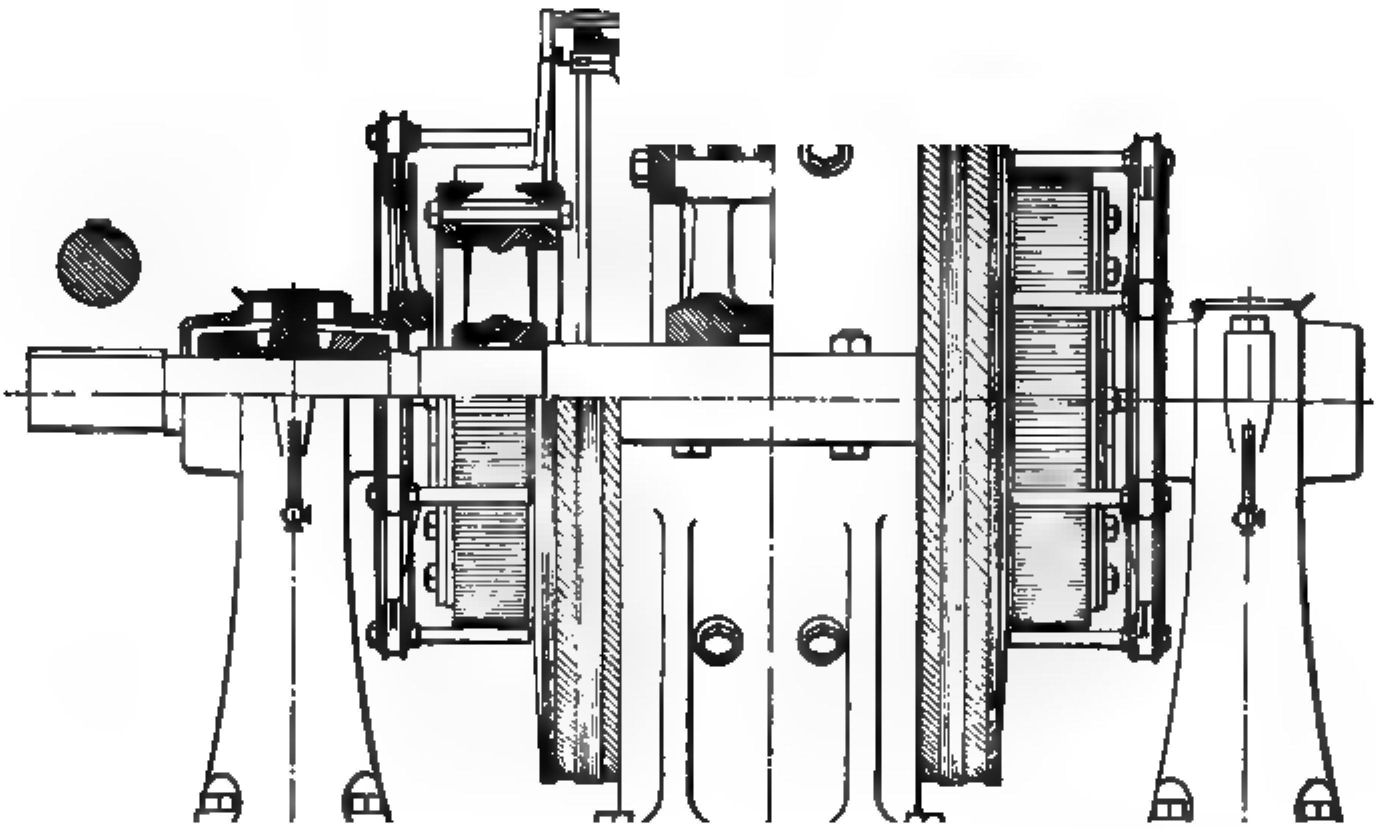


FIGS. 153, 154 and 155.—Front Elevation, Side Elevation, and Plan o

WEIGHTS AND COSTS OF THE EFFECTIVE MATERIA

	Weights in Kilogrammes.	Specific Costs in Pence.	Costs in Shillings.
Armature laminations	1600	3·5	470
Armature copper	130	24	260
Commutator segments, and mica between segments	300	24	600
Magnet cores	1450	4·5	540
Pole shoes	140	4·5	50

The total weight of the motor is 10,300 kilogra



400 Horse-power Motor, designed by A. V. Clayton (see page 147).

s IN 400 HORSE-POWER ROLLING MILL MOTOR.

	Weights in Kilogrammes.	Specific Costs in Pence.	Costs in Shillings.
Yoke	2900	4·5	1090
Shunt copper on magnet spools	730	24	1460
Series copper on magnet spools	160	24	320
Total effective material	7410	..	4790
Effective material per horse-power ..	18·5	..	12·0

mes, or 25·7 kilogrammes per horse-power.

TABLE XXIV.—WEIGHTS AND COSTS OF NET EFFECTIVE MATERIALS

	Weight in Kilogrammes.	Assumed Cost in Pence per Kilogramme.	Total Cost in Shillings.
Armature copper	29	24	58
Commutator copper	35	24	70
Field copper	77	24	154
Armature laminations	83	3·6	25
Magnet core laminations	70	3·6	21
Cast-steel yoke, including feet	220	4·5	82
Total cost of net effective material	...	...	410
Ditto per rated horse-power	...	...	15·2
Total weight of net effective material	514	...	...
Ditto per rated horse-power	19	...	...
Complete weight of motor, without slides and pulleys	300	...	...
Ditto per rated horse-power	11	...	...

§ 8. **400 Horse-Power Eight-Pole Rolling Mill Motor.**—The properties of compound wound motors are discussed theoretically in Chapter IV., p. 52. Such motors are finding a wide application for special processes, such as for mining plants and for rolling mills, where exacting requirements as to speed and power are encountered.

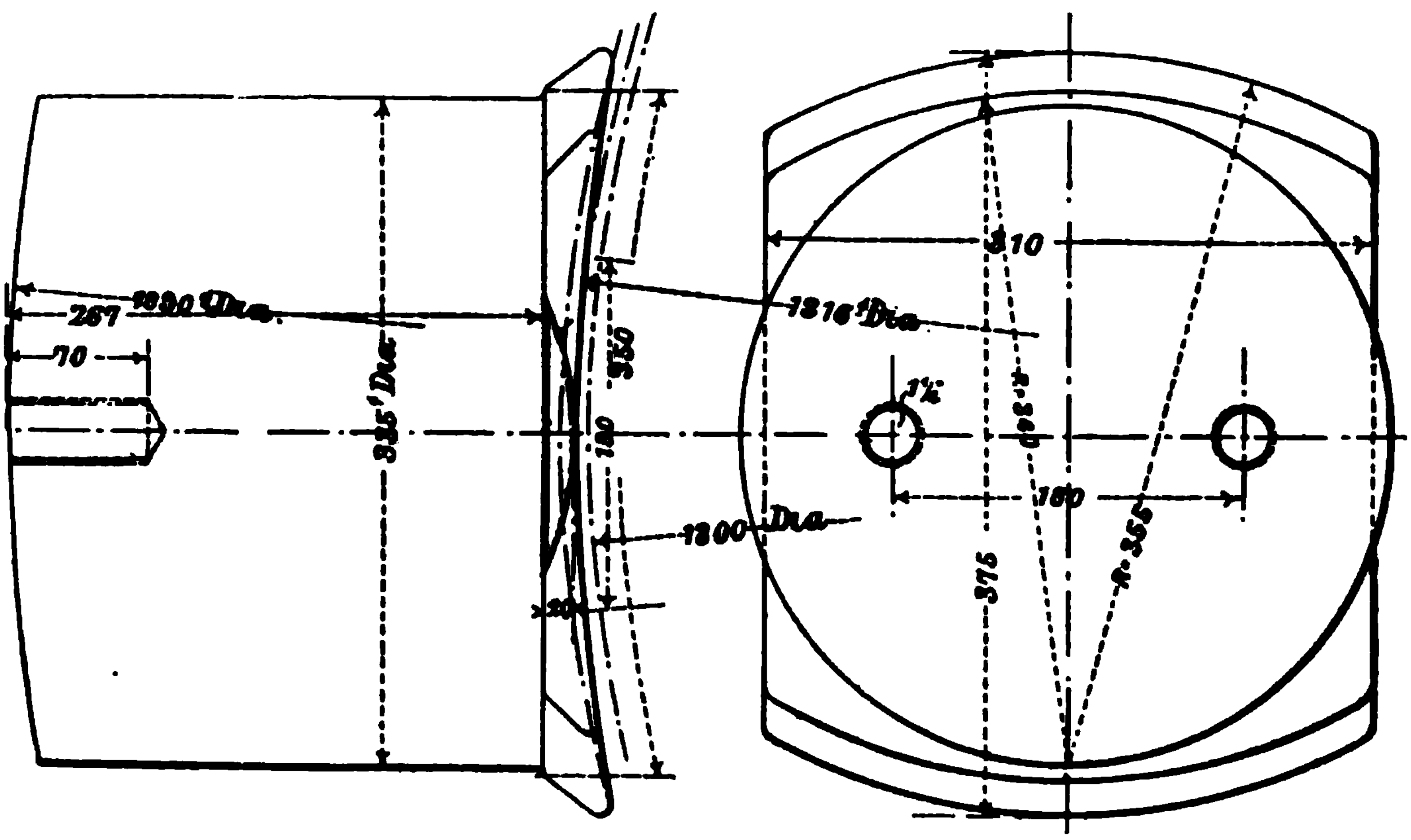
The use of continuous current motors usually affords the best solution in such cases, since by series parallel control and field adjustment, high efficiency at all speeds may be attained. Formerly, owing to incorrect understanding of the principles of design, to secure good commutation, it was believed that wide regulation by increased speed by weakening of the field was incompatible with good commutation. This is not the case; a machine designed with sufficiently low reactance voltage per segment will run sparklessly with constant brush position at all strengths of field.

Figs. 153 to 174, pp. 148 and 149, and Plates 9 and 11, are detailed drawings of an eight-pole 400 horse-power motor specially built for driving a rolling mill; Figs. 175 and 176, Plate 8, are general views of this motor.

The machine is chiefly interesting because of its wide speed regulation, and it is probably one of the largest compound wound motors extant having such a degree of regulation by shunt control. The machine is built with two commutators, and two windings on the armature, which for the low speeds are coupled in series,

and for the high speeds in parallel, by means of an ordinary series parallel controller. At low speeds the rating of the motor is only half that at the higher speeds, but more is not required, as the torque is the same, or greater, and this is what the service demands.

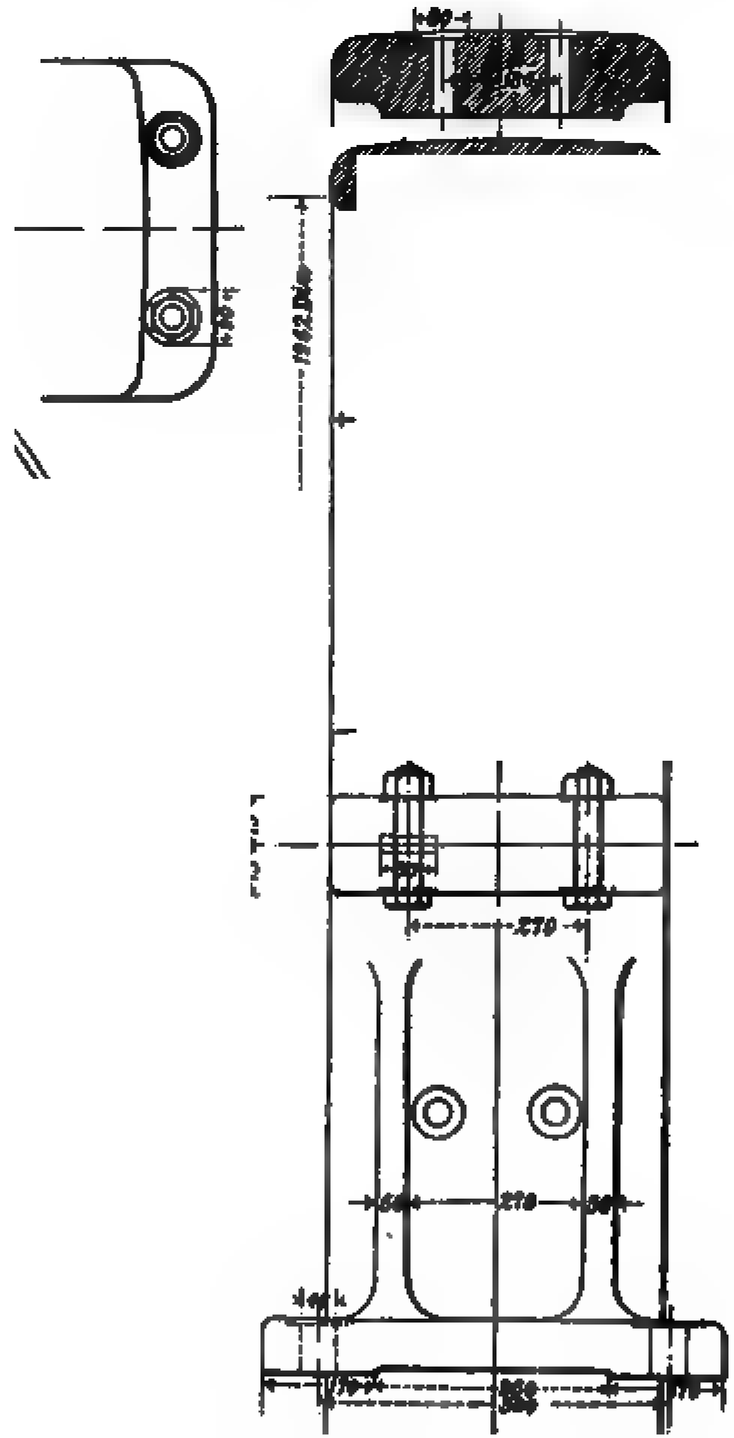
It drives a steel wire rolling mill, and for this purpose was specially built to withstand the enormous strains occasioned in this class of work. The motor is supplied with current from two 175-kilowatt turbine-driven generators situated about half a kilometre away. These generators are over-compounded so as not only to compensate for the voltage drop in the line, but also to raise the voltage at the motor end of the line from 500 at no



FIGS. 158 and 159.—Pole Piece for 400 Horse-power Motor.

load, to about 600 at full load. The motor itself is only compounded for 500 volts at no load to 550 volts at full load, and hence increased speed results with high loads. The object of this arrangement is to automatically increase the speed towards the end of the rolling, this being the time of heaviest load. The highest speed the machine was originally intended to run at was 450 revolutions, but it has been found necessary to go up to even 500 revolutions per minute when rolling some classes of steel which contain a large percentage of carbon.

The conditions of service are very severe; when running at the highest speed the load varies abruptly from 30 horse-power up to over 200 horse-power, then falls below 200 horse-power for a few seconds, and again rises in two or three abrupt steps to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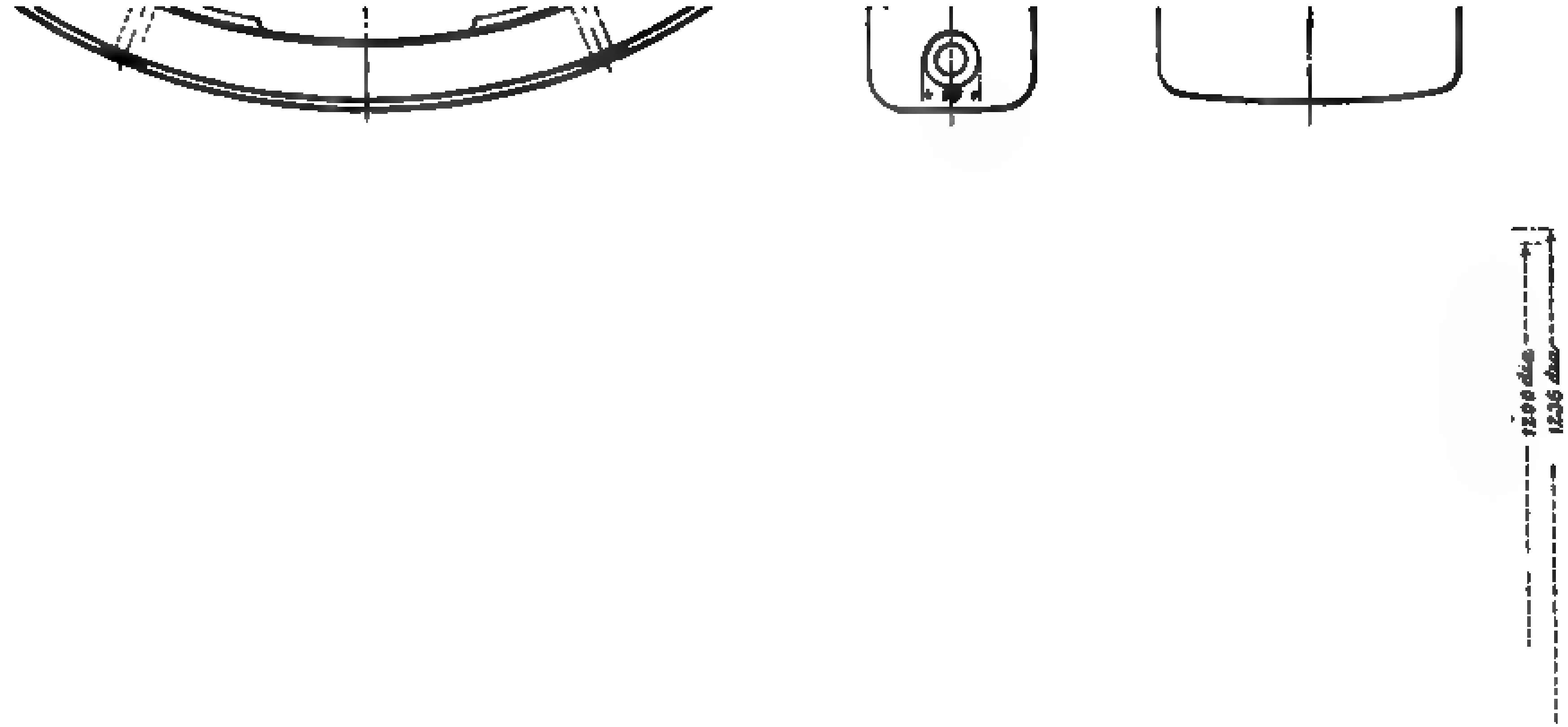
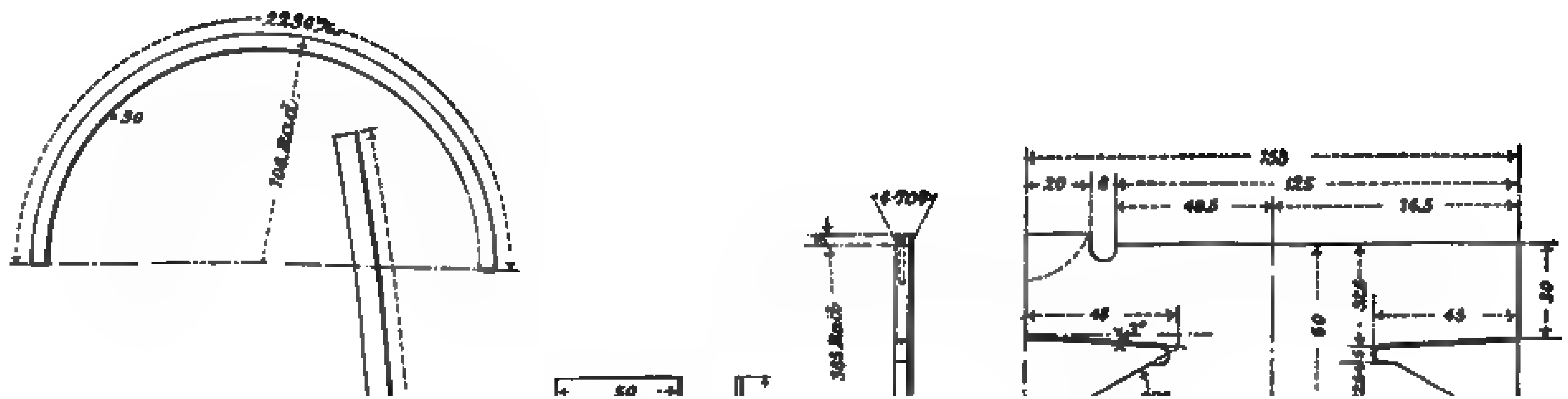
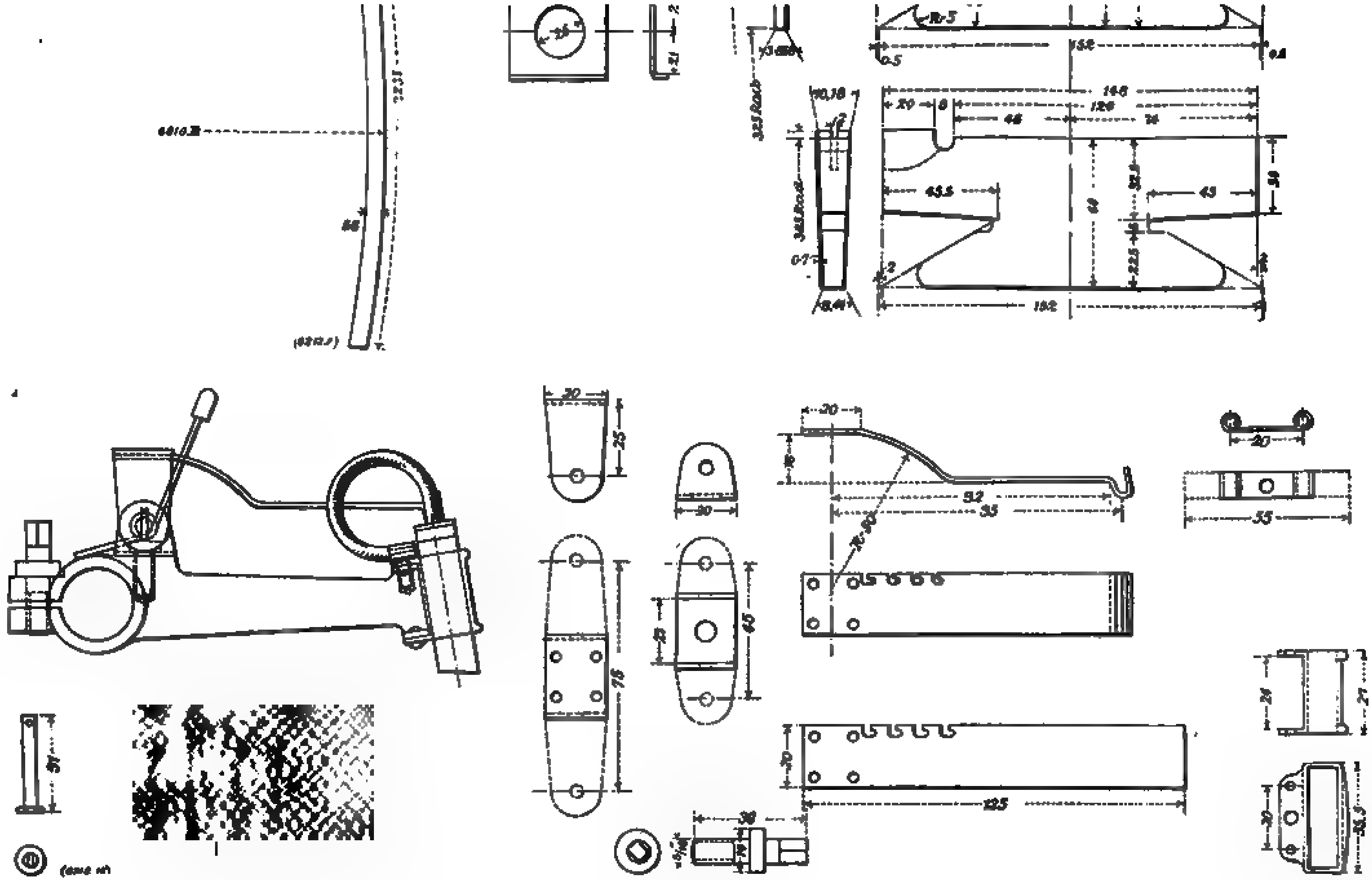


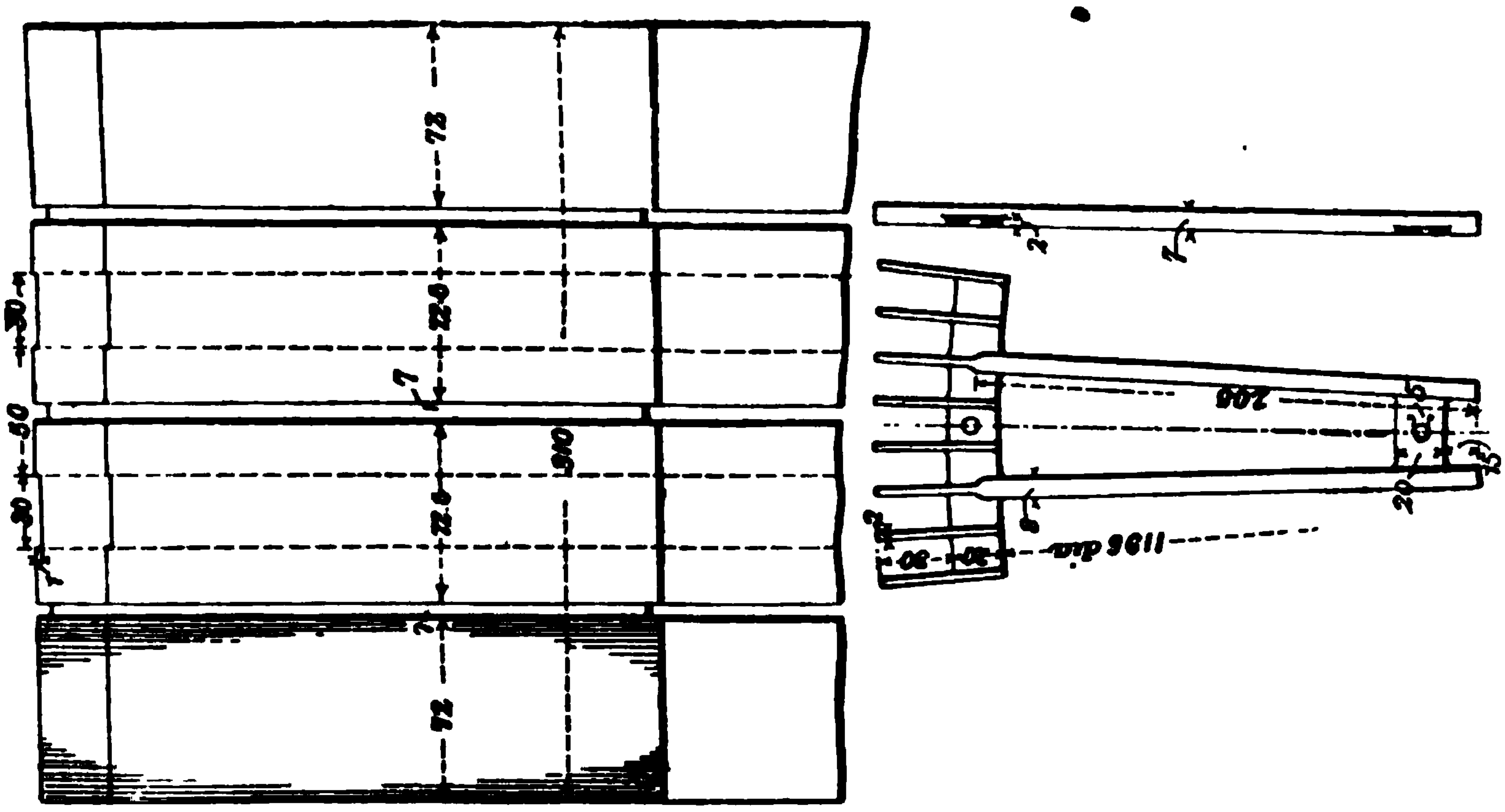
FIGS. 156, 157, 160 and 161.—Details of Frame and Armature for 400 Horse-power Motor, by A. V. Clayton (see page 147).


104 Rad.
50
150
20
125
48.5
76.5
60
45
48

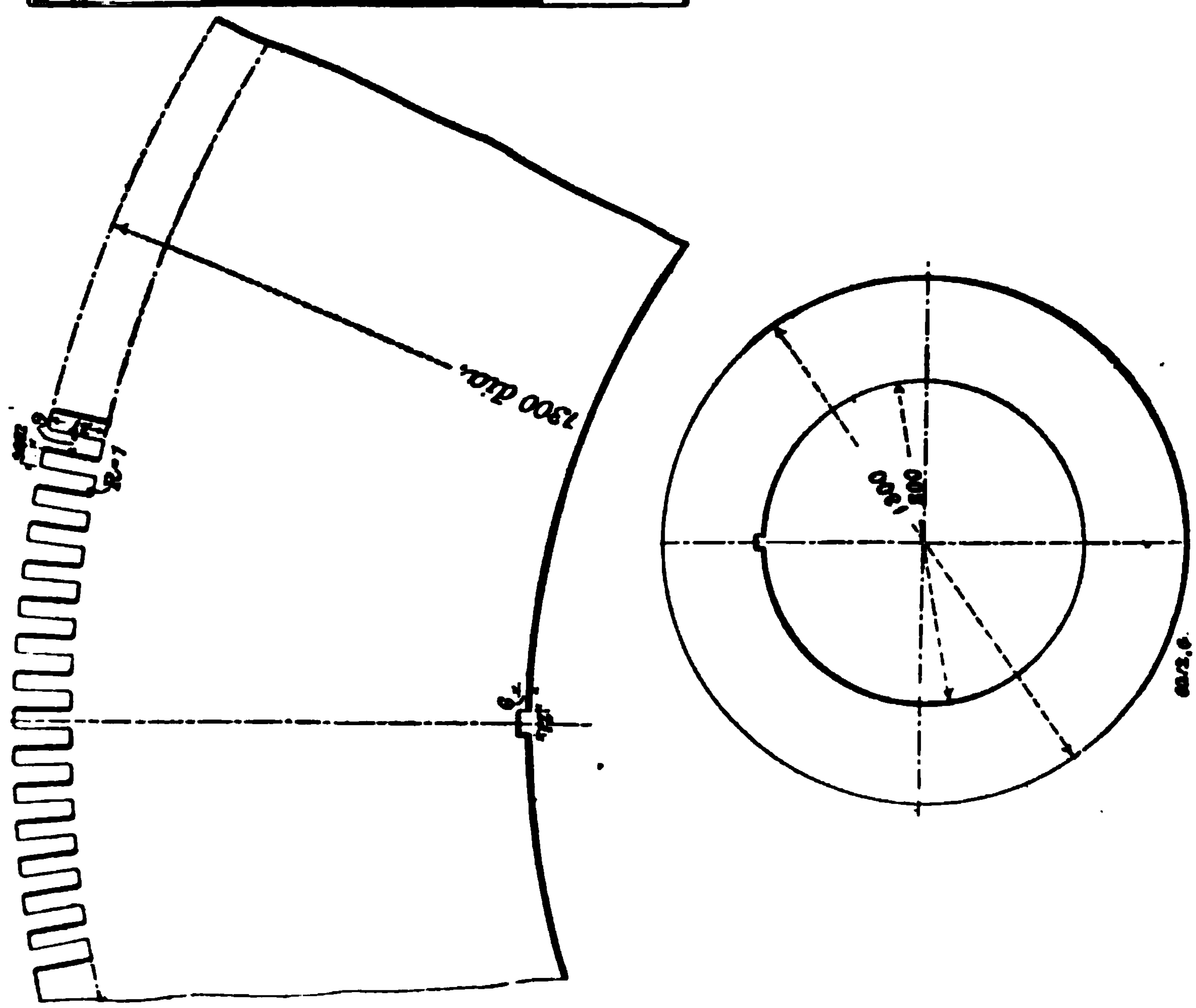


FIGS. 162, 163, 164, 165, 166, 171, 172, 173 and 174.—Details of Commutator and Brush Holders for 400 Horse-power Motor, by A. V. Clayton (see page 147).

over 400 horse-power, this cycle being repeated about every three minutes. It also frequently happens, after stopping the rolls from any cause, that the wire has become too cold, and when the



FIGS. 169 and 170.—Armature Ventilating Distance Pieces for 400 Horse-power Motor



FIGS. 167 and 168.—Armature Stamping for 400 Horse-power Motor.

stoppage has been freed and the wire fed into the rolls again, the load goes abruptly from less than 100 horse-power up to 600 horse-power.

This motor was designed for the Elektriska Aktiebolaget Magnet, of Ludvika, Sweden, by Mr A. V. Clayton, who has

kindly furnished the writer with the data and tests required for the following tabulated description:—

TABLE XXV.—SPECIFICATION FOR EIGHT-POLE, 400 HORSE-POWER, 135 TO 450 REVOLUTIONS PER MINUTE, 550-VOLT MOTOR.

Number of poles	8
Kilowatts input at full load	318
Normal rating in horse-power	400
Speed varies from 135 to 500 revolutions per minute; the normal speed is	450
Periodicity in cycles per second, at normal speed	30
Terminal voltage full load, 550 to 600; normal voltage is taken as	550
Terminal voltage, no load	500
Amperes imput, full load	580

(Dimensions in millimetres.)

Armature:—

External diameter	1300
Axial length of the winding	650
Diameter at the bottom of the slots	1246
Internal diameter of the laminations	800
Axial length of core between flanges	310
Effective length of core (magnetic iron)	265
Depth of the slot	27
Width of tooth + slot at periphery	18·2
Width of tooth + slot at bottom of slot	17·5
Width of the slot	9·0
Number of slots	224
Number of slots per pole	28
Width of tooth at periphery	9·2
Minimum width of tooth	8·5
Average width of tooth	8·9
Radial depth of the laminations—total	250
Radial depth of the laminations below slots	223
Number of ventilating ducts	3
Width of each duct	7
Effective length of core ÷ total length of core	·854
Height of uninsulated conductor	10·0
Width of " "	0·9
Cross section bare conductor, square centimetre	·09
Amperes per conductor	36·3
Amperes per square centimetre	404
Conductors per slot	8
Total copper cross section per slot	·72
Width × depth of slot, square centimetres	2·43
"Space factor" of slot	·30

Magnet Core :—

Length of the pole face parallel to the shaft	310
Diameter of the bore of the pole face	1316
Mean length of the pole arc	350
Ratio of pole arc to pitch	·68
Thickness of the pole shoe at the centre of arc	20
Radial length of the magnet core	267
Magnet core diameter	335
Radial length of the air gap, at centre	8
" " at pole tips	16
" " average	9
Distance between pole tips	166

Yoke :—

External diameter	2140
Internal diameter	1920
Thickness of yoke	110
Axial width	510
Radial thickness of the pole seat	15

Commutator :—

Diameter	770
Number of segments	448
Thickness of segment + insulation at periphery	5·4
Thickness of the insulation between segments	0·7
Thickness of a segment at the periphery	4·7
Length from external end to commutator connection	125

ELECTRICAL AND MAGNETIC DATA

Armature :—

Total induced voltage, full load	540
Terminal voltage, full load	550
Number of face conductors in both windings	1792
Number of slots	224
Number of conductors per slot	8
Total amperes to both commutators in parallel	580
Number of circuits	16
Amperes per circuit	36·3
Number of conductors in series between brushes	112
Mean length of a single turn, centimetres	183
Number of turns in series between brushes	56
Total length of conducting path between brushes, centimetres	10,250
Cross section of one conductor, square centimetres	·092
Cross section of all parallel conductors, square centimetres, one winding	0·72
Specific resistance at 60° Cent., ohm	·0000020
Resistance of winding from + to − at 60° Cent., one winding, ohm	·0284
IR loss in armature at 60° Cent., volts	8 2
IR loss in the series spools at 60° Cent., volts	1·1

Armature—continued.

IR loss at the brush contact surfaces, volt	0·9
Total internal IR loss, volts	10·2
Kilogrammes armature copper in both windings	132

Calculation of Sparking Constants:—

Volts per segment	9·8
Peripheral speed commutator in metres per second (A) ...	18·2
Length of the arc of brush contact	16
Length of the arc of brush contact + width one segment (B) ...	21
Frequency of commutation, cycles per second $=\left(\frac{1000\text{A}}{2\text{B}}=n\right)$	430
Width of a segment at the periphery, including insulation ...	5·4
Maximum number of coils short circuited under a brush ...	3
Turns per coil (q)	1
Maximum number of simultaneously commutated conductors per group (r)	12
Mean length of one turn, centimetres	183
Effective length of core, centimetres	26·5
"Free length" per turn (s)	130
"Embedded length" per turn (t)	53
Lines per ampere turn per centimetre of "free length" (u) ...	0·8
Lines per ampere turn per centimetre of "embedded length" (v)	4·0
Lines per ampere turn for "free length" ($u \times s$)	104
Lines per ampere turn for "embedded length" ($v \times t$) ...	212
Lines per ampere for "free length" $\left(\frac{r}{2} \times u \times s\right)=o$	624
Lines per ampere for "embedded length" $(r \times v \times t)=p$ ...	2540
Total lines linked with short-circuited coil per ampere $(o+p)$	3164
Inductance per segment, $\frac{q \times (o+p)}{10^8}=l$, henry	·0000316
Reactance per segment ($2\pi\, n\, l$) ohm	·085
Reactance voltage, volts	3·1

The reactance voltage in the above calculation was based on the assumption that the brushes on both commutators occupied the same angular position, but in actual practice the machine has the brushes for one commutator set permanently with about one segment backward lead, and the brushes of the other commutator with about 1·5 segments backward lead. It is claimed that this arrangement causes the currents in the short-circuited coils of the two windings to partly neutralise one another so far as relates to setting up a flux of self-induction, and that the reactance voltage is thereby materially decreased. No rocking gear is provided, the brushes being bolted fast in this position, and the machine has been loaded up to 600 horse-power at 500 revolutions per minute, the severest test for commutation, and throughout a long series

of tests no signs of sparking were observed. This was also the case during a one and a quarter hours' run at 640 amperes and

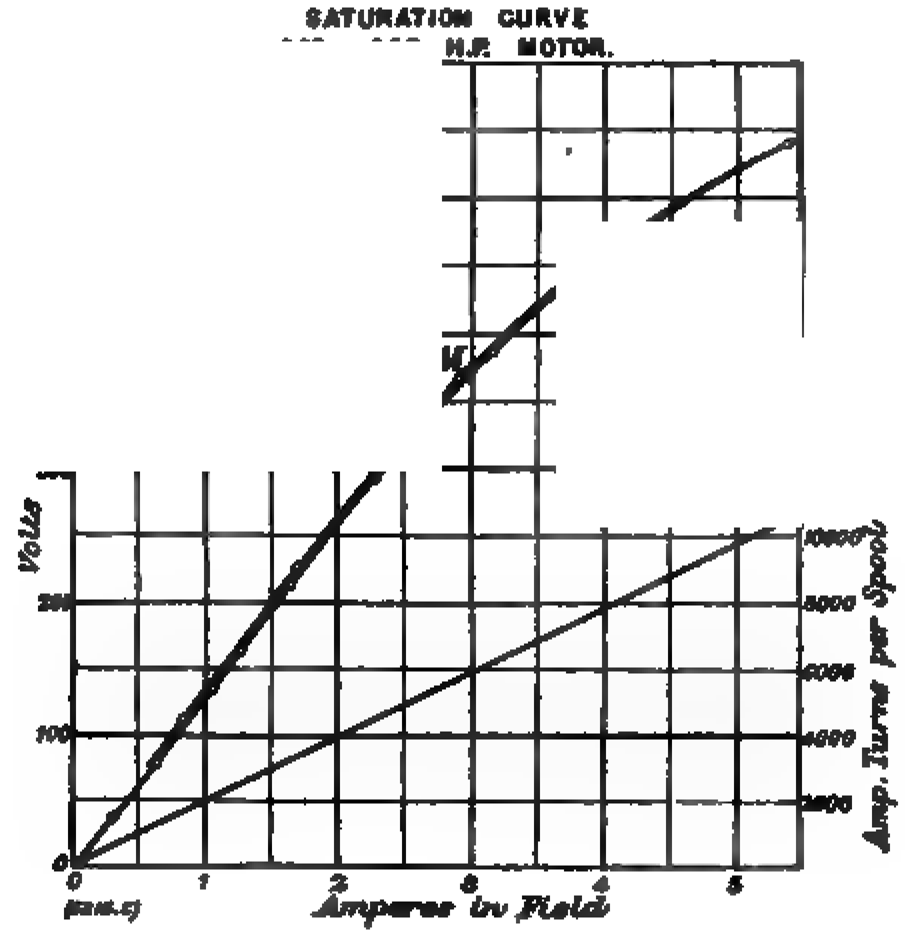


FIG. 177.

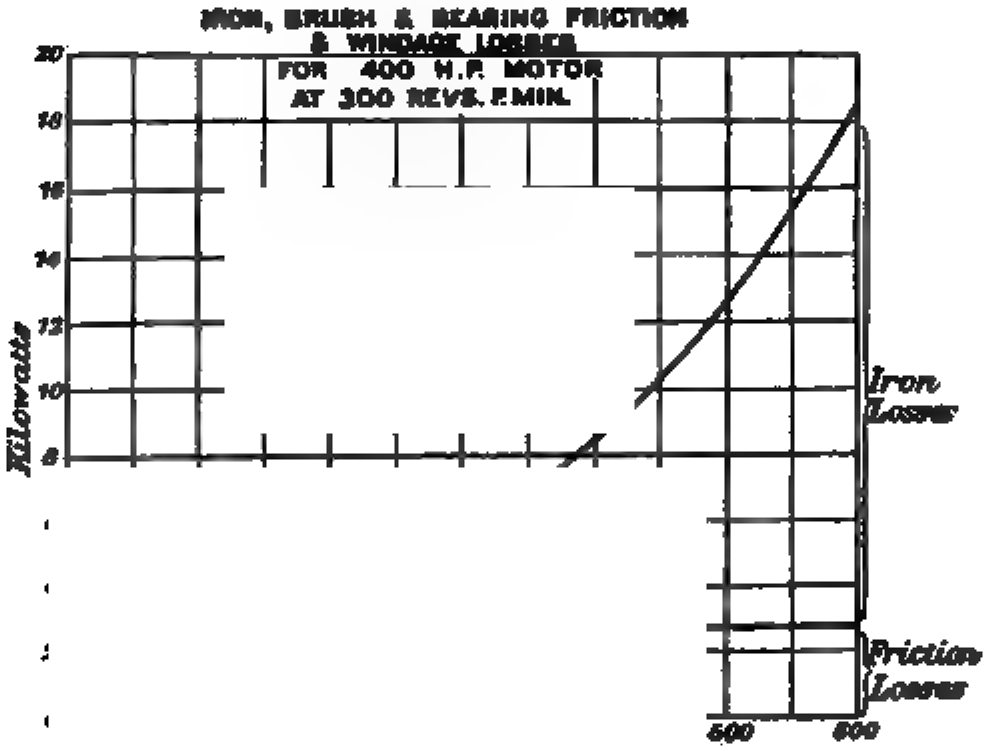


FIG. 178.

450 revolutions per minute, with only 450 volts. Figs. 177 to 181 are curves of saturation losses and efficiency.

The magnetic circuit calculations are carried out in parallel columns for 450 revolutions per minute and 300 revolutions per minute, with 550 terminal volts, and the two commutators in parallel. These are also equivalent to the commutators in series and 225 and 150 revolutions per minute.

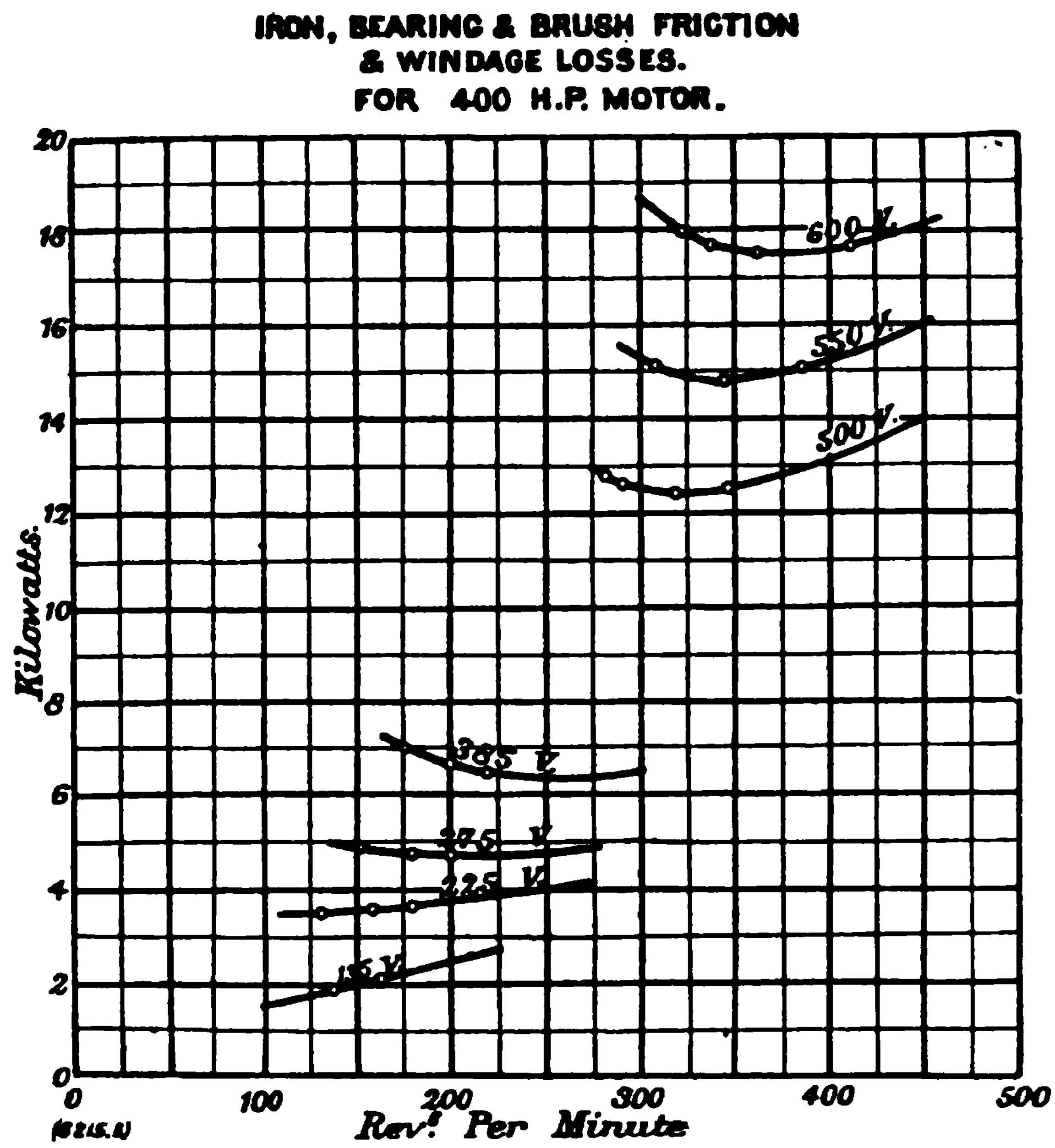


FIG. 179.

TABLE XXVI.—PARTICULARS OF MAGNETIC CIRCUIT.

	450 Revs. per Minute.	300 Revs. per Minute.
Flux entering armature per pole, full load megalines	8·05	12·10
Corresponding internal voltage	539	539
Corresponding terminal voltage	550	550
Leakage factor	1·10	1·10
Flux generated per pole, full load, megalines ...	8·85	13·3

Armature:—

Cross section of the core, square centimetres ...	1180	1180
Density, full load, c.g.s. lines	6800	10,300
Ampere turns per centimetre, full load	3	5
Magnetic length per pole, centimetres	20	20
Ampere turns, full load	60	100

	450 Revs. per Minute	300 Revs. per Minute.
Teeth:—		
Number of teeth per pole	28	
Number of teeth directly below a mean pole arc ...	19	
Percentage increase allowed for spread	10	
Total number of flux carrying teeth per pole ...	21	
Cross section of one tooth at root, square centimetres	22·5	
Total cross section at the bottom of these teeth, square centimetres	470	

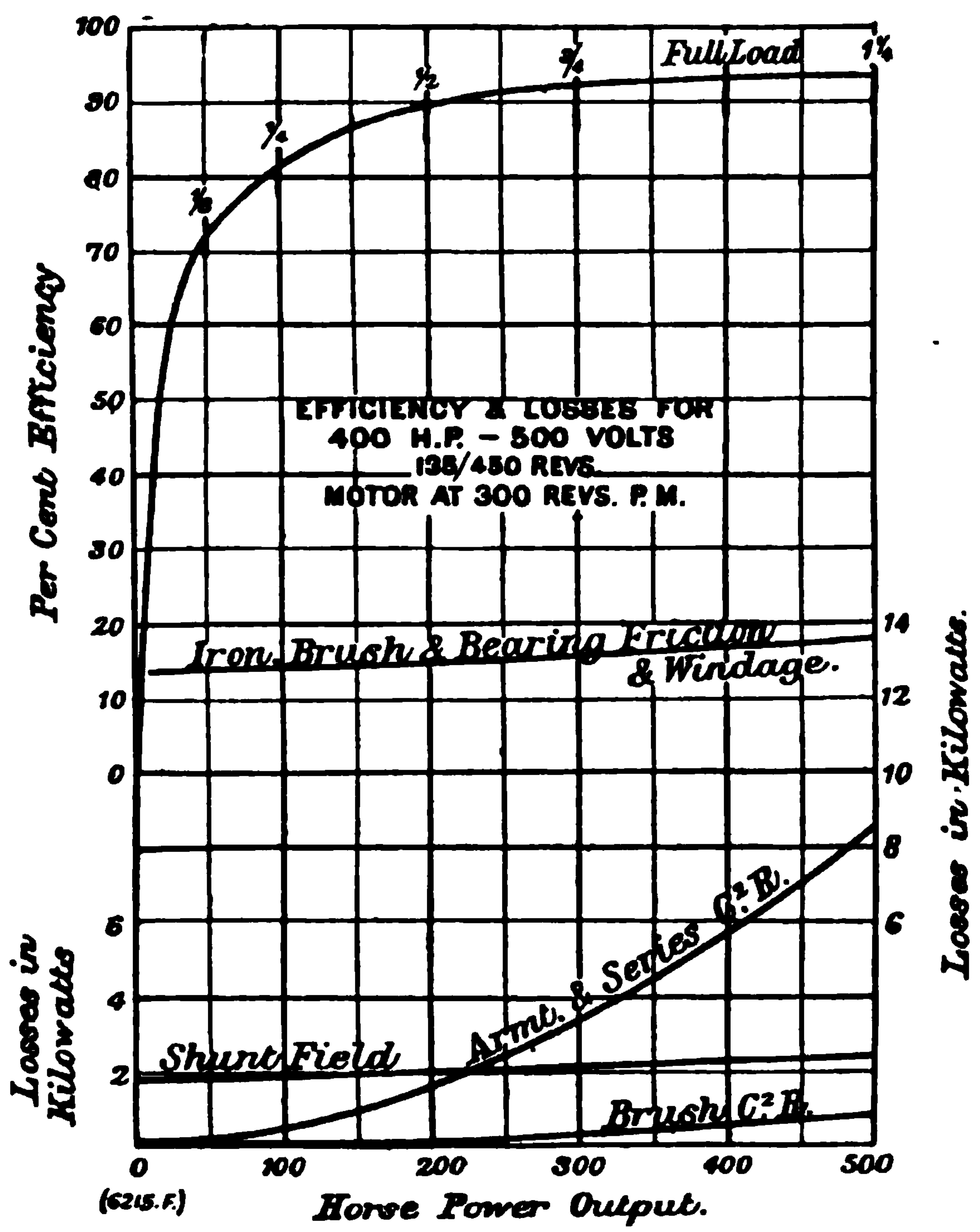


FIG. 180.—Efficiency Curves for 400 Horse-power Motor.

Apparent density, full load, c.g.s. lines	17,200	25,800
Mean width of tooth, millimetres	8·9	
Width of slot, millimetres	9·0	
Mean width of tooth ÷ width of slot	1·0	
Corrected density, full load, c.g.s. lines	17,200	22,800
Ampere turns per centimetre, full load	70	900
Length, centimetres	2·7	2·7
Ampere turns, full load	190	2430

		450 Revs. per Minute.	300 Revs. per Minute.
Air Gap:—			
Cross section at pole face, square centimetres	...	1090	1090
Density at pole face, full load, c.g.s. lines ...	...	7400	11,100
Mean length of air gap, iron to iron, centimetres	...	·9	·9
Ampere turns, full load	...	5300	8000
Magnet Core:—			
Cross section, square centimetres	...	885	885
Density, full load, c.g.s. lines	...	10,000	15,000
Ampere turns per centimetre, full load ...	...	8	29
Magnetic length, centimetres	...	29	29
Ampere turns, full load	...	230	840

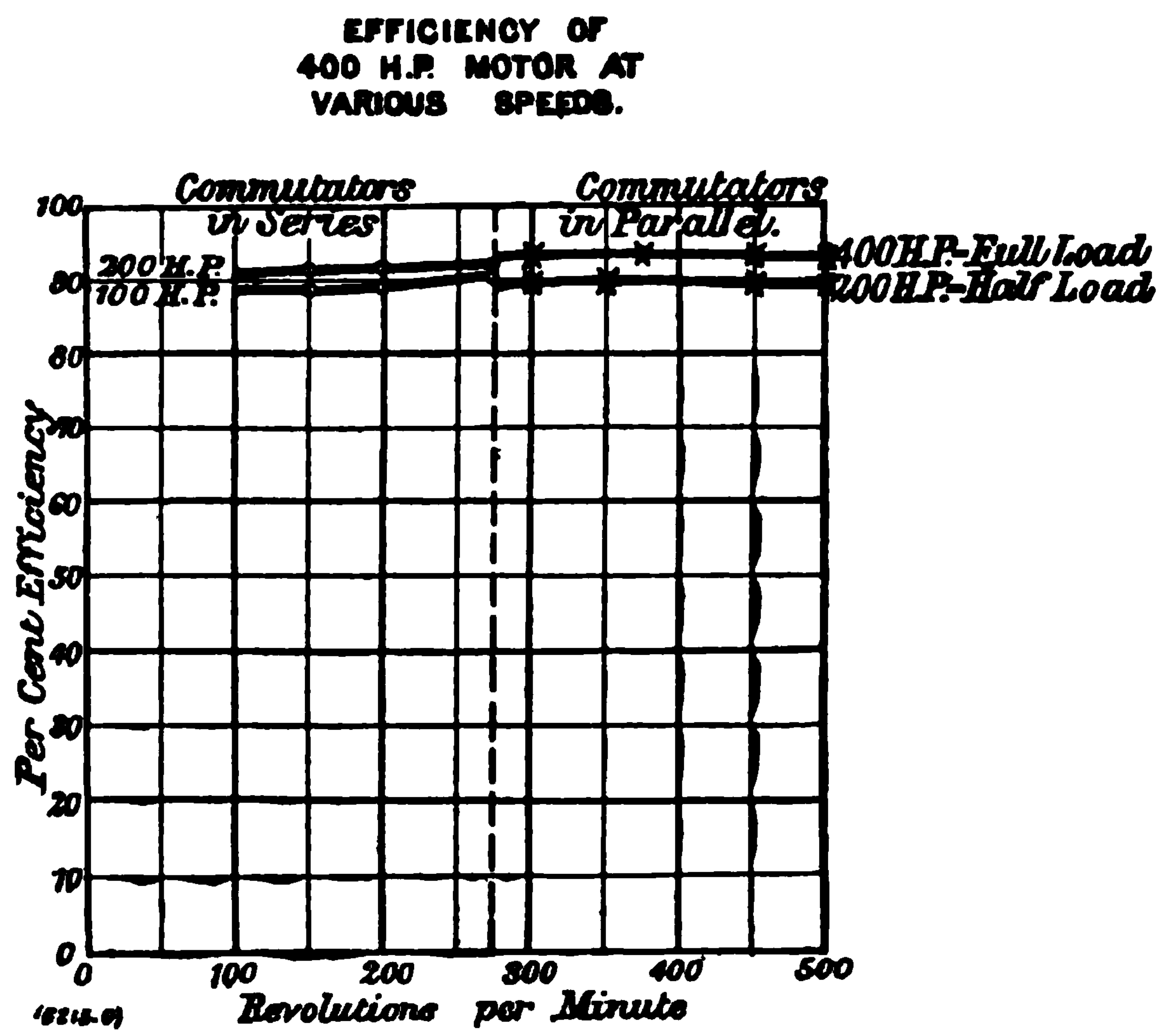


FIG. 181.

Yoke:—			
Cross section, square centimetres	...	1120	1120
Density, full load, c.g.s. lines	...	7900	11,900
Ampere turns per centimetre, full load ...	...	6	11
Magnetic length, per pole, centimetres ...	...	39	39
Ampere turns, full load	...	230	430
Ampere Turns per Spool:—			
Armature core	...	60	100
Armature teeth	...	190	2,430
Air gap	...	5300	8,000
Magnet core	...	230	840
Yoke	...	230	430
Total number of ampere turns per spool ...	...	6010	11,800
Observed values (see Fig. 206)	...	5400	10,500

The following calculations for the shunt spool winding relate to the strongest excitation attainable at 550 volts:—

TABLE XXVII.—SHUNT SPOOL WINDING CALCULATIONS.

Volts available at shunt terminals	550
Voltage per shunt spool at 60° Cent.	69
Internal diameter spool, centimetres	33·5
Radial depth of winding, „	7·0
External diameter of spool, „	47·5
Internal periphery of spool, „	105·5
External periphery of spool, „	150·0
Mean length of one shunt turn, metres (a)	1·28
Ampere turns per shunt spool (b)	11,200
a b	14,300
$\cdot 000176 \times a^2 b^2$	36,000
Axial length of shunt spool, centimetres	19
Cross section of shunt spool winding, square centimetres (r)	133
"Space factor" of shunt spool (s)	·60
Cross section copper in shunt spool ($t = r \times s$)	80
Cubic centimetres copper in shunt spool (100 a t)	10,200
Kilogrammes copper per shunt spool (1 cubic centimetre copper = ·0089 kilogramme)	91
Watts per shunt spool $\left(\text{watts} = \dfrac{\cdot 000176 \times a^2 b^2}{\text{weight in kgs.}}\right)$	396
External cylindrical surface per spool, square decimetres ...	28·5
Watts per square decimetre of external cylindrical spool surface	13·9
Amperes per shunt spool (watts ÷ volts per spool)	5·75
Turns per shunt spool	1940
Cross section copper per turn, square centimetres	·0415
Current density in amperes per square centimetre	140
Diameter of bare copper conductor	2·30
Diameter of insulated copper conductor	2.45
Insulation employed on conductor	S.C.C.
Watts in all shunt spools at 60° Cent.	3160
Weight total shunt copper in kilogrammes, all spools ...	728
Resistance of eight shunt spools at 60° Cent., ohms	96

There are two separate series windings, each connected to its own commutator, so as to secure the full compounding whether the commutators are coupled in series or in parallel. Each winding is, for the sake of simplicity in construction, carried out on every other pole.

SPOOL WINDING (SERIES).

Ampere turns (series), full load	1800
Total amperes of the machine, full load per winding	290
Amperes in the series diverter rheostat...	90
Amperes in the series winding	200
Turns per spool (series)	9

Dimensions of the conductor, millimetres	30 × 2
Number in parallel	3
Total cross section, square centimetres	1·8
Amperes per square centimetre	111
Mean length of turn, centimetres	140
Total length of conductor per series spool, centimetres ...	1260
Resistance per spool at 60° Cent.	·0014
Watts lost per spool at 60° Cent.	56
Cylindrical surface, square decimetres	12·5
Watts lost per square decimetre at 60° Cent.	4·5
Resistance of four series spools at 60° Cent.	·0056
Watts lost in eight series spools at 60° Cent.	448
Weight of copper per series spool, kilogrammes	20
Total weight of series copper, kilogrammes	160
Total cross section of copper per series spool	16·2
Total cross section of winding space per series spool	40
"Space factor" of the series spool	·40

Armature Copper Loss:—

Resistance of the winding from + to − at 60° Cent., ohm ...	·0284
Total amperes to commutator	290
Watts lost in armature copper at 60° Cent., one winding ...	2380

Core Loss:—	450 Revs. per Minute.	300 Revs. per Minute.
Weight of the armature teeth, kilogrammes	110	
Weight of the armature core, „ ...	1490	
Total weight of armature laminations, kilogrammes	1600	
Flux density in the core, kilolines (D) ...	6·8	10·3
Periodicity, cycles per second (N)	30	20
$D \times N \div 100$	2·04	2·06
Watts lost in iron per kilogramme (from Fig. 21)	5·1	5·3
Total core loss (estimated), watts	8150	8450
Total core loss (observed), „		11,700
Armature copper loss		4760
Armature iron loss		11,300
Total armature loss		16,060
Circumference, decimetres		40·8
Axial length of the winding, decimetres		6·5
Peripheral surface, square decimetres		265
Watts per square decimetre of peripheral surface		60

Commutator Losses:—

Length of brush contact arc, millimetres ...	16
Width of brush, millimetres	32
Contact surface per brush, square centimetres	5·1

	450 Revs. per Minute.	300 Revs. per Minute.
Commutator Losses—continued.		
Number of brushes per pole per commutator	3	
Total number of positive brushes	12	
Contact surface of all positive brushes, square centimetres	61	
Current strength of the machine, amperes per commutator	290	
Amperes per square centimetres of brush contact surface	4·75	
I^2R loss in watts per ampere, from Table XVIII., p. 104	1·8	
I^2R loss at brush contacts, positive plus negative per commutator	520	
Peripheral speed of commutator in metres per second	18·2	12·2
Brush friction loss in watts per ampere, from Table XIII., p. 104	3·0	2·0
Brush friction loss in watts	870	580
Total commutator loss, watts per commutator	1390	1100
Commutator Temperature Increase:—		
Circumference, decimetres	24·2	
Length of commutator surface, decimetres	1·25	
Cylindrical surface of commutator, square decimetres per commutator	30·2	
Watts per square decimetre of cylindrical surface	43·5	36·5
Efficiency at 60° Cent.:—		
Iron loss, watts	11,300	11,700
Watts lost in armature copper	4,760	4,760
Watts lost at the brush contact resistance at the commutator	1,040	1,040
Brush friction loss at the commutator ...	1,740	1,160
Friction loss at bearings and windings ...	1,700	1,540
Watts lost in shunt winding	400	1,500
Watts lost in series winding	450	450
Watts lost in shunt rheostat	700	700
Watts lost in series diverter rheostat ...	200	200
Total of all losses	22,290	23,050
Output at full load, watts	295,000	295,000
Input at full load, watts	317,290	318,050
Commercial efficiency at full load	93·0	92·6

CHAPTER IX

FORM WOUND ARMATURE COILS

§ 1. **The Design of Coils by the Alioth Company.**—In the descriptions of these various motors, considerable differences will have been noticed in the shapes of the form wound coils, and

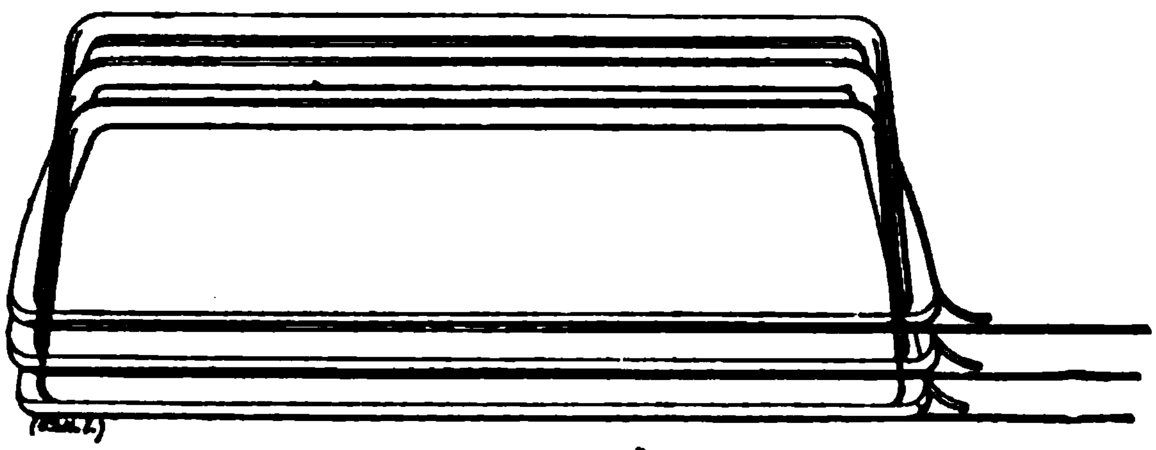
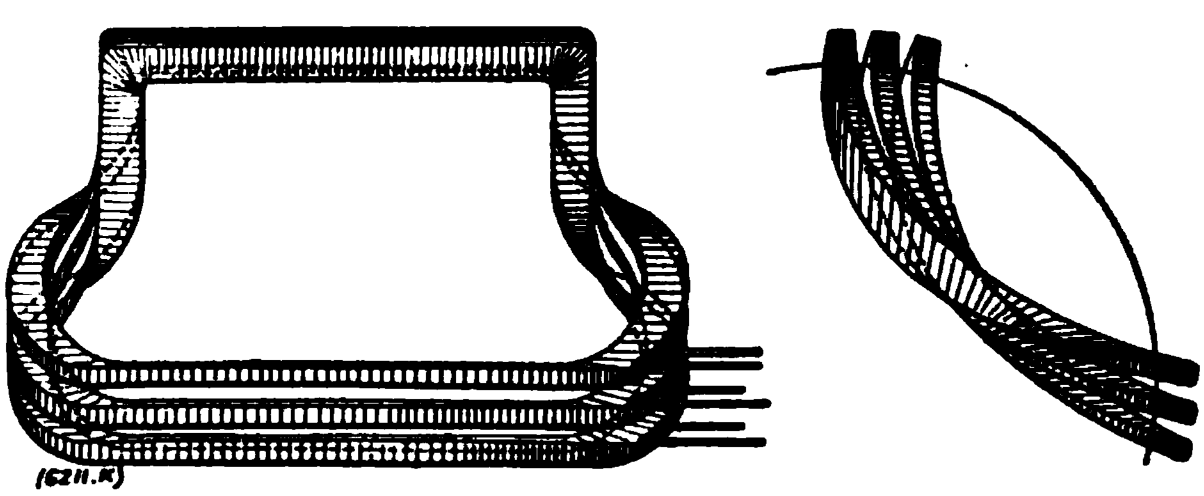


Fig. 182.—Alioth's Form Winding.

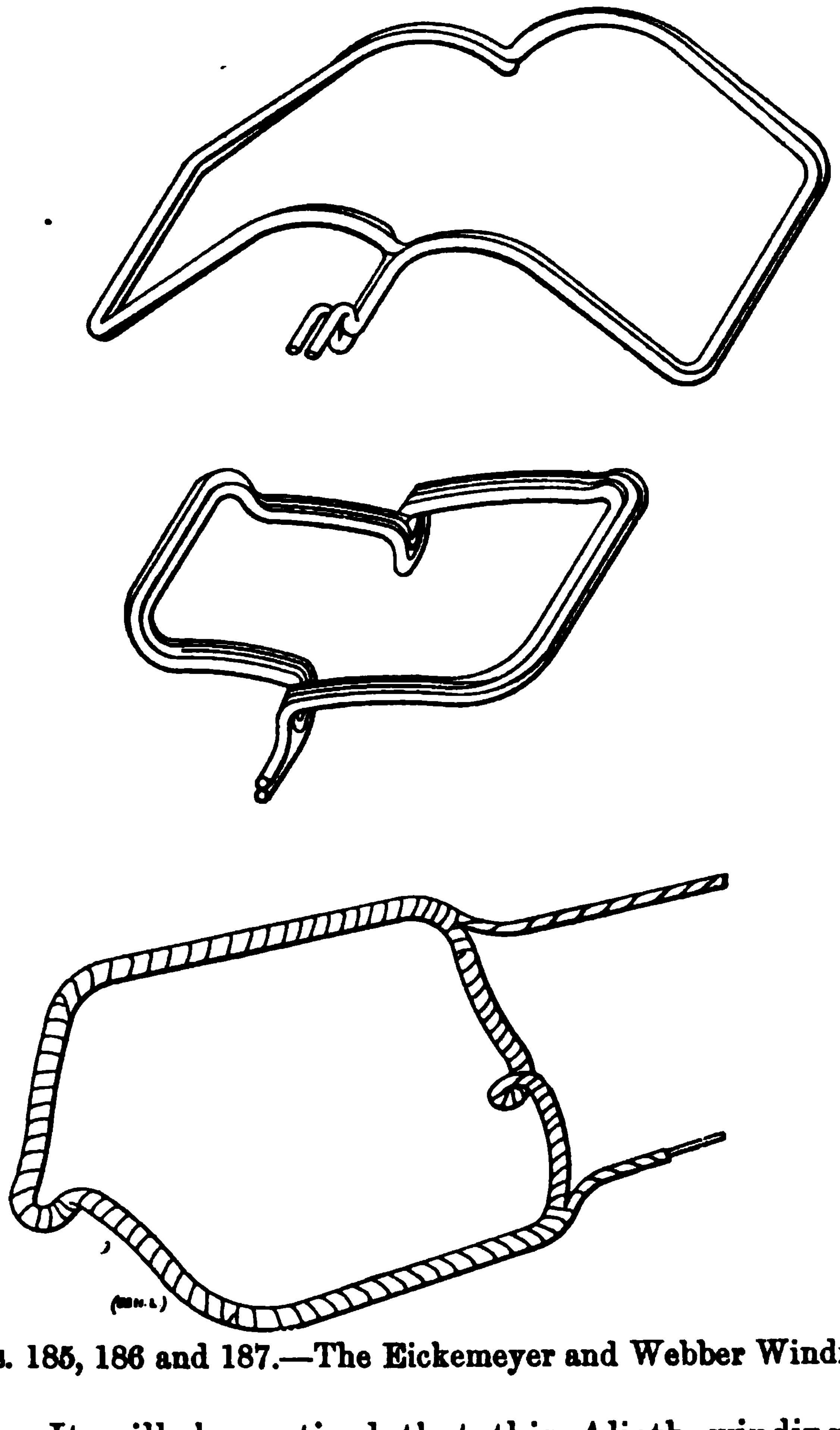
in the details of their arrangement. The design and construction of these coils is an exceedingly important detail of small motors, especially of those for such speeds and voltages as have more than



Figs. 183 and 184.—Alioth's Form Winding.

one turn per commutator segment. One of the earliest methods of form winding is described in the Alioth Company's D.R.P. 34,783, of March 17th, 1885. The illustration in the patent is reproduced in Fig. 182. A different and more clear idea of the winding is, however, obtained from Fig. 184, which is reproduced

from Arnold's *Ankerwicklungen und Ankerkonstruktionen,* page 323 of the third edition. Arnold states that it is the winding requiring the least length of wire. This, of course, is an important point, since it contributes to increased efficiency, decreased inductance, decreased weight of copper, and more compact con-



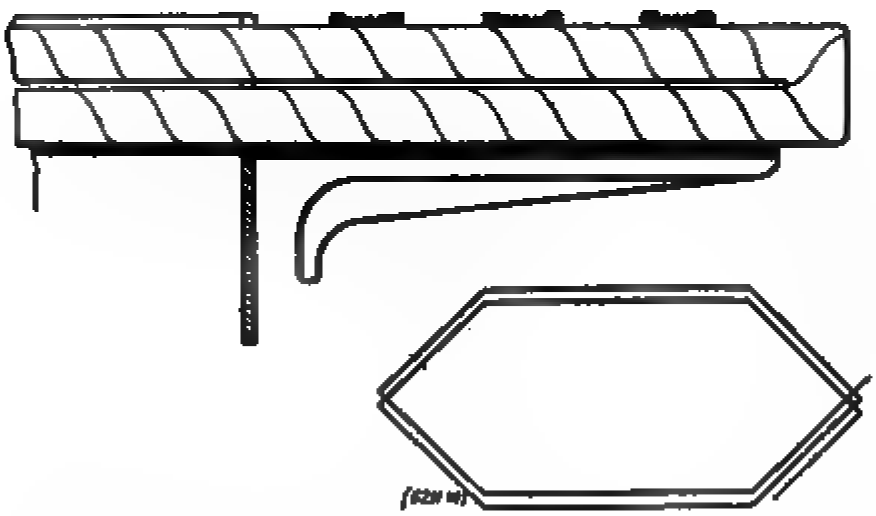
FIGS. 185, 186 and 187.—The Eickemeyer and Webber Windings.

struction. It will be noticed that this Alioth winding is more free from sharp bends than many of the other form windings.

§ 2. **Eickemeyer's Form Wound Coils.**—The Eickemeyer winding (D.R.P. 45,413, of February 14th, 1888) is the best known, and for many years most widely used, form winding. Figs. 185 and 186 are reproduced from the patent, and represent perspective views of coils for two and four-pole motors. The Eickemeyer

patent describes winding forms for use in winding such coils. One of the many, more modern, methods is described in Webber's U.S.A. patent No. 561,636, of 1896. A perspective view of a coil, as wound by this method, is shown in Fig. 187.

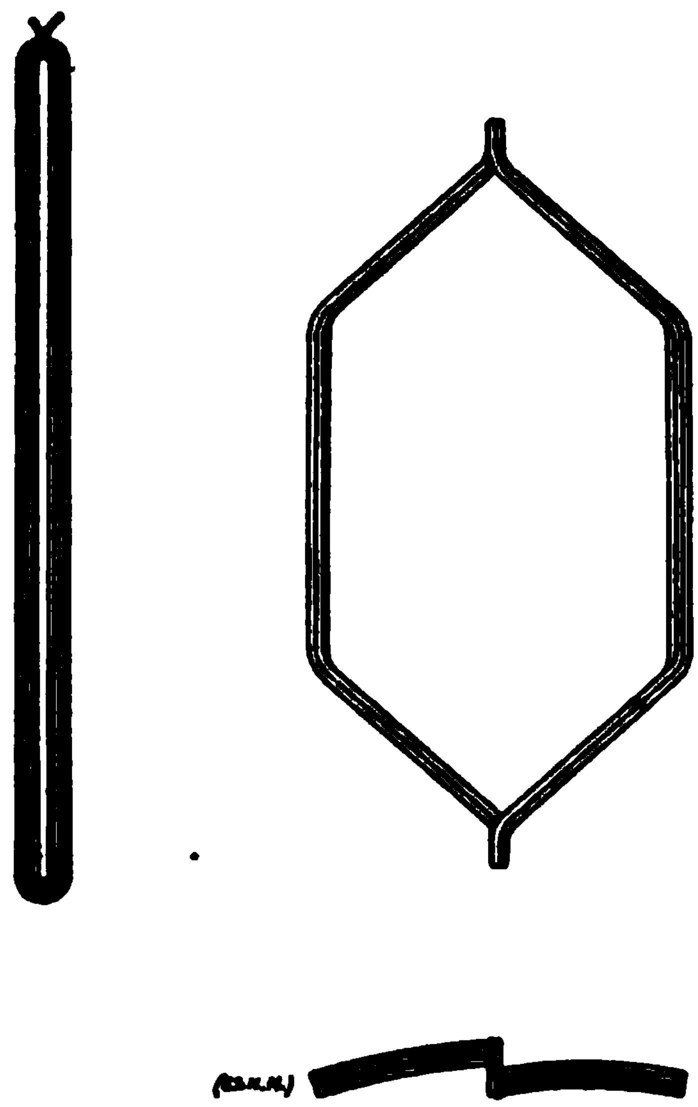
§ 3. **Recent Improvements in Form Wound Coils Design.** — Other methods of constructing form wound coils are described in Langdon-Davies & Soames' British patent 7373, of 1900, and in Rothert's U.S.A. patent No. 660,659.



Figs. 188, 189 and 190.—Eickemeyer Type of Winding, End Connections.

The end connections of windings of the Eickemeyer type, as well as of bar windings, were at first generally arranged in planes, approximately normal to the armature surface, as shown in Fig. 188. By that arrangement one often encountered limitations with reference to the space available for the end connections, these becoming crowded at their lower ends, especially on armatures of small diameter. In a modified type which has been frequently employed, this difficulty is partly overcome by arranging the end

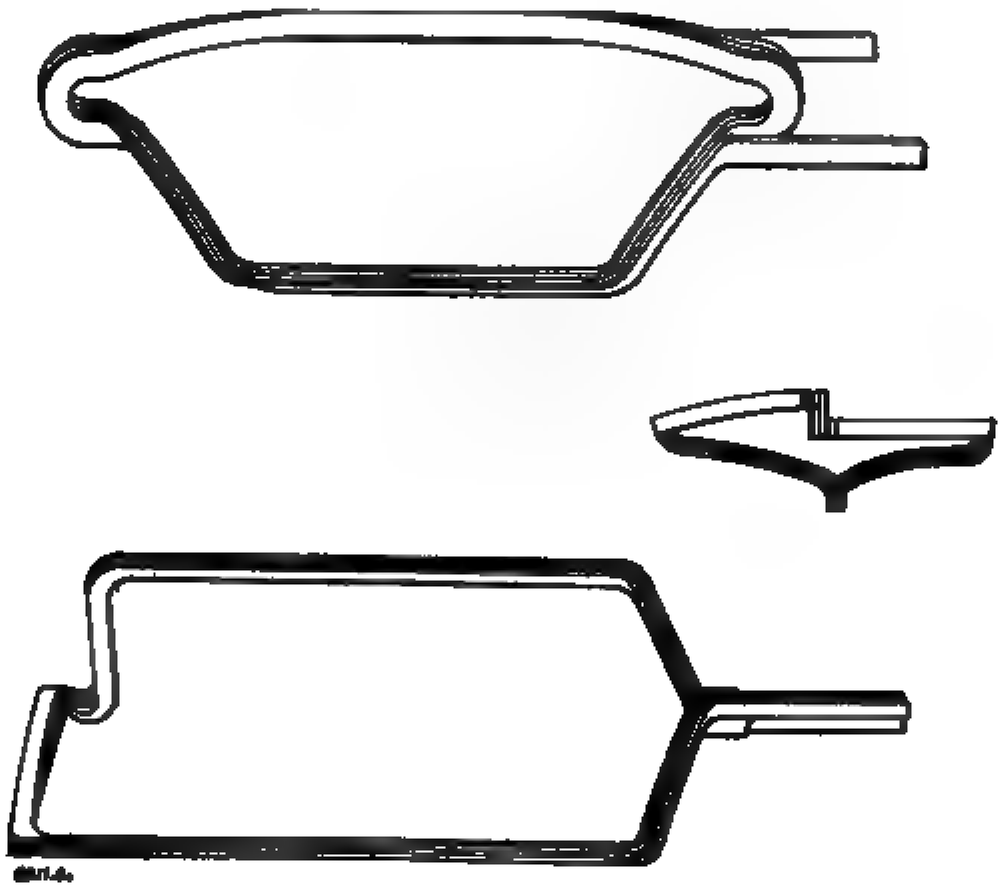
connections on a conical surface, as shown in Fig. 189, thus giving more room for the lower ends. Nowadays the most frequently-used construction is that shown in Fig. 190, where the end connections are arranged on the same cylindrical surface as the face conductors. This leads to a better mechanical and thermal design. Moreover, it has not been found necessary to devote much more space, lengthwise, to the armature winding, although the tendency is to that result. An objection is that it is unsuitable for one layer windings. Figs. 191, 192, and 193 show a coil as wound by the method described in Persson & Thomson's U.S.A.



Figs. 191, 192 and 193.—Persson and Thomson's Winding.

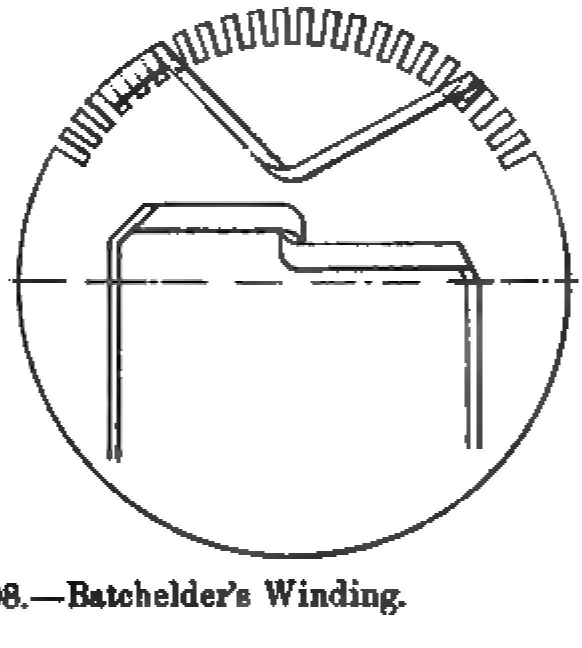
patent No. 539,881. A Swiss patent, No. 21,731, of 1900, of Mallett's, illustrates such a coil as wound with flat strip. Fig. 194 is a perspective view of a 4-turn coil by this method. Fig. 195, taken from the same patent, is also a perspective view of a coil wound from flat strip. Fig. 196 is an end view of this coil, which is, at the commutator end, wound in an extension of the armature surface, and at the other end has the end connections in planes normal to the armature surface. A modern type intermediate between Figs. 188 and 190 was devised by Batchelder, U.S.A. patent No. 596,136, of 1897. This is shown in Figs. 197 and 198. The end connections start off similarly to the winding

of Fig. 190, bend round, and ultimately return to the lower conductors similarly to the winding of Fig. 188.



FIGS. 194, 195 and 196.—Mallett's Flat Strip Winding.

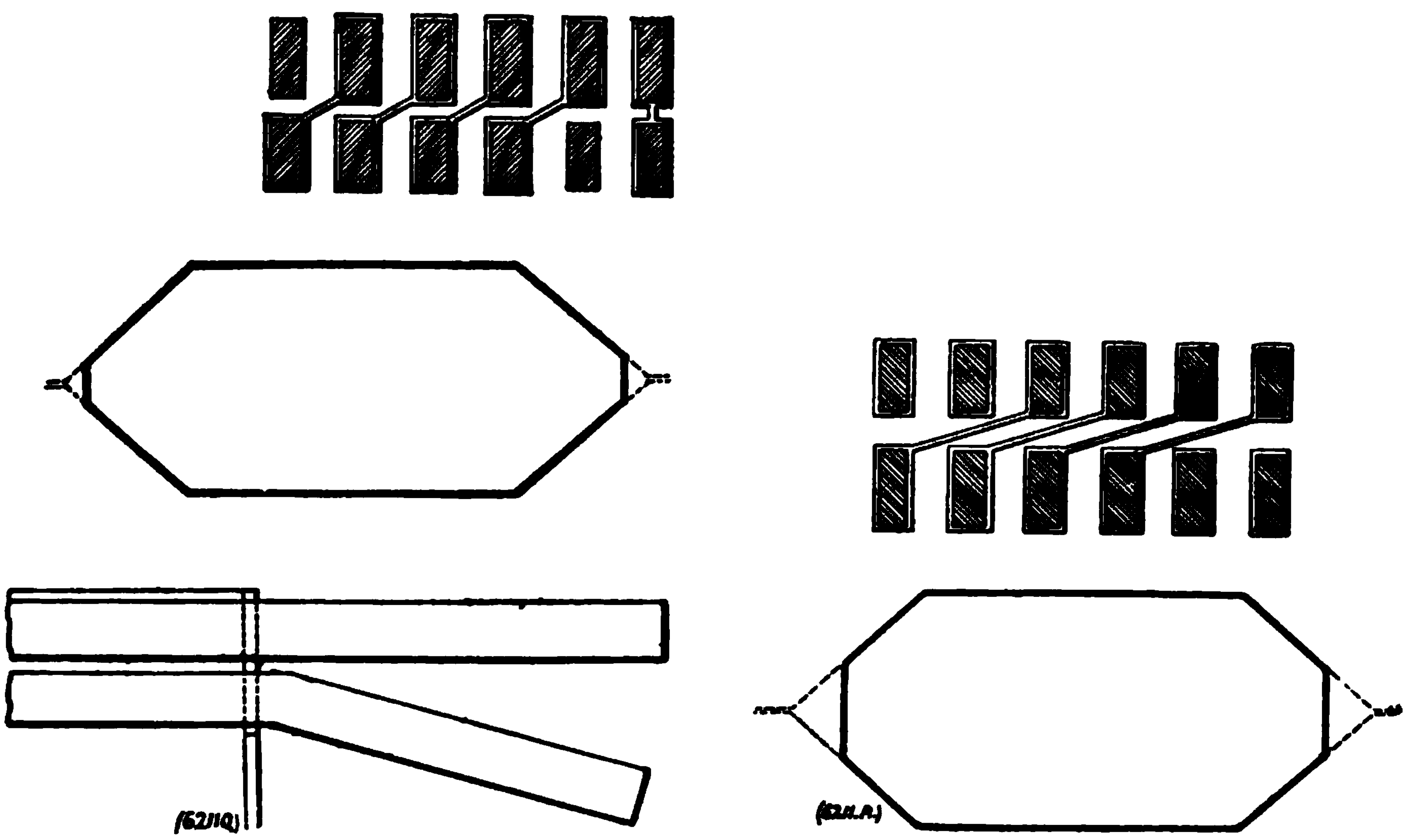
In the windings of Figs. 188 to 198 the outgoing and returning sides of the end connections lie close together; no considerable



FIGS. 197 and 198.—Batchelder's Winding.

internal space is enclosed by them; all are, moreover, distinguished by the presence of a quirk at the extreme end where they bend back.

If, as in Fig. 199, a short diagonal ligament is used to connect the top to the bottom conductor in windings of the type shown in Fig. 190, the length which the winding occupies in a direction parallel to the shaft is decreased. This is shown diagrammatically in Fig. 200. But to make the winding very much shorter by increasing the length of these ligaments, it is necessary, in order to provide the requisite room for them, to bend down the lower conductor, as shown in Fig. 201. This permits of making the ligament considerably longer, as shown in Fig. 202, and the length of the winding parallel to the shaft is still shorter, as shown in Fig. 203.



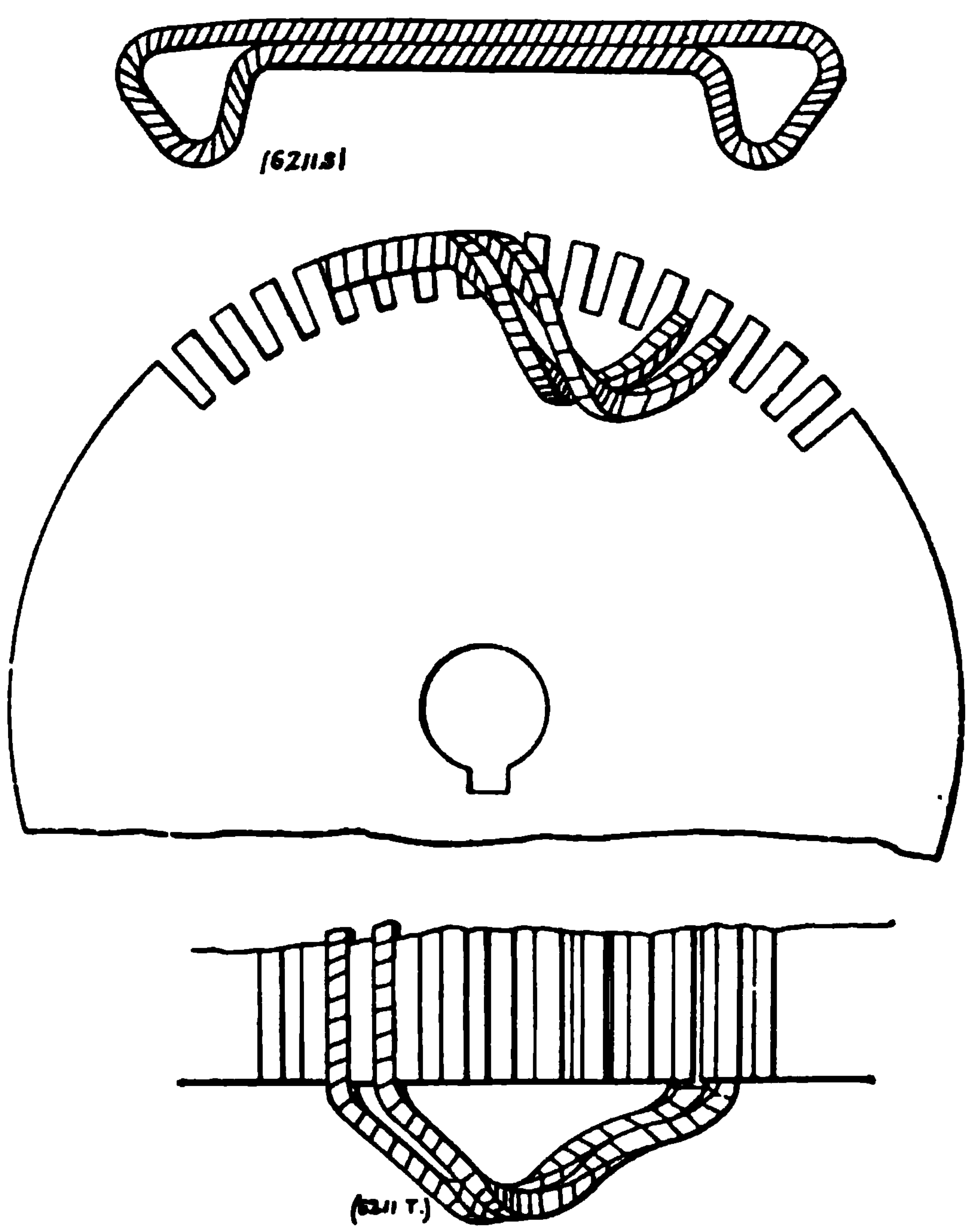
FIGS. 199, 200, 201. Diagonal Ligament Connections. FIGS. 202, 203.

Windings employing these principles to a greater or less extent have been proposed from time to time (compare also the Alioth winding, Fig. 184), and one of their characteristics is that an annular space is formed by the inner contour of the end connections when the coils are all in place. They are also generally characterised by a smooth gradual bend or twist at the end, as distinguished from the abrupt quirk of the other types. They may also be adapted to use as "one-layer" windings when required.

§ 4. **The Shortest Possible Coil.**—It has occurred to the writer (see British Patent No. 17,489, of 1901) that the shortest coil would have its connections formed on the principle of an

equilateral triangle bent out of its original plane, and with the corners rounded off and otherwise modified to adapt it to joining the two corresponding face conductors by an essentially three-sided equilateral end connection.

For the case of a two-layer winding it might sometimes be convenient to first wind the coils in one plane, and of the shape shown in Fig. 204. After opening out the coils, and assembling them on the armature core, they will, in end and plan views, have



FIGS. 204, 205 and 206.—Hobart Winding.

the appearance shown in Figs. 205 and 206. From the diagrammatical representation in Fig. 207, it will be seen that the intermediate member, B, of the three equal components of the end connection, lies in principle on a conical surface, as does also the member C. This and the inclination of line B, as seen in vertical projection, together with the general tendency to conform to the proportions of an equilateral triangle, serve to differentiate this winding from others, amongst which one sometimes finds instances where the intermediate link is generally shorter than the others, and lies in a plane perpendicular to the shaft, the intermediate

link itself being sometimes radial and sometimes inclined. In other windings sometimes occurring, the component B lies in a conical surface; but the component C lies in a plane perpendicular to the plane of the shaft, and in still another the component A constitutes a straight prolongation of the face conductor. The dotted lines in Fig. 207 show the length which would have been occupied by the winding had it been of the type shown in Fig. 190.

In windings of the type illustrated in Figs. 204 to 206, the annular space may be utilised to contain a solid or hollow annular ring of good conducting material, as indicated in Fig. 208; and

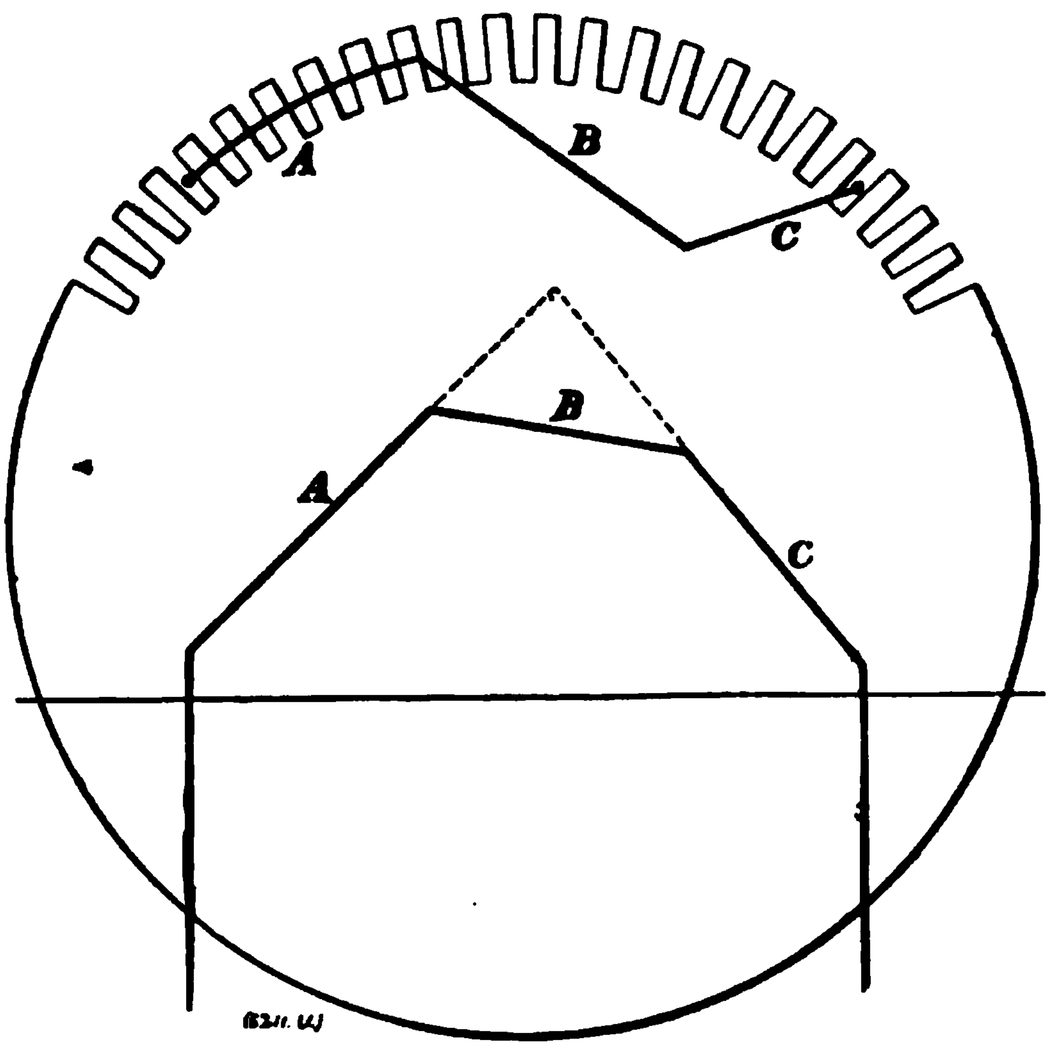


FIG. 207.—Two-layer Winding.

windings of the type shown in Fig. 190 may have secondary circuits arranged as shown in Figs. 209 and 210. In Fig. 209 the secondary circuits are completed at the inner end by rivets between the end connections. One of these is shown dotted in the figure. Whether internal or external to the end connections, as in Figs. 208 and 209, or interleaved with them, as in Fig. 210, these secondary circuits serve the purpose of decreasing the inductance of the armature coils when undergoing short circuit at the commutator brushes, and thus improve the commutation of the motor. It may at first occasion surprise that it should be maintained that a decrease in the inductance of the end connections alone will materially decrease the total induction per coil. This

is the case, however, in many modern motors. As already explained, rough average values for the magnetic flux set up per ampere turn per centimetre of length of coil are 4·0 and 0·8 c.g.s. lines for "embedded" and "free" length respectively, the former being, therefore, only about five times the latter. For reasons relating solely to the most economical attainment of a given result, one frequently finds cases where the "free" length per turn is from four to eight times the "embedded" length. If, therefore,

Figs. 208, 209 and 210.—Form Winding Secondary Circuits.

by such methods as those illustrated in Figs. 208 to 210, one can, say, halve the inductance of the end connections, one has, in the case of such motors, decreased the reactance voltage per segment by from 14 per cent. to 22 per cent. This arbitrarily-chosen illustration is merely cited to illustrate that the proposition should lead to no inconsiderable gain. By the choice of proportions suited to employing the method to best advantage, 20 per cent. to 30 per cent. decrease in the inductance may be regarded as a conservative estimate of the advantage to be gained. It is not the writer's opinion that the extra expense of such constructions would make it desirable to adopt them in any cases except the

extremes where commutating consideratious impose a limit. With increase in rated output, speed, and voltage, limits are reached where sparkless commutation presents difficulties.

§ 5. **The Economical Co-ordination of Horse-power, Speed, Voltage, and Thermal Capacity.**—Thus a satisfactory single commutator shunt motor for operation in either direction with fixed brush position—that is, for operation with the brushes in the neutral point—is, for a rated output of 600 horse-power at 600 volts and 600 revolutions per minute, a relatively expensive machine. If, however, any one of these three values is halved, a satisfactory design is at once possible without excessive cost. Thus, ratings of 600 horse-power at 600 volts and 300 revolutions per minute; 600 horse-power at 300 volts and 600 revolutions per minute; 300 horse-power at 600 volts and 600 revolutions per minute, present no especial difficulties as regards cost and quality, and good motors for 600 horse-power at 300 volts and 300 revolutions per minute; 300 horse-power at 300 volts and 600 revolutions per minute; 300 horse-power at 600 volts and 300 revolutions per minute, are relatively still cheaper. And the same holds true in still greater measure when all three quantities are halved, giving a rating of 300 horse-power at 300 volts and 300 revolutions per minute.

For all the above comparisons, the extra cost of the commutator copper for transmitting the larger currents corresponding to the lower voltages is not taken into consideration, and the conclusions arrived at as to relative cost have to be modified to the extent that the cost of the commutator copper affects the total cost of the machine.

Considerations relating to commutation thus still lead to restrictions in motor design to the extent that, for large capacities and high voltages, the choice of such moderately high speeds as would from other standpoints often be desirable, presents difficulties, and even for smaller motors of lower voltages there exist speeds at which these difficult conditions are encountered whenever it becomes desirable to employ them.

Hence it will be understood that the cost of a motor for a given output and voltage will at first decrease with increased rated speed, reach a minimum, and then increase. The higher the motor's rated output and voltage the lower is this most economical speed. While the great majority of motors are required for speeds much below this maximum economical speed, and hence, when properly designed, are rated at their thermal limit, modern re-

quirements are tending rapidly in the direction of higher speeds, and motors for speeds at or near these economical limits are becoming more frequently required. In these cases good commutation is the controlling consideration in the design, and methods of decreasing the inductance are of value in decreasing the cost of machines or of raising the limiting economical speed. When, at speeds well below these limits, the commutating properties of a machine are poor, it is an indication of incorrect design, which may be remedied in more economical ways than by resorting to methods such as those described.

CHAPTER X

COMPARATIVE DESIGNS FOR 35 H.P. MOTOR

§ 1. **Traditional Lines of Design.**—From the large number of examples of modern motors which have been given in the last few articles, one could, of course, readily work out a safe design for a motor of any required speed, output, voltage, and specified performance. Unfortunately this is the process generally followed, and but little originality is employed so far as relates to the general proportions adopted. The consequence is that the design of the continuous current motor has for some time followed traditional lines, and designers, finding such a general agreement in the proportions adopted by different manufacturers, are often content to let well enough alone, and confine their original work to detailed modifications in which no unknown elements are involved.

The object of the present chapter is to demonstrate that amongst the heretofore neglected proportions there is a wide field for advantageous choice; to discuss the considerations involved in the choice of proportions for any particular motor; and especially to contrast the possible results with those obtained by the designs nowadays generally employed.

§ 2. **Four Designs for Open Type Motors 600 R.P.M.**—Suppose we wish to design a 35 horse-power, 600 revolutions per minute, 220-volt, open-type shunt motor, we may employ a large magnetic flux, and but relatively few armature turns, as is generally done; or we may go to the other extreme; or we may adopt intermediate proportions. It is quite impossible to do justice to the question in all its bearings by any less thorough process than by carrying through a series of comparative calculations. Otherwise some important considerations are certain to be overlooked.

In columns A, B, C, and D of the following tabular specification are rough preliminary calculations for this motor, the

armature strength and magnetic flux per pole face in the four cases being:—

	A	B	C	D
Armature strength per pole, ampere turns	2260	3550	4750	6120
Magnetic flux per pole, megalines ...	3·94	2·46	1·78	1·41

TABLE XXVIII.—ARRANGEMENT OF PRELIMINARY COMPARTIVE DESIGNS FOR FOUR-POLE, OPEN-TYPE, SHUNT WOUND 35 HORSE-POWER MOTOR FOR 220 VOLTS AT 600 REVOLUTIONS PER MINUTE (FIGS. 212 TO 219, PP. 182, 183).

Normal Rating.	A	B	C	D
Number of poles	4	4	4	4
Kilowatts input as motor	28·7	28·9	29·8	30·3
Normal rating in horse-power	35			
Speed in revolutions per minute ...	600			
Speed in revolutions per second ...	10			
Periodicity in cycles per second ...	20			
Terminal voltage	220			
Amperes, input full load	131	132	135·5	138
Amperes input, no load	7·3	5·7	5·5	5·5
Watts input, no load	1600	1250	1220	1200
(Dimensions are in millimetres.)				
Armature:—				
External diameter	360	395	430	460
Axial length of the winding	460	390	360	330
Diameter at the bottom of the slots ...	304	337	370	392
Internal diameter of laminations ...	180	200	225	240
Insulation between laminations, per cent.	10			
" " " material	Varnish			
Thickness of each lamination	0·5			
Depth of the slot	28	29	30	34
Width of the slot as stamped (see Fig. 211)	10·1	7·9	11·0	9·2
Width of the slot, assembled	9·8	7·6	10·7	8·9
Number of slots	45	71	57	71
Width of tooth at periphery as stamped	15·1	9·7	12·7	11·2
Minimum width of tooth, as stamped	11·1	7·0	9·4	8·1
Average width of tooth, as stamped ...	13·1	8·4	11·1	9·7
Radial depth of the laminations ...	90·0	97·5	102·5	110·0
Ditto, below slots	62·0	68·0	72·5	76·0
Number of intermediate ventilating ducts	2	1	1	1
Width of the intermediate ducts ...	10	10	10	10
Total axial length occupied by intermediate ducts	20	10	10	10
Total axial length occupied by insulation	28	16	11	8
Effective length of magnetic iron ...	252	155	110	82
Axial length between flanges, total ...	300	181	131	100
Ratio axial length to diameter... ...	·835	·460	·304	·247

Dimensions of Armature Conductors:—

		A	B	C	D
Height of uninsulated conductor	...	11·0	3·7	12·0	6·0
Width of uninsulated conductor	...	2·0	5·2	1·2	2·6
Height of insulated conductor ...	...	11·3	4·0	12·3	6·3
Width of insulated conductor ...	...	2·3	5·5	1·5	2·9

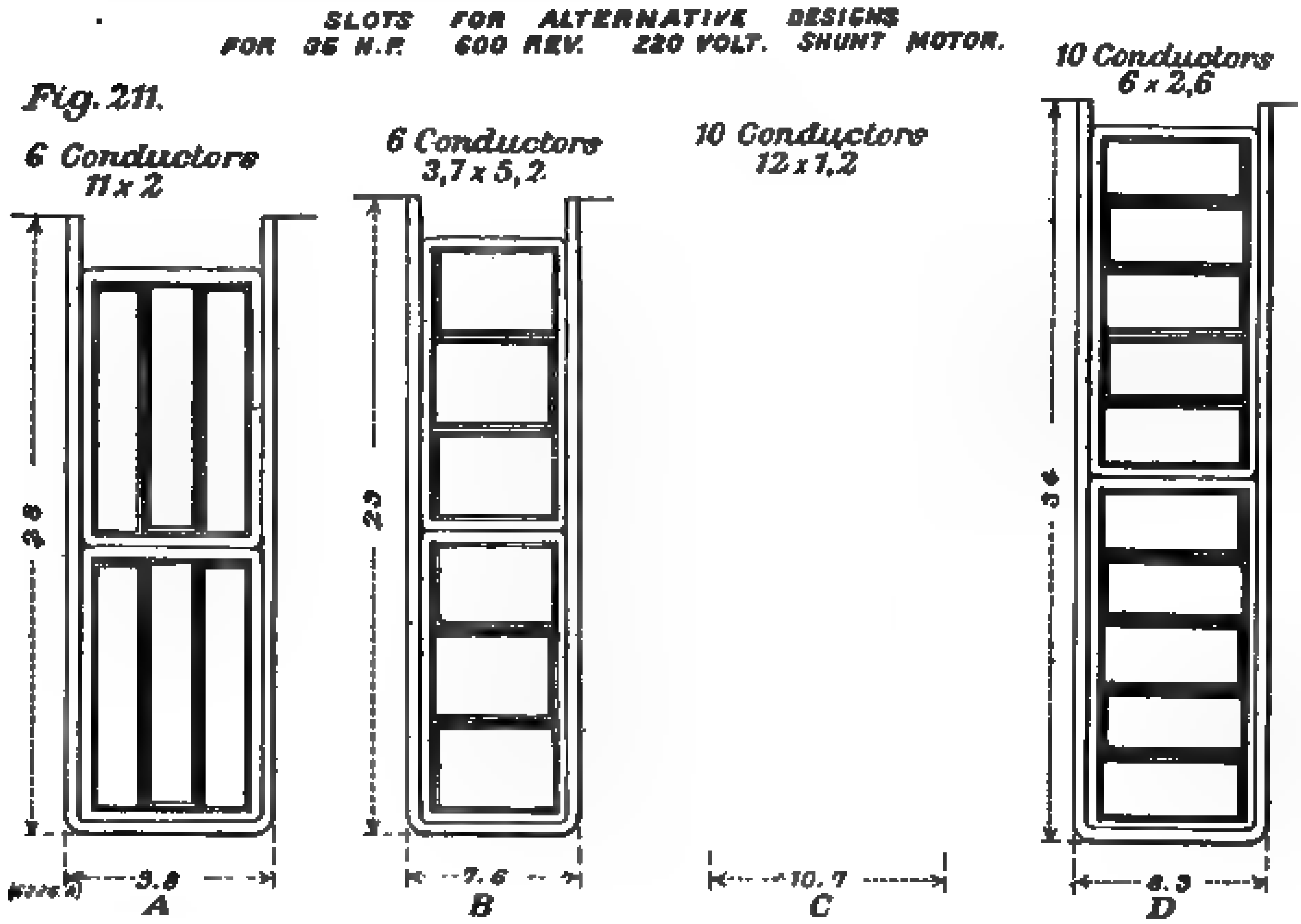


Fig. 211.

Calculation of "Space Factor" of Armature Slot, and of Current Density in Conductors:—

Cross section bare conductor, square centimetres	·220	·193	·144	·156
Amperes per conductor	65·5	66·0	67·8	69·0
Amperes per square centimetre ...	305	342	470	442
Conductors per slot	6	6	10	10
Total copper cross section per slot ...	1·32	1·16	1·44	1·56
Width by depth of slot, square centimetres	2·82	2·29	3·30	3·13
"Space factor" of slot	·467	·505	·435	·496

Dimensions of Magnet Core:—

Material.	Steel.	Steel.	Iron.	Iron.
Length of the pole face parallel to the shaft	300	181	131	100
Diameter of the bore of the pole face ...	368	403	438	468
Pitch at the bore of the pole face ...	290	317	344	367
Mean length of the pole arc	200	220	240	255
Ratio of pole arc to pitch	·695	·695	·695	·695

Dimensions of Magnet Core—continued.

	A	B	C	D
Thickness of the pole shoe at the centre of the arc	11	11	11	11
Radial length of the magnet core ...	100	100	140	140
Magnet core diameter	200	155	130	114
Radial depth of the air gap	4	4	4	4
Distance between pole tips	90	97	104	112

Dimensions of Yoke:—

Material.	Steel.	Steel.	Iron.	Iron.
External diameter	760	760	930	930
Internal diameter	640	665	770	770
Thickness of yoke, exclusive of ribs ...	60	47·5	80	70
Axial width	320	250	300	260
Radial thickness of the pole seat ...	25	20	15	10

NOTE.—In designs A and B, cast steel is employed for the yoke, as the section required for the large magnetic flux employed would, with cast iron, be rather in excess of convenient dimensions; but in designs C and D, where the flux is much smaller, cast iron is employed. The magnet cores in designs A and B are of steel, and are cast in one piece with the yoke, but in designs C and D the magnet cores are of Swedish charcoal iron, around which the yoke is cast. Swedish charcoal iron gives the best magnetic permeability at these densities of any material as yet commercially attainable. This is practicable in small motors, and avoids the high local magnetic reluctance in the cast iron due to the small section of the butt joint when the magnet cores are bolted to the yoke. The pole shoes are of laminations in all four designs.

Dimensions of Commutator:—

Diameter	250	300	350	400
Circumference	785	942	1100	1260
Number of segments	135	213	285	355
Thickness of segment + insulation at periphery	5·81	4·42	3·86	3·54
Thickness of the insulation between segments	0·70	0·65	0·56	0·54
Thickness of a segment at the periphery	5·11	3·77	3·30	3·00
Length from external end to commutator connection	100	92	83	75

Dimensions of Brushes:—

Number of sets	4	4	4	4
Number per set	4	3	3	3
Width of the brushes	22	25	22	20
Length of the arc of contact	15	18	21	24
Contact surface per brush, square centimetres	3·3	4·5	4·6	4·8
Material of the brushes	Carbon			
Amperes per set of brushes	65·5	66·0	68·0	69·0
Amperes per brush	16·4	22·0	22·7	23·0
Amperes per square centimetre ...	4·95	4·90	4·95	4·80

Electrical and Magnetic Data.

Armature:—	A	B	C	D
Terminal voltage	220	220	220	220
Number of face conductors	270	426	570	710
Number of slots	45	71	57	71
Number of conductors per slot... ...	6	6	10	10
Arrangement of the conductors in the slot	3 × 2	1 × 6	5 × 2	1 × 10
Style of winding...	Two-circuit single winding			
Total amperes from commutator ...	131	132	135·5	138
Number of circuits through armature...	2	2	2	2
Amperes per circuit	65·5	66·0	68·0	69·0
Mean length of a single turn, centimetres	130	123	120	120
Total number of turns	135	213	285	355
Number of turns in series between brushes...	67·5	106·5	142·5	177·5
Total length of conducting path between brushes, centimetres	8770	13,100	17,100	21,300
Cross section of one conductor, square centimetres	·220	·183	·144	·156
Cross section of all parallel conductors, square centimetres	·440	·386	·288	·312
Specific resistance at 60° Cent., ohm ...	·0000020			
Resistance of winding from + to − at 60° Cent.	·040	·068	·12	·14
IR loss in armature at 60° Cent., volts	5·2	9·0	16·1	18·9
IR loss in the brush contact surfaces, volts	1·8	1·8	1·8	1·8
Total internal IR loss, volts	7·0	10·8	17·9	20·7
Total induced voltage, full load ...	213·0	209·6	202·1	199·3

Commutator (Sparking Constants).

	A	B	C	D
Number of poles	4	4	4	4
Number of segments	135	213	285	355
Number of segments per pole ...	33·8	53·3	71·3	88·8
Voltage	220			
Volts per segment	6·5	4·1	3·1	2·5
Number of slots	45	71	57	71
Number of slots per pole	11 3	17·8	14·3	17·8
Total number of face conductors...	270	426	570	710
Number of conductors per slot ...	6	6	10	10
Armature turns per pole	33·8	53·3	71·3	88·8
Armature turns per segment ...	1	1	1	1
Total current strength	131	132	135·5	138
Style of winding	Two-circuit single winding			

	A	B	C	D
Amperes per circuit	65·5	66·0	68·0	69·0
Armature ampere turns per pole...	2260	3550	4750	6120
Diameter of the commutator, metres	·250	·300	·350	·400
Periphery of the commutator, metres	·785	·942	1·10	1·26
Revolutions per second	10	10	10	10
Peripheral speed in metres per second (=A)	7·85	9·42	11·0	12·6
Length of the arc of contact (=B), mm.	15	18	21	24
Frequency of commutation (cycles per second $\left(=\frac{1000\,A}{2B}=n\right)$ ...	262	262	262	262
Width of a segment at the periphery (including insulation), millimetres	5·81	4·42	3·86	3·54
Maximum number of coils short circuited under a brush... ...	3	5	6	7
Turns per coil (q)	1	1	1	1
Maximum number of simultaneously commutated conductors per group (r)	6	10	12	14
Mean length of one turn, centimetres	130	123	120	120
Effective length of core, centimetres	25·2	15·5	11·0	8·2
"Free length" per turn (s) ...	79·6	92·0	98·0	103·6
"Embedded length" per turn (t)...	50·4	31·0	22·0	16·4
Lines per ampere turn per centimetre of "free length" (u) ...	0·8			
Lines per ampere turn per centimetre of "embedded length" (v)	4·0			
Lines per ampere turn for "free length" ($u \times s$)	64	74	78	83
Lines per ampere for "embedded length" ($v \times t$)	200	124	88	65·5
Lines per ampere for "free length" $\left(\frac{r}{2} \times u \times s\right)=o$	192	370	468	580
Lines per ampere for "embedded length" $(r \times v \times t)=p$	1200	1240	1050	920
Total lines linked with short circuited coil per ampere $(o+p)$ ...	1392	1610	1518	1500
Inductance per segment, henry ...	·0000139	·0000161	·0000152	·0000150
Reactance per segment, ohm ...	·0229	·0265	·0250	·0247
Number of sets of brushes employed (for wave winding)	4	4	4	4
Minimum series reactance of short circuit conductors (for wave winding)	·0229	·0265	·0250	·0247
Reactance voltage, volts	1·50	1·75	1·70	1·71

MAGNETIC CIRCUIT.

	A	B	C	D
Flux entering armature per pole, full load, megalines	3·94	2·46	1·78	1·41
Corresponding internal voltage ...	213·0	209·6	202·1	199·3
Corresponding terminal voltage ...	220	220	220	220
Leakage factor	1·2	1·2	1·2	1·2
Flux generated per pole, full load, megalines	4·73	2·95	2·14	1·69
Armature:—				
Cross section of the core, square centimetres	312	218	160	125
Density, full load, c.g.s. lines	12600	11300	11100	11300
Ampere turns per centimetre, full load	8	6	6	6
Magnetic length per pole, centimetres	10	11	12	13
Ampere turns, full load...	80	70	70	80
Teeth:—				
Number of teeth per pole	11·3	17·8	14·3	17·8
Number of teeth directly below a mean pole arc	7·9	12·3	10·0	12·3
Percentage increase allowed for spread	10 per cent.			
Total number of flux carrying teeth per pole	8·7	13·5	11·0	13·5
Cross section of one tooth at root, square centimetres	28·0	10·8	10·4	6.7
Total cross section at the bottom of these teeth, square centimetres	241	146	115	90
Density, full load, c.g.s. lines	16300	16800	15600	15800
Ampere turns per centimetre, full load	29	45	21	23
Length, centimetres	2·8	2·9	3·0	3·2
Ampere turns, full load...	80	130	60	70
Air Gap:—				
Cross section at pole face, square centimetres	600	398	314	455
Density at pole face, full load, c.g.s. lines	6550	6200	5650	5500
Length of air gap, iron to iron, centimetres	0·4	0·4	0·4	0·4
Ampere turns, full load...	2100	1980	1810	1770
Magnet Core:—				
Cross section, square centimetres ...	315	190	132	103
Density, full load, c.g.s. lines	15000	15500	16200	16400
Ampere turns per centimetre, full load	29	35	29	30
Magnetic length, centimetres	10	10	14	14
Ampere turns, full load	290	350	410	420

The investigations of Professor W. F. Barrett have already shown that aluminium steel and silicon steel of suitable proportions are superior in

permeability even to the purest charcoal iron, and, since the amounts required for magnet cores in designs such as C and D are very small, a considerably higher price could be economically paid for such material, since a small reduction in the mean length of turn of the field spool would lead to a more than proportional saving in copper. (See "Researches on the Electrical Conductivity and Magnetic Properties of upwards of one hundred different Alloys of Iron," by Professor W. F. Barrett, *Journal of the Institute of Electrical Engineers*, vol. xxxi., 1901–1902, page 674.)

Yoke:—	A	B	C	D
Cross section, square centimetres ...	384	238	480	364
Density, full load, c.g.s. lines	12300	12400	4450	4650
Ampere turns per centimetre, full load	13	13	19	20
Magnetic length, per pole, centimetres	28	29	32	33
Ampere turns, full load	360	380	610	660
Ampere Turns per Spool:—				
Terminal voltage	220	220	220	220
Internal voltage	213·0	209·6	202·1	199·3
Armature core	80	70	70	80
Armature teeth	80	130	60	70
Air gap	2100	1980	1810	1770
Magnet core	290	350	410	420
Yoke	360	380	610	660
Total number of ampere turns per spool	2910	2910	2960	3000
Total ampere turns to be provided per spool	3200			

SHUNT SPOOL WINDING CALCULATIONS.

	A	B	C	D
Volts available at shunt terminals ...	220			
Residual voltage per shunt spool at 60° Cent.	55			
Internal diameter spool, centimetres ...	20·0	15·5	13·0	11·4
Radial depth of winding, " ...	4·5	4·4	2·6	2·5
External diameter of spool, " ...	29·0	24·3	18·2	16·4
Internal periphery of spool, " ...	62·8	48·8	41·0	35·8
External periphery of spool, " ...	91·0	76·5	57·2	51·6
Mean length of one shunt turn, metres (a)	·769	·627	·491	·437
Ampere turns per shunt spool (b) ...	3200	3200	3200	3200
a b	2460	2010	1570	1400
$\cdot 000176 \times a^2b^2$	1060	710	433	345
Axial length of shunt spool, centimetres	10	10	14	14
Cross section of shunt spool winding, square centimetres (r)	45·0	44·0	36·4	35·0
"Space factor" of shunt spool (s) ...	0·50	0·50	0·45	0·45
Cross section copper in shunt spool ($t = r \times s$)	22·5	22·0	16·4	15·8

	A	B	C	D
Cubic centimetres copper in shunt spool (100 a t)	1730	1380	805	690
Kilogrammes copper per shunt spool (1 cubic centimetre copper weighs ·0089 kilogramme)	15·4	12·3	7·15	6·15
Watts per shunt spool $\left(\text{Watts} = \frac{\cdot000176 \times a^2 b^2}{\text{weight in kilogrammes}}\right)$ ...	69·0	58·0	60·5	56·0
External cylindrical surface per spool, square decimetres	9·10	7·65	8·00	7·25
Watts per square decimetre of external cylindrical spool surface	7·6	7·6	7·6	7·7
Amperes per shunt spool (watts÷volts per spool)	1·25	1·05	1·10	1·02
Turns per shunt spool	2560	3050	2910	3140
Cross section copper per turn, square centimetre	0·0088	0·0072	0·0056	0·0050
Current density in amperes per square centimetre	142	146	196	204
Total watts in shunt circuit at 60° Cent.	276	232	242	224
Weight total shunt copper in kilogrammes, all spools	62	49	29	25

ARMATURE.

Armature Copper Loss :—

Resistance of the winding from + to − at 60° Cent., ohm	0·040	0·068	0·12	0·14
Total amperes from commutator ...	131	132	135·5	138
Watts lost in armature copper at 60° Cent.	680	1180	2180	2600

Core Loss :—

Weight of the armature teeth, kilogs.	31	21	18	16
Weight of the armature core, kilogs. ...	93	60	55	50
Total weight of armature laminations, kilogrammes	124	81	73	66
Flux density in the core, kilolines (D)	12·6	11·3	11·1	11·3
Periodicity, cycles per second (N) ...	20	20	20	20
D × N ÷ 100	2·52	2·26	2·22	2·26
Watts lost in iron per kilogramme (obtained from Fig. 21, page 29) ...	6·4	5·6	5·5	5·6
Total core loss (estimated), watts ...	790	460	400	370

Armature Thermal Constants.

Armature copper loss	680	1180	2180	2600
Armature iron loss	790	460	400	370
Total armature loss	1470	1640	2580	2970

	A	B	C	D
Circumference, decimetres	11·35	12·45	13·50	14·50
Axial length of the winding, decimetres	4·6	3·9	3·6	3·3
Peripheral surface, square decimetres...	52·0	48·5	48·5	48·0
Watts per square decimetre of peripheral surface	28	34	53	62

COMMUTATOR.

	A	B	C	D
Current strength of the machine, amperes	131	132	135·5	138
Amperes per square centimetre of brush contact surface	4·85	4·90	4·95	4·80
I^2R loss in watts per ampere (from Table XVIII., page 104)	1·8	1·8	1·8	1·8
Total I^2R loss at brush contacts, watts	236	240	244	250
Peripheral speed of commutator in metres per second	7·85	9·42	11·0	12·6
Brush friction loss in watts per ampere	1·0	1·2	1·35	1·5
Brush friction loss in watts	131	160	184	207

COMMUTATOR THERMAL CONSTANTS.

	A	B	C	D
Total commutator loss, watts	367	400	428	457
Circumference, decimetres	7·85	9·42	11·0	12·6
Length of commutator surface, decimetres	1·00	0·92	0·83	0·75
Cylindrical surface of commutator, square decimetres	7·85	8·66	9·15	9·40
Watts per square decimetre of cylindrical surface	46·5	46·0	46·8	48·5

EFFICIENCY AT 60° CENT.

	A	B	C	D
Iron loss, watts	790	460	400	370
Watts lost in armature copper	680	1180	2180	2600
Watts lost at the brush contact resistance at the commutator	240	240	240	250
Brush friction loss at the commutator	130	160	180	210
Friction loss at bearings and air friction	400	400	400	400
Watts lost in shunt winding	280	230	240	220
Constant losses	1,600	1,250	1,220	1,200
Variable losses	920	1,420	2,420	2,850
Total of all losses	2,520	2,670	3,640	4,050
Output at full load, watts	26,200	26,200	26,200	26,200
Input at full load, watts	28,720	28,870	29,840	30,250
Commercial efficiency at $1\frac{1}{4}$ load ...	91·4	90·5	86·8	85·2
" " full load ...	91·0	90·6	87·8	86·5
" " $\frac{3}{4}$ " ...	90·3	90·5	88·1	87·4
" " $\frac{1}{2}$ " ...	87·6	89·0	87·6	87·1
" " $\frac{1}{4}$ " ...	79·7	83·0	82·7	82·6

WEIGHTS AND COSTS.

Weights of the Effective Materials (in kilogrammes).

	A	B	C	D
Armature laminations	124	81	73	57
Armature copper	34	45	44	60
Commutator segments	20	21	23	24
Magnet cores	118	71	70	54
Pole shoes, sheet iron	20	12	8	6
Yoke, with allowance for feet	335	213	508	390
Shunt copper on magnet spools ...	62	49	29	25
Total effective material...	713	492	755	616
Effective material per horse-power ...	20·4	14·1	21·6	17·6

Specific Costs of the Effective Materials (pence per kilogramme).

Armature copper	24
Commutator copper	24
Spool copper	24
Laminations	3·6
Cast iron	2·2
Cast steel	4·5
Wrought iron	3·0

Total Cost of Effective Material (shillings).

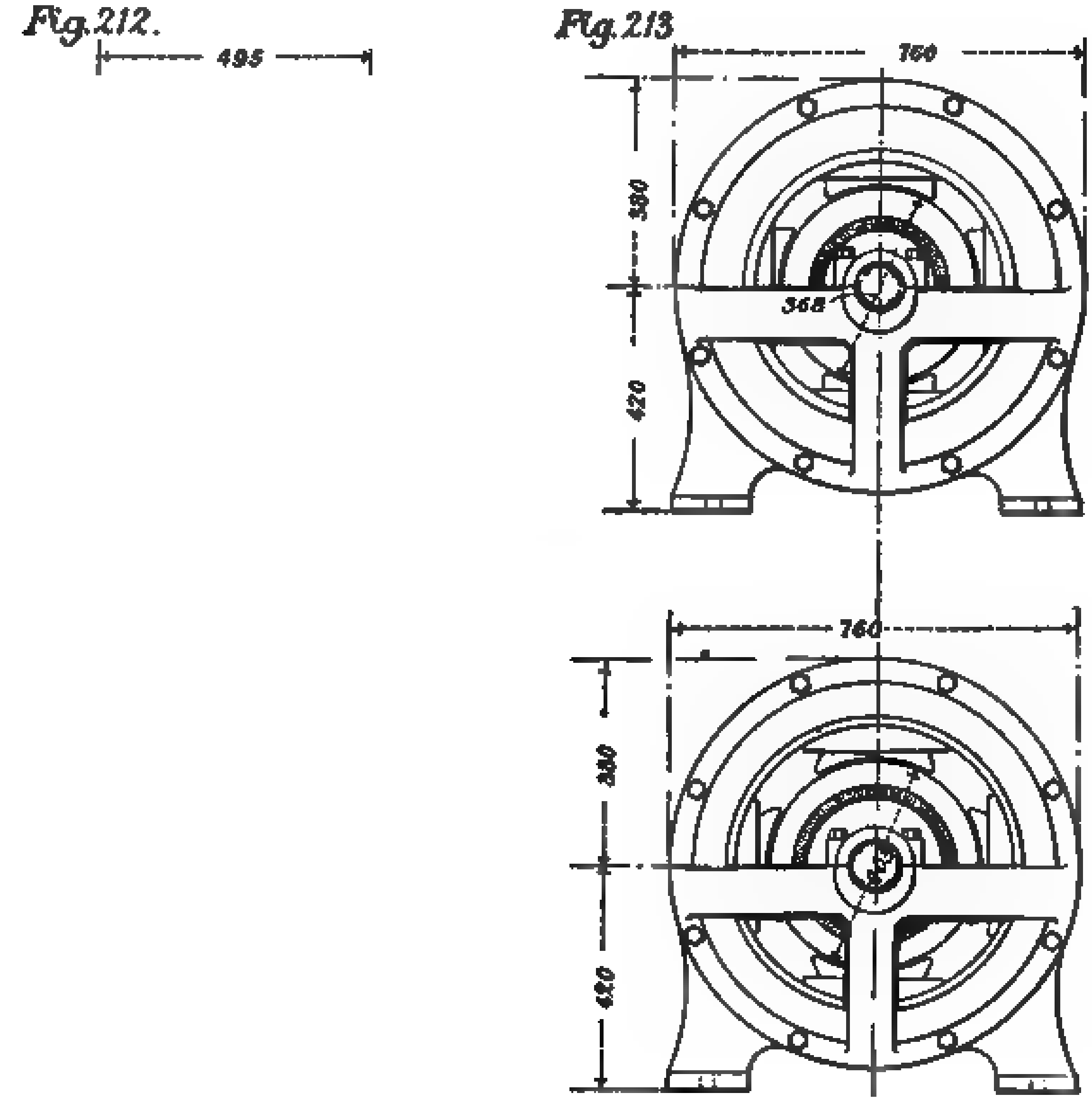
Armature copper	68	90	88	120
Commutator copper	40	42	46	48
Spool copper	124	98	58	50
Armature laminations	37	24	22	17
Pole shoe laminations	6	4	3	2
Cast iron	...	...	93	72
Cast steel	170	107	...	...
Wrought iron	...	...	18	14
Total effective material	445	365	328	323
Effective material per horse-power ...	12·7	10·5	9·4	9·2

Estimate of the Total Weight of Materials.

Weight of non-active material, kilogrammes	250	250	250	250
Total weight	963	742	1005	866
Total weight per horse-power, kilogrammes	27·5	21·2	28·6	24·8

The four designs are illustrated in Figs. 212 to 219, pages 182 and 183.

The results for cost and efficiency are of assistance in deciding as to the best proportions to be followed in making the final design.

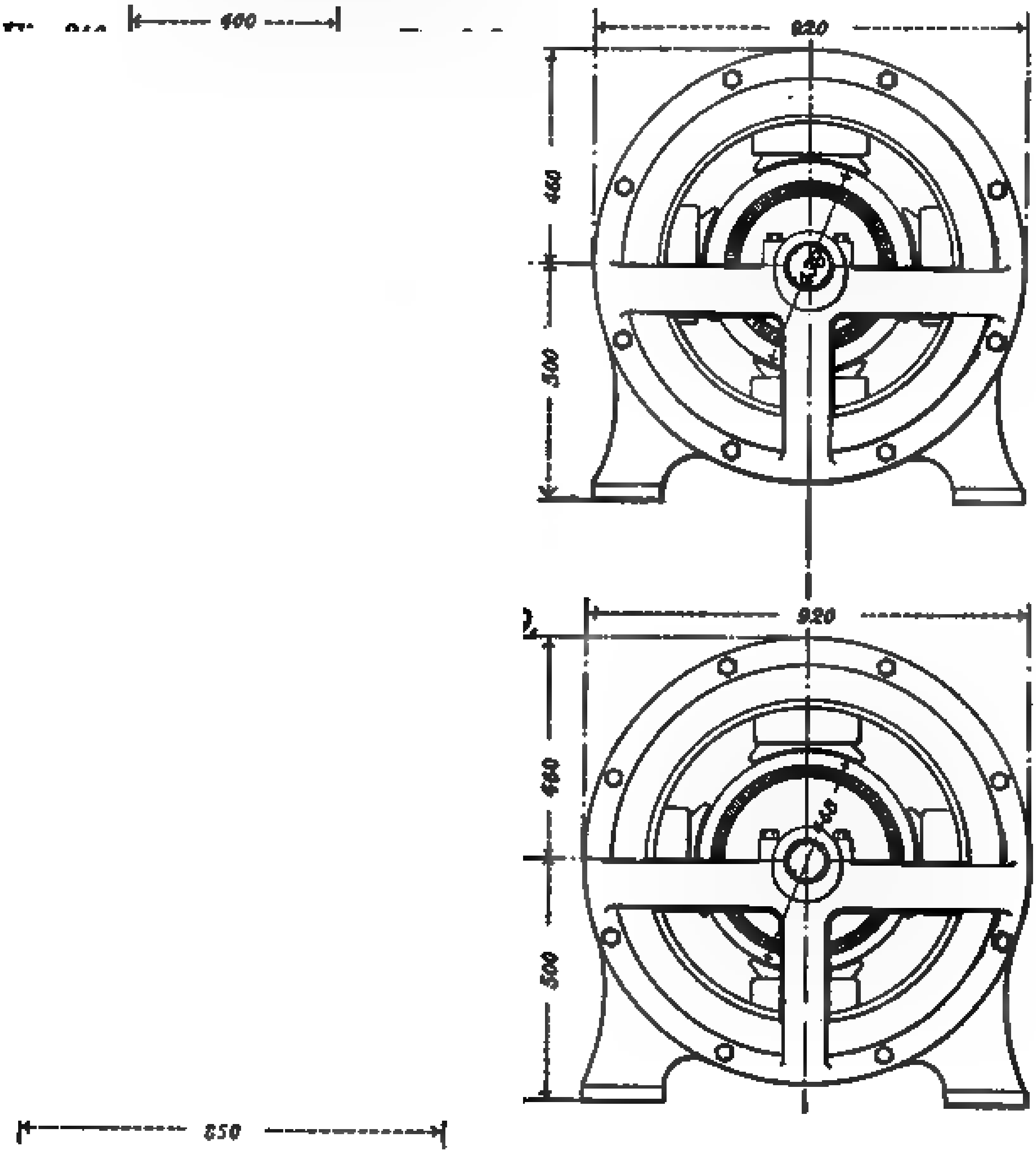


FIGS. 212, 213, 214 and 215.—Designs for 35 Horse-power, 600 Revolutions, 220-volt, Shunt Motors.

§ 3. The Rating of these Designs when totally Enclosed.—But there is still another consideration, namely, the rating and performance of these designs when totally enclosed. It will be practicable to allow 7·5 watts of total internal loss per square decimetre of external surface of case. This will result in a thermometric temperature rise of not over 60° Cent. The following calculation is on this basis:—

TABLE XXIX.—ESTIMATION FOR TOTALLY ENCLOSED MOTORS.

	A	B	C	D
Width over end shields, decimetres ...	6·4	5·6	5·2	4·7
Diameter of case, decimetre	7·6	7·6	9·2	9·2
External radiating surface, sq. decimetres	243	224	285	270
Permissible watts per square decimetre	7·5	7·5	7·5	7·5



FIGS. 216, 217, 218 and 219.—Designs for 35 Horse-power, 600 Revolutions, 220-volt, Shunt Motors.

Total internal loss (a)	1820	1680	2140	2020
Constant loss (b), same as for open motor	1600	1250	1220	1200
Variable loss ($a-b$)	220	430	920	820
Variable loss as open motor	920	1420	2420	2850
Ratio of variable losses, enclosed and open	·238	·225	·381	·288
Square root of above ratio	·488	·475	·618	·537

	A	B	C	D
Horse-power rating as totally enclosed motor...	17·1	16·6	21·6	18·8
Cost of net effective material, shillings	445	365	328	332
Ditto per horse power, shillings ...	26·0	22·0	15·2	17·2
Efficiency as totally enclosed motor at ¼ load	66·4	70·9	75·7	73·6
" " " ½ "	79·4	82·5	84·7	83·4
" " " ¾ "	85·1	86·3	87·4	86·5
" " " full "	87·4	88·1	88·2	87·5
" " " 1¼ "	89·1	88·9	88·4	87·6

These results, plotted in Figs. 220 to 226, pages 184 to 187, show that the best design for 600 revolutions per minute, when cost, efficiency, and rating and performance when totally

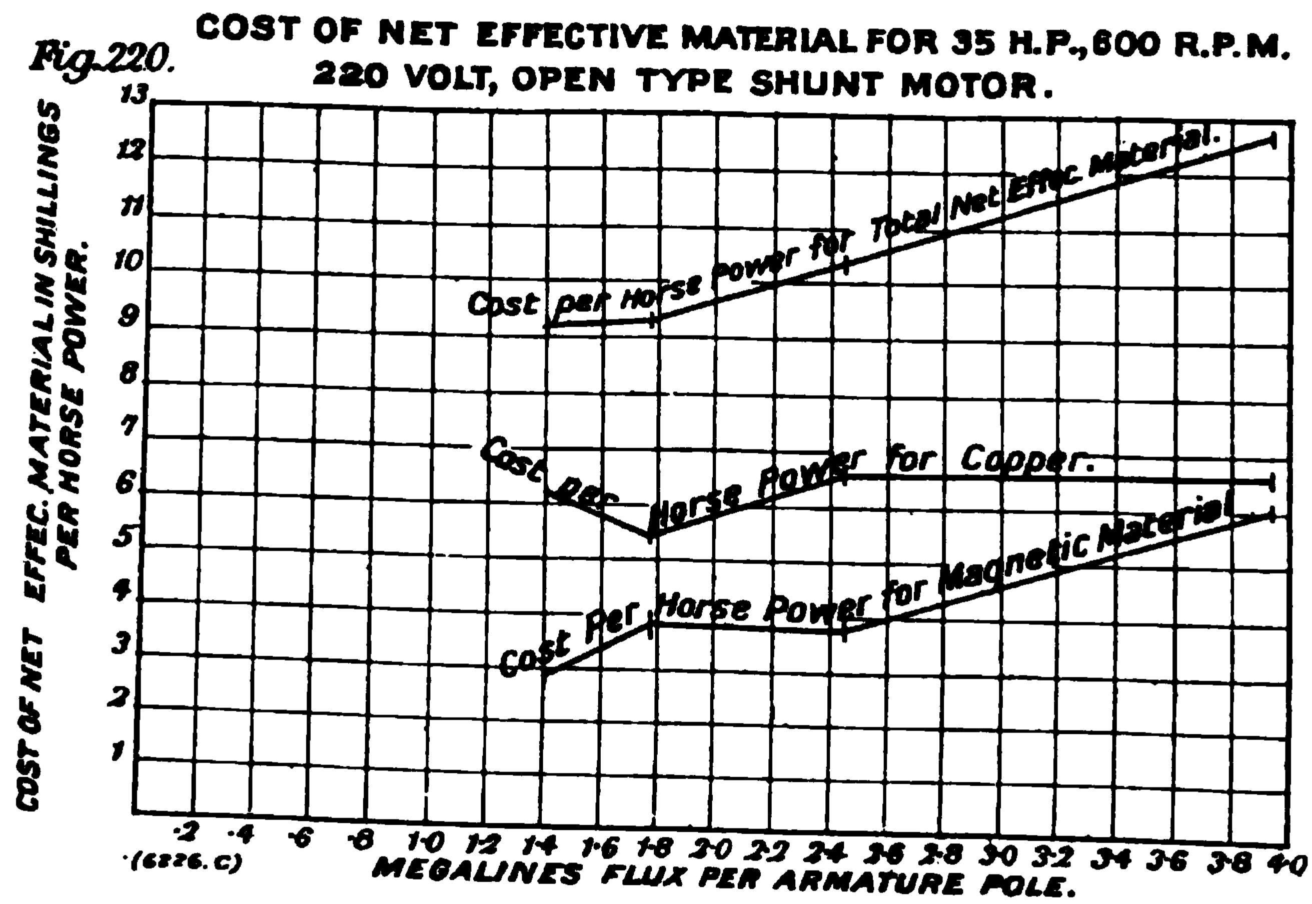


FIG. 220.

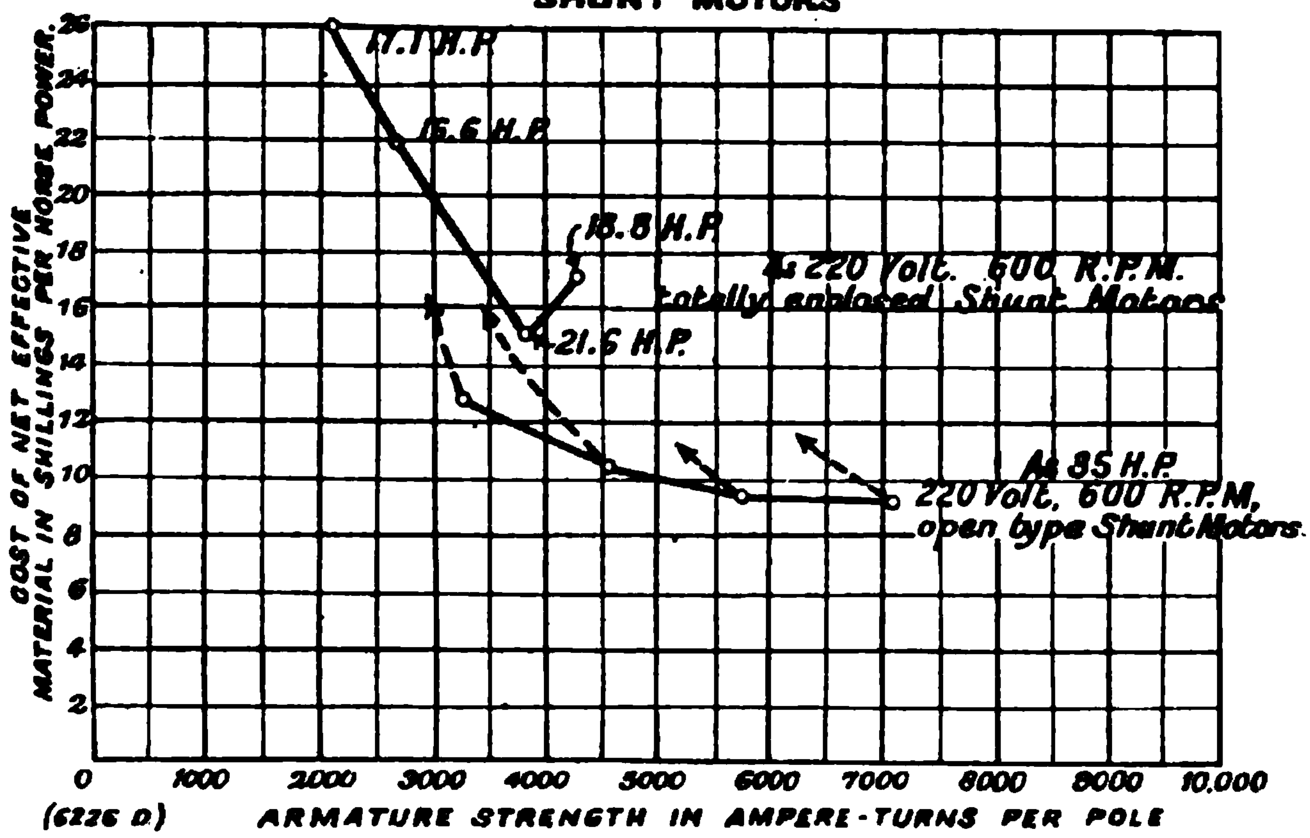
enclosed, are considered, should have a magnetic flux of about 2 megalines per pole face, as against the general use of from 3·5 to 5·0 megalines in motors for this rating. Hence it is evident that great economy over present practice in open motors, and especially in totally enclosed motors, is obtainable, and that the totally enclosed motors are, at the same time, greatly improved. But one cannot generally consider a single design by itself; we must keep in mind ratings for other speeds and voltages.

§ 4. **The Adaptation of same Designs to other Speeds.**—Having for a motor of given capacity arrived at the most economical proportions for a given speed, it is very practicable to use the same design for other speeds, merely changing the length of laminations between flanges in inverse proportion to the speed

required. This is a much better plan than that followed by some designers, of using larger diameters for lower speeds. This latter plan is dictated by the consideration that the use of a high

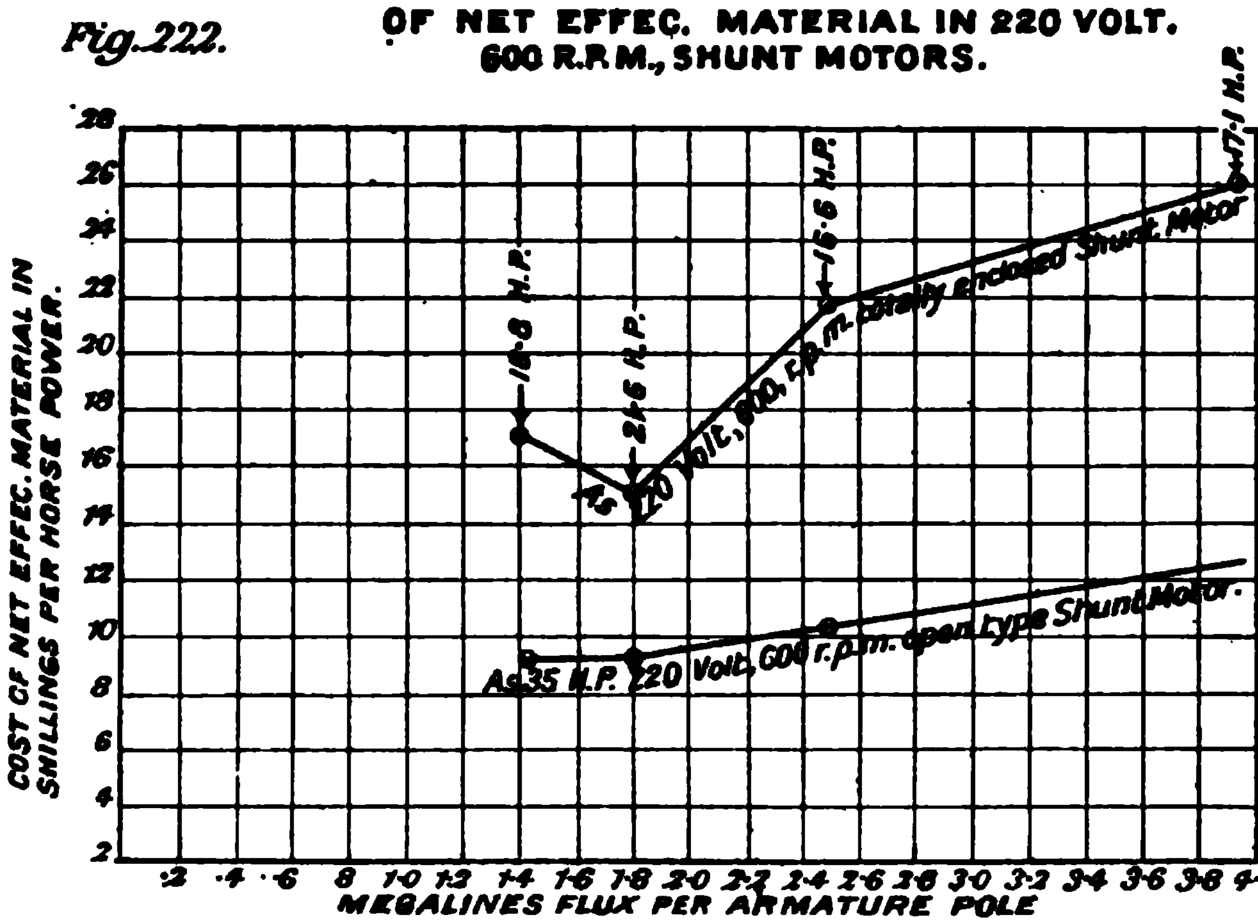
CURVES OF COST OF NET EFFECTIVE MATERIAL IN 220 VOLT, 600 R.P.M. SHUNT MOTORS

Fig. 221.



CURVES OF COST OF NET EFFEC. MATERIAL IN 220 VOLT, 600 R.P.M., SHUNT MOTORS.

Fig. 222.



FIGS. 221 and 222.

peripheral speed is associated with the most efficient use of a given amount of active material. But the attainment of the latter result is in practice controlled by many other considerations, and the writer is of opinion that the best commercial results will be obtained by employing the same diameters for all rated speeds,

the peripheral speed thus being inversely proportional to the angular speed. This plan is in accordance with the requirements for good commutation. Machines for high-rated speed must, from commutating considerations, be narrow, but the lower the rated

Fig. 223.

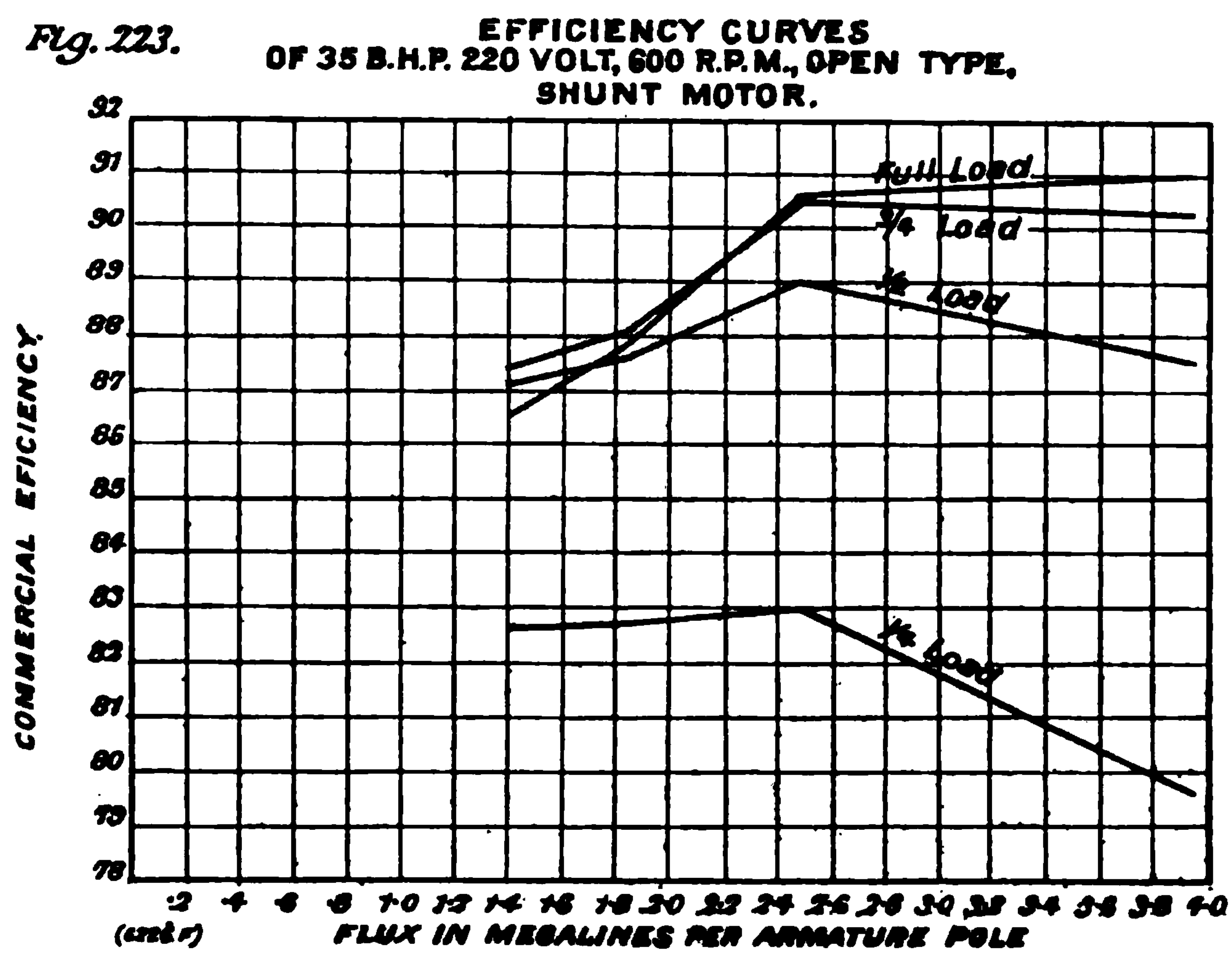
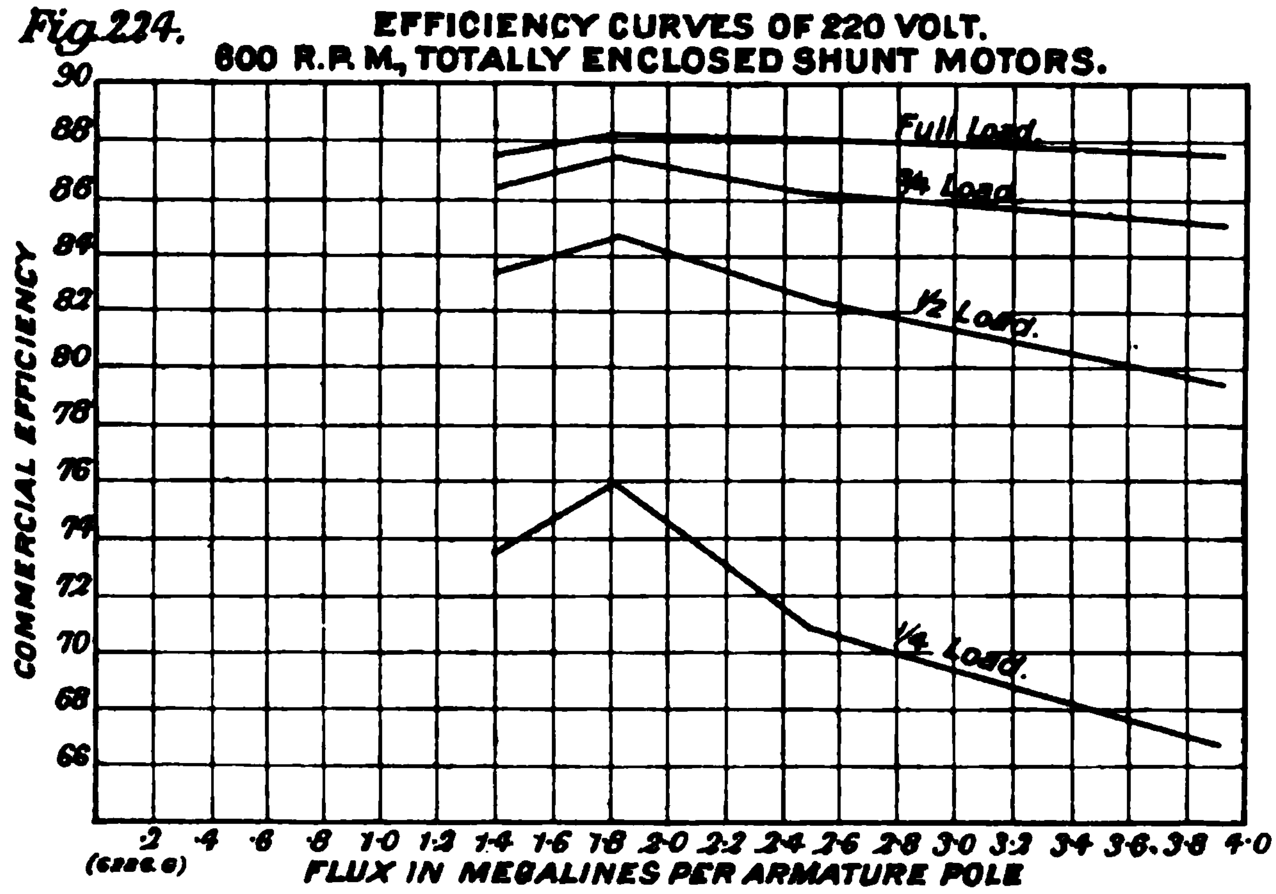


Fig. 224.

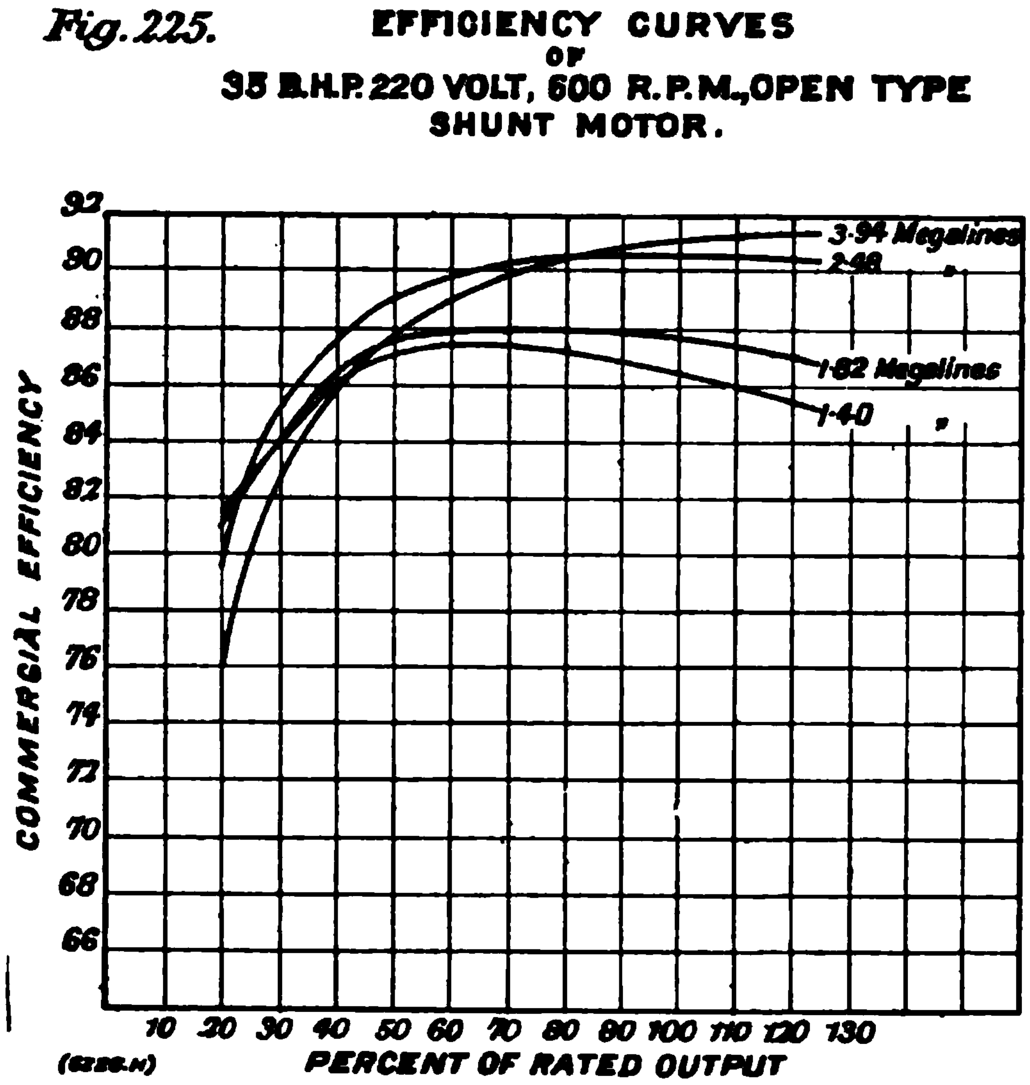


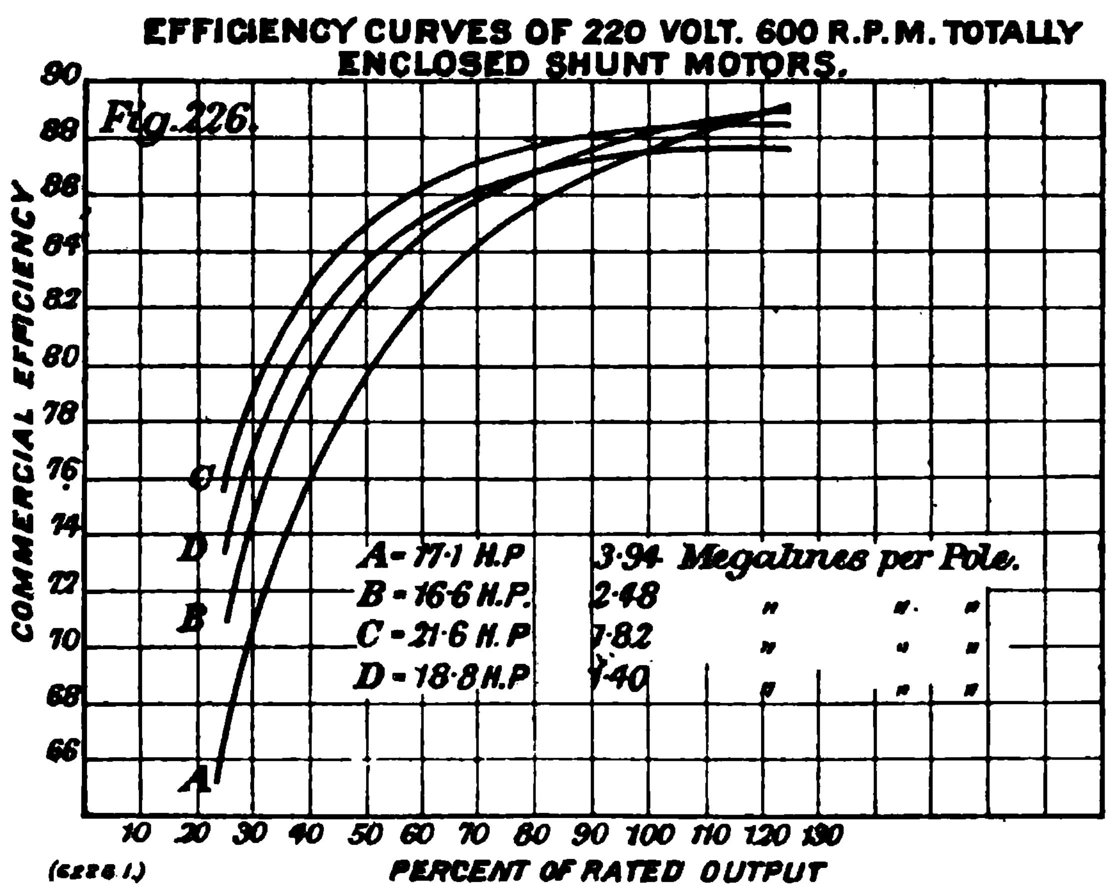
Figs. 223 and 224.

speed, the lower will be the frequency of commutation, and the wider may be the machine without obtaining an undesirably high reactance voltage.

The 35 horse-power motor, recalculated for 600 revolutions per minute in column B of the following table, is, in column A,

widened out 100 per cent. for a speed of 300 revolutions per minute, and in column C it is narrowed up 33·3 per cent. for 900 revolutions per minute. These three motors are for 220 volts. For these





Figs. 225 and 226.

particular designs the reactance voltages are so low that the same cores wound with two-turn coils would give good 440-volt motors for the same speeds. Generally, however, it is preferable to use longer armature cores and weaker armatures, as expressed in ampere turns per armature pole, for higher voltages.

Conversion factor mm to inches = 25.4 mm = 1 in.

TABLE XXX.—DESIGNS FOR FOUR-POLE, OPEN-TYPE, SHUNT WOUND, 35 HORSE-POWER, 220-VOLT MOTORS FOR 300, 600, AND 900 REVOLUTIONS PER MINUTE (Figs. 227 to 230, page 190).

Normal rating	A	B	C
Number of poles		4	
Normal rating in kilowatts, kilowatts input as motor	30·3	29·7	29·7
Normal rating in horse-power	35	35	35
Speed in revolutions per minute ...	300	600	900
Speed in revolutions per second ...	5	10	15
Periodicity in cycles per second ...	10	20	30
Terminal voltage		220	
Amperes input, full load	138·0	135·0	134·5
Amperes input, no load...	5·5	5·7	6·0
Watts input, no load	1200	1250	1330

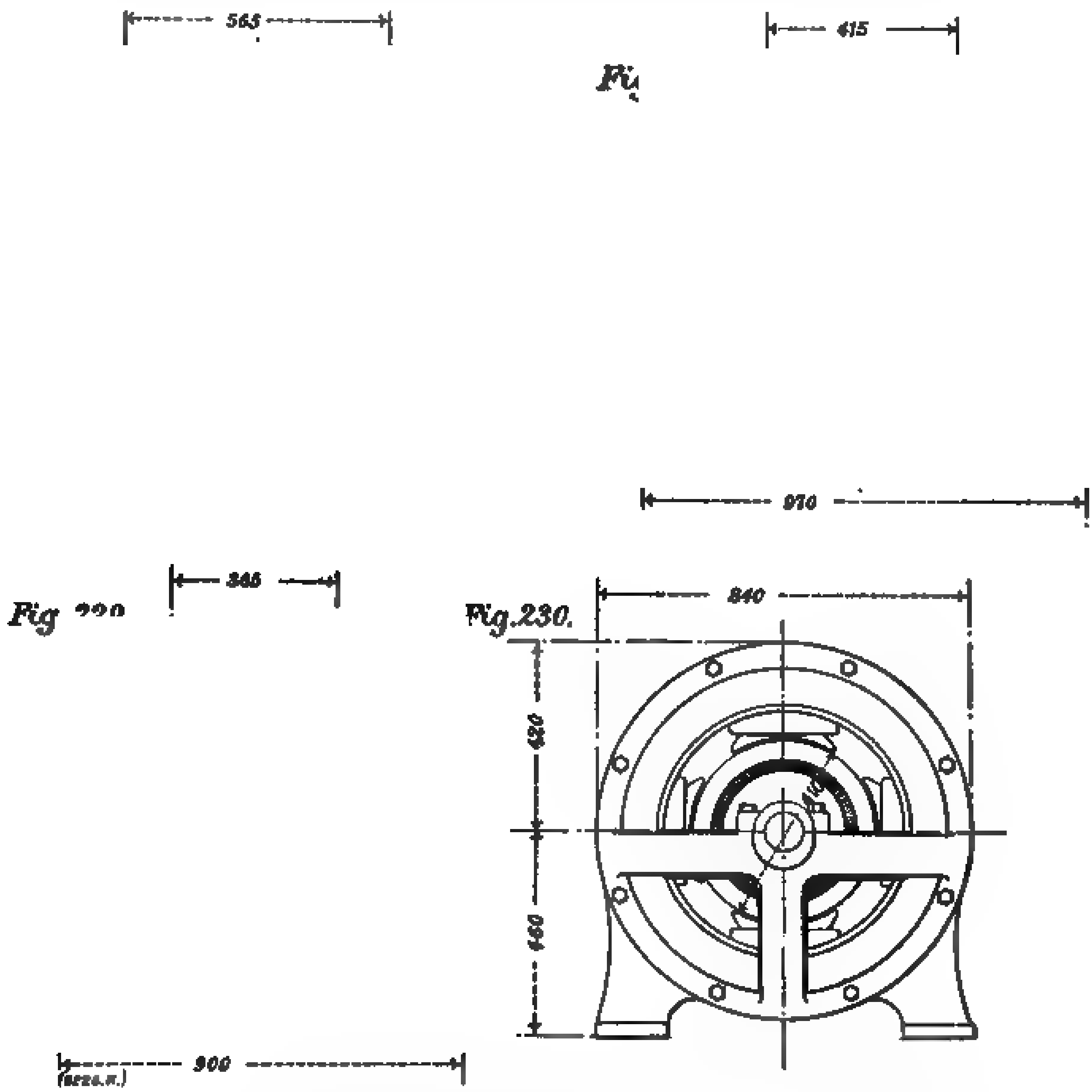
(Dimensions in millimetres.)

	A	B	C
Armature:—			
External diameter		410	
Axial length of the winding	535	385	335
Diameter at the bottom of the slots ...		338	
Internal diameter of the laminations ...		200	
Depth of the slot		36	
Width of the slot, as stamped		8·3	
Width of the slot assembled		8·0	
Number of slots		51	
Width of tooth at periphery, as stamped		16·9	
Minimum width of tooth, as stamped		12·5	
Average width of tooth, as stamped ...		14·7	
Radial depth of the laminations ...		105	
Radial depth of the laminations below slots		69	
Number of intermediate ventilating ducts	4	2	1
Width of the intermediate ducts... ...	10	10	10
Total axial length occupied by ducts ...	40	20	10
Total axial length occupied by insulation	26	13	9
Effective length of magnetic iron ...	234	114	81
Axial length between flanges, total ...	300	150	100
Ratio axial length to diameter ...	0·73	0·37	0·24

	A	B	C
Dimensions of Armature Conductors:—			
Height of uninsulated conductor ...		5·0	
Width of uninsulated conductor ...		2·7	
Height of insulated conductor... ...		5·3	
Width of insulated conductor		3·0	
"Space Factor" and Current Density Calculations:—			
Cross section bare conductor, square centimetres		·135	
Amperes per conductor...	69·0	67·5	67·3
Amperes per square centimetre ...	510	500	500
Conductors per slot		10	
Total copper cross section per slot ...		1·35	
Width × depth of slot, square centimetres		2·99	
"Space factor" of slot		0·45	
Magnet Core:—			
Length of the pole face parallel to the shaft	300	150	100
Diameter of the bore of the pole face ...		418	
Circumference at bore of the pole face		1320	
Pitch at the bore of the pole face ...		330	
Mean length of the pole arc		230	
Ratio of pole arc to pitch		·695	
Thickness of the pole shoe at the centre of the arc		11	
Radial length of the magnet core ...		130	
Magnet core, diameter	192	137	112
Radial depth of the air gap		4	
Distance between pole tips		100	
Yoke:—			
External diameter		890	
Internal diameter		730	
Thickness of yoke		80	
Axial width		310	
Radial thickness of the pole seat ...		15	
Material of yoke...	Cast steel	Cast iron	Cast iron

NOTE.—Motors for 300, 600, and 900 revolutions per minute, according to these dimensions, are sketched in Figs. 227 to 230, except that in so far as

relates to the yoke, the three motors are shown as of cast iron, with widths inversely proportional to the speeds, whereas the yoke dimensions above given are identical for all three speeds, the first being of cast steel and the other two of cast iron. From commercial considerations this might be the best plan to adopt.



Figs. 227, 228, 229 and 230.—Designs for 35 Horse-power, 300, 600 and 900 Revolutions, 220-volt Shunt Motors.

Commutator :—	A	B	C
Diameter		325	
Circumference		1020	
Number of segments		255	
Thickness of segment + insulation at periphery		4·00	
Thickness of the insulation between segments		0·50	

	A	B	C
Commutator—continued.			
Thickness of a segment at the periphery		3·50	
Length from external end to commutator connection		85	
Brushes :—			
Number of sets		4	
Number per set		3	
Width of the brushes		22	
Length of the arc of contact		21	
Contact surface per brush, square centimetres		4·6	
Material of the brushes...		Carbon	
Amperes per set of brushes	69·0	67·5	67·3
Amperes per brush	23·0	22·5	22·4
Amperes per square centimetre, brush contact	5·0	4·9	4·9

ELECTRICAL AND MAGNETIC DATA.

	A	B	C
Armature :—			
Terminal voltage		220	
Number of face conductors		510	
Number of slots...		51	
Number of conductors per slot ...		10	
Arrangement of the conductors in the slot		1 × 10	
Style of winding		Two-circuit single winding	
Total amperes from commutator ...	138·0	135·0	134·5
Number of circuits through armature	2	2	2
Amperes per circuit	69·0	67·5	67·3
Mean length of a single turn, centimetres	150	120	110
Total number of turns		225	
Number of turns in series between brushes		127·5	
Total length of conducting path between brushes, centimetres	19100	15300	14000
Cross section of one conductor, square centimetres		·135	
Cross section of all parallel conductors, square centimetres		·270	
Specific resistance at 60° Cent., ohm ...		·0000020	
Resistance of winding from + to − at 60° Cent, ohm	·141	·113	·104

Armature—continued.

	A	B	C
IR loss in armature at 60° Cent., volts	19·0	15·2	13·9
IR loss in brush contact surfaces, volts	1·8	1·8	1·8
Total internal IR loss, volts	20·8	17·0	15·7
Total induced voltage, full load ...	199·2	203·0	204·3

COMMUTATOR (SPARKING CONSTANTS).

Number of poles		4	
Number of segments		255	
Number of segments per pole		63·8	
Voltage		220	
Volts per segment		3·45	
Number of slots		51	
Number of slots per pole		12·75	
Total number of face conductors ...		510	
Number of conductors per slot ...		10	
Armature turns per pole		63·7	
Armature turns per segment		1	
Total current strength	138·0	135·0	134·5
Style of winding		Two-circuit single	
Number of circuits		2	
Amperes per circuit	69·0	67·5	67·3
Armature ampere turns per pole ...	4400	4300	4290
Diameter of the commutator, metres ...		·325	
Periphery of the commutator, metres		1·02	
Revolutions per second	5	10	15
Peripheral speed in metres per second (=A)	5·1	10·2	15·3
Length of the arc of contact (=B), millimetres		21	
Frequency of commutation (cycles per second) $\left(\frac{1000\text{ A}}{2\text{B}} = n\right)$	121	242	363
Width of a segment at the periphery (including insulation), millimetres...		4·00	
Maximum number of coils short circuited under a brush...		6	
Turns per coil (q)		1	
Maximum number of simultaneously commutated conductors per group (r)		12	

	A	B	C
Mean length of one turn, centimetres...	150	120	110
Effective length of core, centimetres ...	23·4	11·4	8·1
"Free length" per turn (s)	103	97	94
"Embedded length" per turn (t) ...	47	23	16
Lines per ampere turn per centimetre of "free length" (u)		0·8	
Lines per ampere turn per centimetre of "embedded length" (v)		4·0	
Lines per ampere turn for "free length" ($u \times s$)	82	78	75
Lines per ampere turn for "embedded length" ($v \times t$)	188	92	64
Lines per amperes for "free length" $\left(\frac{r}{2} \times u \times s\right) = o$	492	468	450
Lines per ampere for "embedded length" $(r \times v \times t) = p$	2260	1102	765
Total lines linked with short-circuited coil per ampere $(o+p)$, ...	2752	1570	1215
Inductance per segment, $\frac{q \times (o+p)}{10} = l$, henry	·0000275	·0000157	·0000122
Reactance per segment, ohm $(2\pi\ n\ l)$ ...	·0209	·0238	·0275
Number of sets of brushes employed (for wave winding)	4	4	4
Minimum series reactance of short-circuited conductors (for wave winding)	·0209	·0238	·0275
Reactance voltage, volts	1·44	1·61	1·85

MAGNETIC CIRCUIT.

	A	B	C
Flux entering armature per pole, full load, megalines	3·92	2·00	1·34
Corresponding internal voltage ...	199·2	203·0	204·3
Corresponding terminal voltage ...	220	220	220
Leakage factor	1·2	1·2	1·2
Flux generated per pole, full load, megalines	4·70	2·40	1·61
Armature :—			
Cross section of the core, square centimetres	320	155	110
Density, full load, c.g.s. lines	12300	12900	12200
Ampere turns per centimetre, full load	7	9	7
Magnetic length per pole, centimetres		11	
Ampere turns, full load	80	100	80

	A	B	C
Teeth:—			
Number of teeth per pole		12·75	
Number of teeth directly below a mean pole arc		8·85	
Percentage increase allowed for spread		10 per cent.	
Total number of flux carrying teeth per pole		9·7	
Cross section of one tooth at root, square centimetres	29·2	14·2	7·7
Total cross section at the bottom of these teeth, square centimetres ...	283	138	75
Apparent density, full load, c.g.s. lines	13900	14500	17900
Corrected density, full load, c.g.s. lines	13900	14500	17600
Ampere turns per centimetre, full load	13	15	80
Length, centimetres		3·6	
Ampere turns, full load	50	50	290
Air Gap:—			
Cross section at pole face, square centimetres	690	345	230
Density at pole face, full load, c.g.s. lines	5680	5800	5820
Length of air gap, iron to iron, centimetres		0·4	
Ampere turns, full load	1820	1860	1860
Magnet Core:—			
Cross section, square centimetres ...	290	148	100
Density, full load, c.g.s. lines	16200	16200	16200
Ampere turns per centimetre, full load	29	29	29
Magnetic length centimetres		13	
Ampere turns, full load	380	380	380
Yoke:—			
Cross section, square centimetres ...		495	
Density, full load, c.g.s. lines	9500	4850	3250
Material of yoke	Cast steel	Cast iron	Cast iron
Ampere turns per centimetre, full load	7	21	13
Magnetic length, per pole, centimetres		31	
Ampere turns, full load	220	650	400

Ampere Turns per Spool:—	A	B	C
Terminal voltage		220	
Internal voltage	199·2	203·0	204·3
Armature core	80	100	80
Armature teeth	50	50	80
Air gap	1820	1860	1860
Magnet core	380	380	380
Yoke	220	650	400
Total number of ampere turns per spool	2550	3040	2800
Total ampere turns to be provided per spool	2800	3300	3000

SHUNT SPOOL WINDING CALCULATIONS.

Volts available at shunt terminals ...		220	
Residual voltage per shunt spool at 60° Cent.		55	
Internal diameter spool, centimetres ...	19·2	13·7	11·2
Radial depth of winding, „ ...	2·4	3·0	2·5
External diameter of spool, centimetres	24·0	19·3	16·2
Internal periphery of spool, „	60·3	43·0	35·2
External periphery of spool, „	75·5	62·0	51·0
Mean length of one shunt turn, metres (a)	·678	·525	·431
Ampere turns per shunt spool (b) ...	2800	3300	3000
a b	1890	1730	1300
$\cdot000176 \times a^2b^2$	628	525	296
Axial length of shunt spool, centimetres	13	13	13
Cross section of shunt spool winding, square centimetres (r)	31·2	39·0	32·5
"Space factor" of shunt spool (s) ...	0·45	0·45	0·45
Cross section copper in shunt spool ($t = r \times s$)	14·0	17·6	14·7
Cubic centimetres copper in shunt spool (100 a t)	950	925	635
Kilogrammes copper per shunt spool (1 cubic centimetre copper = ·0089 kilogramme)	8·45	8·25	5·65
Watts per shunt spool $\left(\text{watts} = \dfrac{\cdot000176 \times a^2b^2}{\text{weight in kgs.}}\right)$	74·5	63·5	52·3
External cylindrical surface per spool, square decimetres	9·80	8·05	6·65

	A	B	C
Watts per square decimetre of external cylindrical spool surface	7·60	7·90	7·86
Amperes per shunt spool (watts ÷ volts per spool)	1·36	1·16	0·95
Turns per shunt spool	2060	2840	3160
Cross section copper per turn, square centimetre	0·0068	0·0062	0·0047
Current density in amperes per square centimetre	200	187	205
Total watts in shunt circuit at 60° Cent.	298	254	209
Weight total shunt copper in kilogrammes, all spools	33·8	33·0	22·6

Armature Losses.

Armature Copper Loss:—

Resistance of the winding from + to − at 60° Cent., ohm	·141	·113	·104
Total amperes from commutator ...	138·0	135·0	134·5
Watts lost in armature copper at 60° Cent.	2660	2050	1880

Core Loss:—

Weight of the armature teeth, kilogrammes	50	24	17
Weight of the armature core, kilogrammes	90	44	31
Total weight of armature laminations, kilogrammes	140	68	48
Flux density in the core, kilolines, (D)	12·3	12·9	12·2
Periodicity, cycles per second (N) ...	10	20	30
D × N ÷ 100	1·23	2·58	3·66
Watts lost in iron per kilogramme [1] ...	3·0	6·5	9·9
Total core loss (estimated), watts ...	420	440	480

Armature Thermal Constants.

Armature copper loss	2660	2050	1880
Armature iron loss	420	440	480
Total armature loss	3080	2490	2360
Circumference, decimetres		12·9	
Axial length of the winding, decimetres	5·4	3·9	3·4
Peripherial surface, square decimetres	69	50	43
Watts per square decimetre of peripheral surface	45	50	55

[1] These values are obtained by means of Fig. 21, page 30.

Commutator Losses.

	A	B	C
Current strength of the machine, amperes	138·0	135·0	134·5
Amperes per square centimetre of brush contact surface	5·0	4·9	4·9
I^2R loss in watts per ampere [1]	1·8	1·8	1·8
Total I^2R loss at brush contacts, watts	250	240	240
Peripheral speed of commutator in metres per second	5·1	10·2	15·3
Brush friction loss in watts per ampere [1]	0·6	1·2	1·8
Brush friction loss in watts	80	160	240

Commutator Thermal Constants.

	A	B	C
Total commutator loss, watts	330	400	480
Circumference, decimetres		10·2	
Length of commutator surface, decimetres		0·85	
Cylindrical surface of commutator, square decimetres		8·7	
Watts per square decimetre of cylindrical surface	37	46	55

Efficiency at 60° Cent.

	A	B	C
Iron loss, watts	420	440	480
Watts lost in armature copper	2660	2050	1880
Watts lost at the brush contact resistance at the commutator	250	240	240
Brush friction loss at the commutator	80	160	240
Friction loss at bearings and air friction	400	400	400
Watts lost in shunt winding	300	250	210
Constant losses	1200	1250	1330
Variable losses	2910	2290	2120
Total of all losses ...	4110	3540	3450
Output at full load, watts	26200	26200	26200
Input at full load, watts	30310	29740	29650
Commercial efficiency at full load ...	86·5	88·0	88·5
„ „ $1\frac{1}{4}$ „ ...	85·3	87·2	87·6
„ „ $\frac{3}{4}$ „ ...	87·4	88·7	88·7
„ „ $\frac{1}{2}$ „ ...	87·2	87·8	87·5
„ „ $\frac{1}{4}$ „ ...	82·5	82·5	81·7

[1] These values are taken from Table XVIII., page 104.

WEIGHTS OF THE EFFECTIVE MATERIALS (in kilogrammes).

	A	B	C
Armature laminations	140	68	48
Armature copper	46	37	34
Commutator segments	22	22	22
Magnet cores, wrought iron	142	72	48
Pole shoes, laminations	20	10	7
Yoke, with allowance for feet	505	505	505
Shunt copper on magnet spools ...	34	33	23
Total effective material	909	747	687
Effective material per horse-power ...	25·9	21·3	19·6

SPECIFIC COSTS OF THE EFFECTIVE MATERIALS (pence per kilogramme).

Armature copper	24
Commutator copper	24
Spool copper	24
Laminations for armature	3·6
Cast iron	2·2
Cast steel	4·5
Wrought iron	3·0

TOTAL COST OF EFFECTIVE MATERIALS (shillings).

Armature copper	92	74	68
Commutator copper	44	44	44
Spool copper	68	66	46
Armature laminations	42	20	14
Pole shoe laminations	6	3	2
Cast iron	—	93	93
Cast steel	190	—	—
Wrought iron	35	18	12
Total effective material ...	477	318	279
Effective material per horse-power, shillings	13·7	9·1	8·0 [1]

ESTIMATE OF THE TOTAL WEIGHT AND COST FOR MATERIALS.

Weight of non-active material, kilogrammes	250	250	250
Total weight of motor	1150	1000	946
Total weight per horse power, kilogrammes	32·8	28·5	27·0 [2]

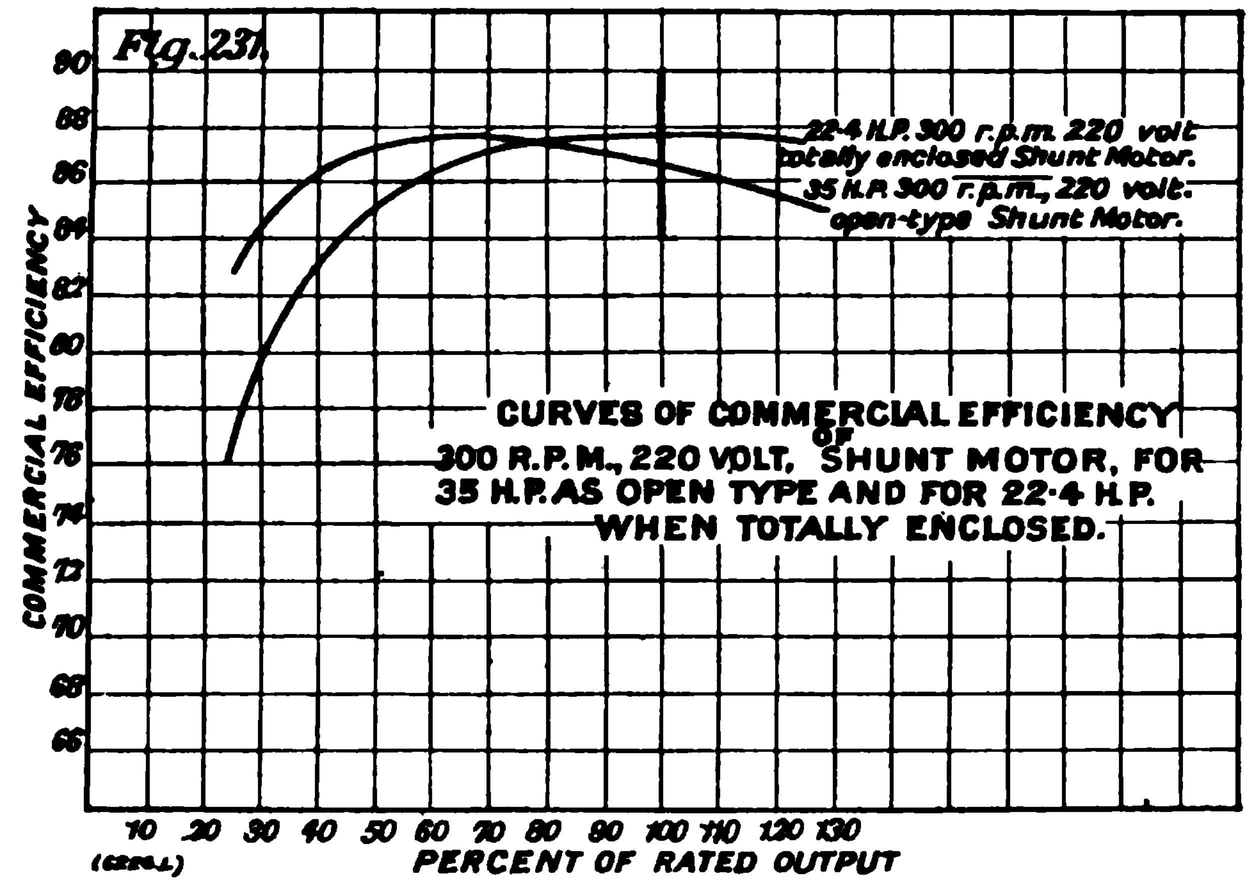
[1] This becomes 7 when the yoke is built of cast iron of the most economical section for a motor of this particular speed.

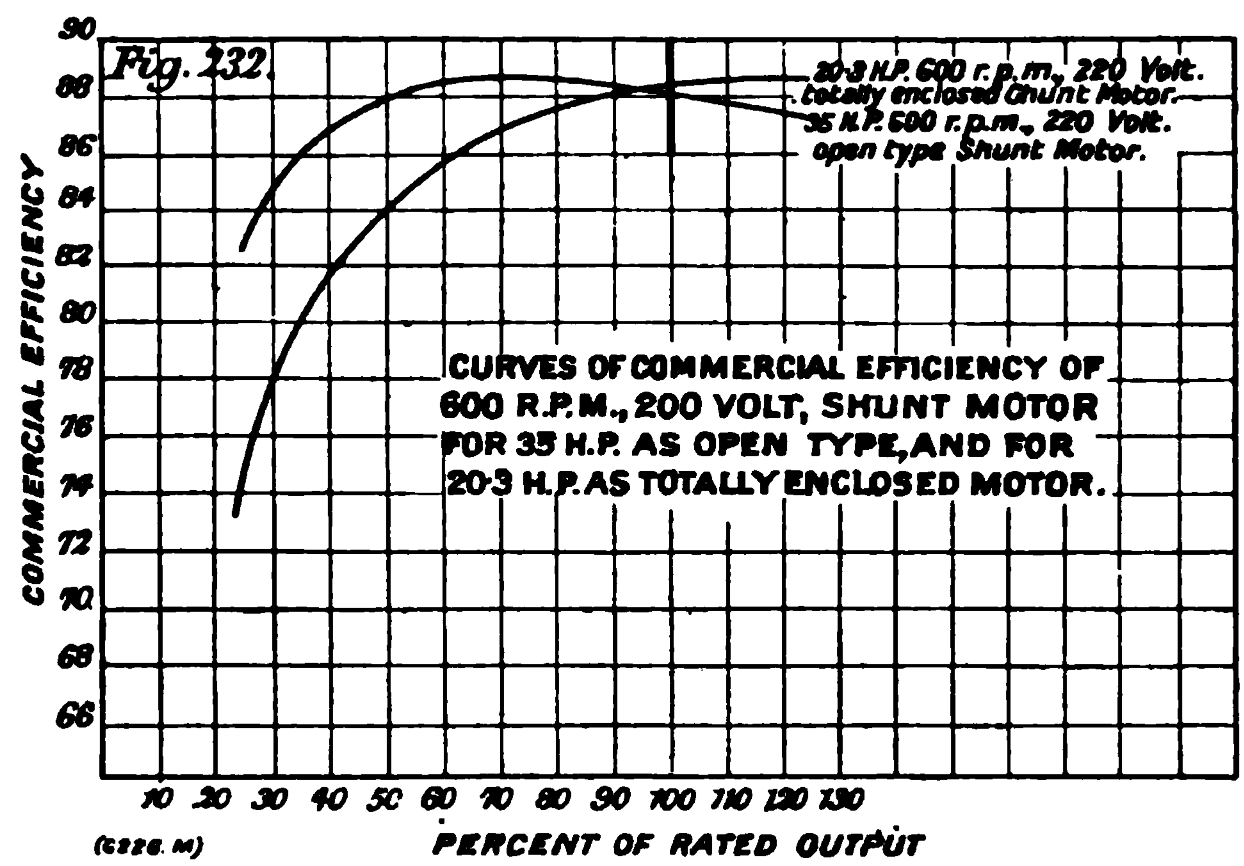
[2] This becomes 22·2 kilogrammes when the yoke is built of cast iron of an economical section for a motor of this particular speed.

§ 5. Totally Enclosed Ratings at other Speeds.

TABLE XXXI.—ESTIMATION FOR TOTALLY ENCLOSED RATINGS.

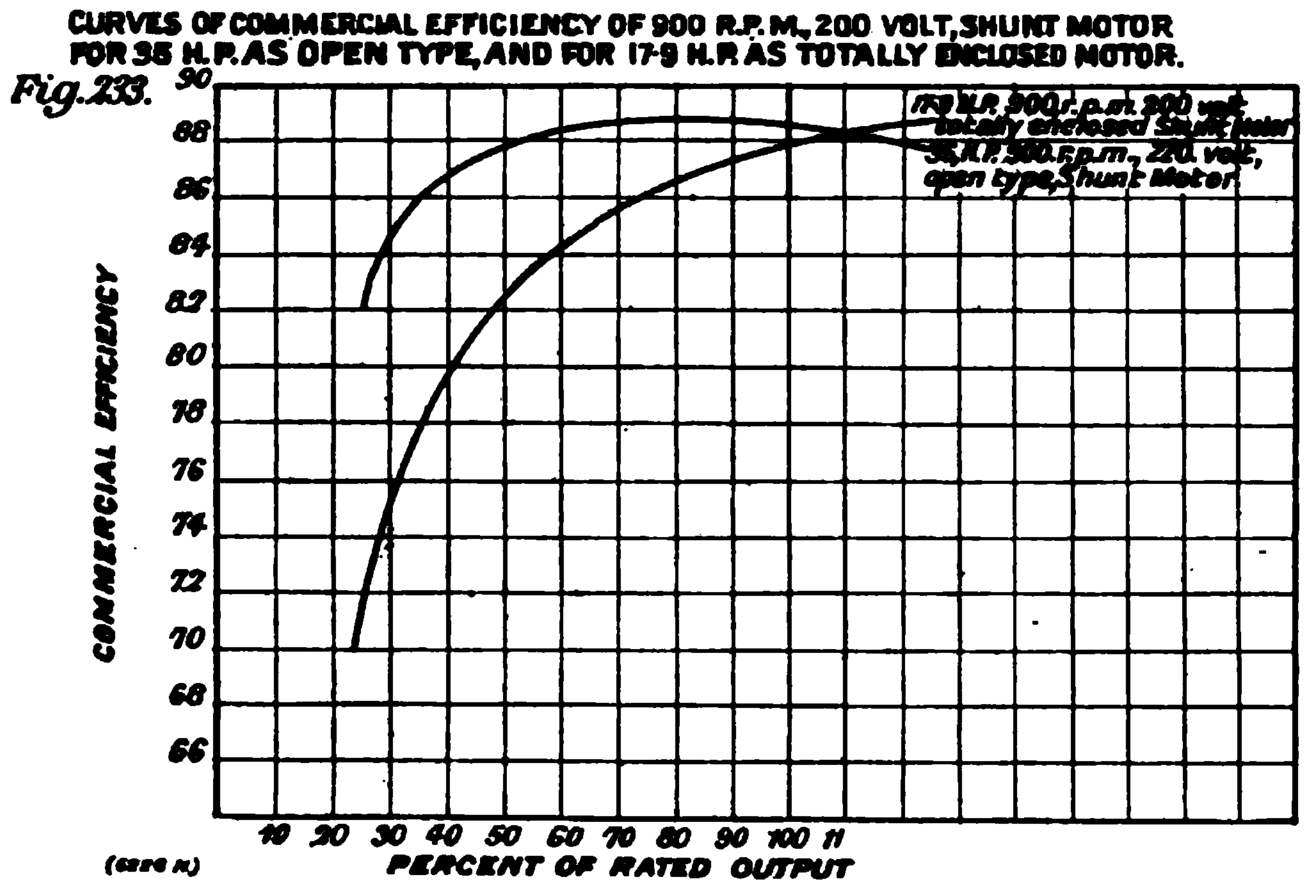
	A	B	C
Total external radiating surface	320	265	250
Watts per square decimetre	7·5	7·5	7·5
Total internal loss (*a*)	2400	2000	1880
Constant loss, same as for open type (*b*) ...	1200	1250	1330
Variable loss (*a*-*b*)	1200	750	550
Variable loss as open motor	2910	2290	2120
Ratio of variable losses, enclosed and open	0·41	0·33	0·26
Square root ditto	0·64	0·58	0·51
Horse-power rating as totally enclosed motor	22·4	20·3	17·9
Cost of net effective material, shillings ...	477	318	279
Ditto per horse-power, shillings	21·3	15·7	17·6

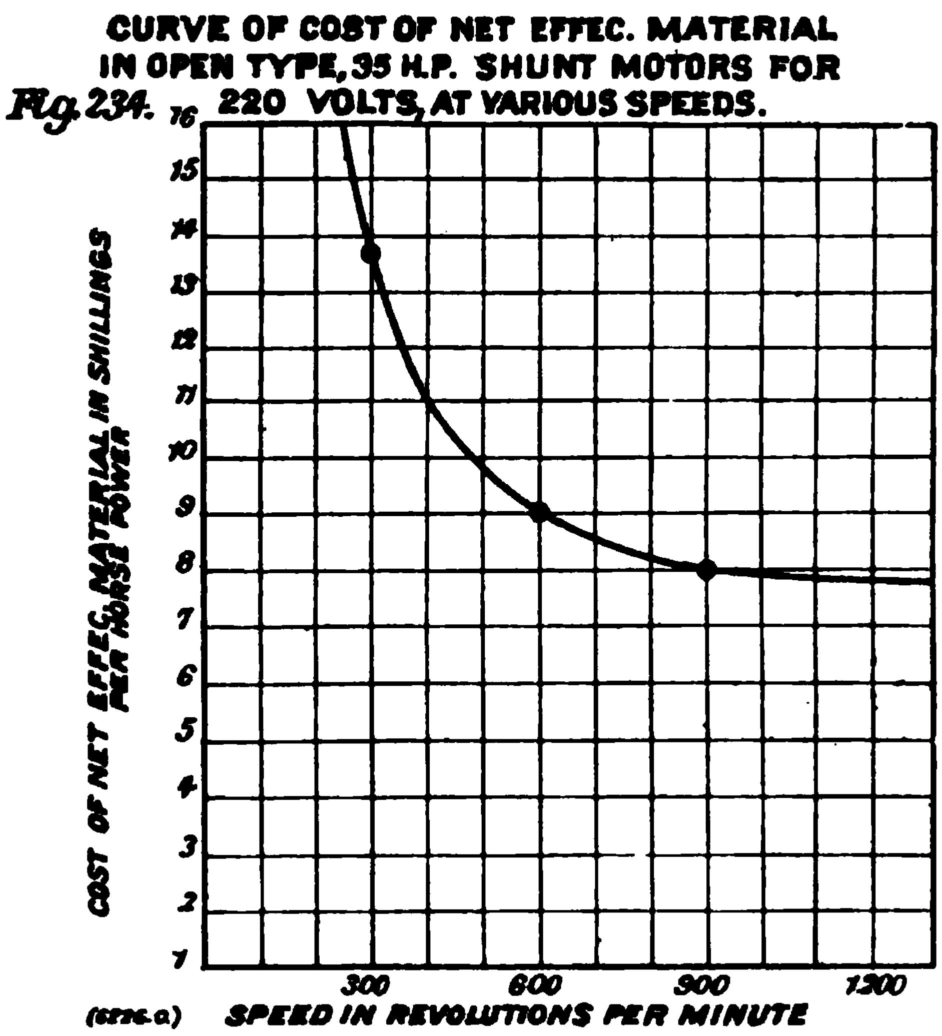




FIGS. 231 and 232.

		A	B	C
Efficiency as totally enclosed motor at	¼ load	77·0	74·3	71·3
" " "	½ "	85·0	84·0	82·2
" " "	¾ "	87·1	87·2	86·0
" " "	full "	87·5	88·3	87·8
" " "	1¼ "	87·3	88·6	88·5





FIGS. 233 and 234.

In Figs. 231, 232, and 233 are given the efficiencies of these three motors both for open and enclosed types. Fig. 234 gives a curve showing the rate of variation of the cost of net effective material with the speed.

CHAPTER XI

VARIABLE SPEED MOTORS

§ 1. Methods of Obtaining Variable Speed with Continuous Current Motors.—With respect to its capacity for economical operation throughout wide ranges of speed, the continuous current motor is not equalled by any type of alternating current motor as yet in commercial use in other than very small sizes. It is true that induction motors are sometimes employed for variable speed work, but their best advocates generally admit their decided inferiority for such purposes. With the continuous current motor, on the contrary, numerous satisfactory and economical methods of operation at variable speed are employed.

Simply by means of a rheostat in the armature circuit, the speed may be varied from full speed to zero; but this is a most wasteful method, and, furthermore, for a given position of the rheostat, the speed will vary according to the current consumed—that is, in proportion to the torque. If the torque required of the motor is increased, the amperes will increase and the speed will diminish, although the rheostat position remains unchanged.

A far more efficient method of speed regulation is by means of a rheostat in the shunt circuit, involving thus only the very slight rheostatic loss of a shunt-adjusting rheostat. This method, however, has generally been pronounced unsatisfactory, and the prevalent opinion is that a motor cannot have its speed varied through a range of more than 30 or 40 per cent. without encountering sparking at the commutator, unless the motor is large and expensive. This is doubtless the case with many motors designed on the customary lines, but the writer would point to the methods which he has advocated in these articles as indicating the correct lines for the design of variable speed and reversible motors. It is hard to see how this can be otherwise than obvious; but this does not appear to be the case, as the most curious interpretations of his line of reasoning have appeared. Almost the only evidence of

a thorough understanding of the writer's purpose comes from Mr Esson,[1] who recently put the case as follows:—

"So far as I am aware, fault is not found with the old theory. The argument, I take it, is this: You probably want some kind of fringe for bringing the current in the section to zero and reversing it, but the magnitude of this fringe must be dependent on the reactance voltage. Make the reactance voltage negligible, and then you can be independent of this fringe; or, in other words, you can do without a positive field, and work even with a negative one. . . . The idea of reactance is not new, and when the matter is looked carefully into, reducing the reactance voltage really means making the commutator sections for a given machine as numerous as possible."

And it could well be added: Making the cross section of the magnetic circuit linked by each turn as small as possible.

Now, the reason that the usual types of design of motors with high reactance voltage generally spark when the field is weakened, *i.e.*, when the speed is increased by field control, is that they are, in virtue of their high reactance voltage, dependent upon electromagnetic commutation—*i.e.*, upon a sufficiently strong "magnetic fringe." At increased speed for a given current such motors are more than ever in need of this, because of the increased reactance voltage due to the increased frequency of reversal under the brush.

§ 2. **Essential Conditions for Variable Speed Motors.** —Hence it is obvious that motors for speed variation by shunt control ought to be designed with the minimum obtainable reactance voltage, *i.e.*, with so low a reactance voltage that they are inherently incapable of sparking within the required range of load and speed, for by this method of control we cannot depend upon the reversing field if we are to obtain a wide range of speed control. Still more must *reversible* motors be independent of magnetic commutation, since the brushes must remain in the neutral position, and, obviously, magnetic reversal is absolutely wanting in this case. No machine designed on normal lines, and which operates satisfactorily with the brushes in the neutral position, is doing so in virtue of electro-magnetic commutation. By carrying far enough the method of design advocated by the writer, the commutation is rendered absolutely independent of the field strength; such a motor will, nevertheless, for a given current input, have a reactance voltage proportional to the speed in virtue of the increasing periodicity of commutation, but it will not have

[1] *Proceedings of the Institution of Electrical Engineers*, vol. xxxii., p. 464.

the superposed handicap of the customary type of motors, a simultaneous *falling off*, just when it should be *increased*, in the very factor upon which reliance is placed for preventing sparking—*i.e.*, the magnetic field. With motors designed on correct lines, the reactance voltage should be made sufficiently low at the highest speed and load required. This condition fulfilled, the motor will operate satisfactorily by shunt control for all speeds and loads from zero up to that limit, and in either direction, and with the brushes in the neutral position.

Were these arguments better grasped, the method of speed variation by shunt control would generally suffice, and there would, probably, rarely be sufficient justification for resorting to

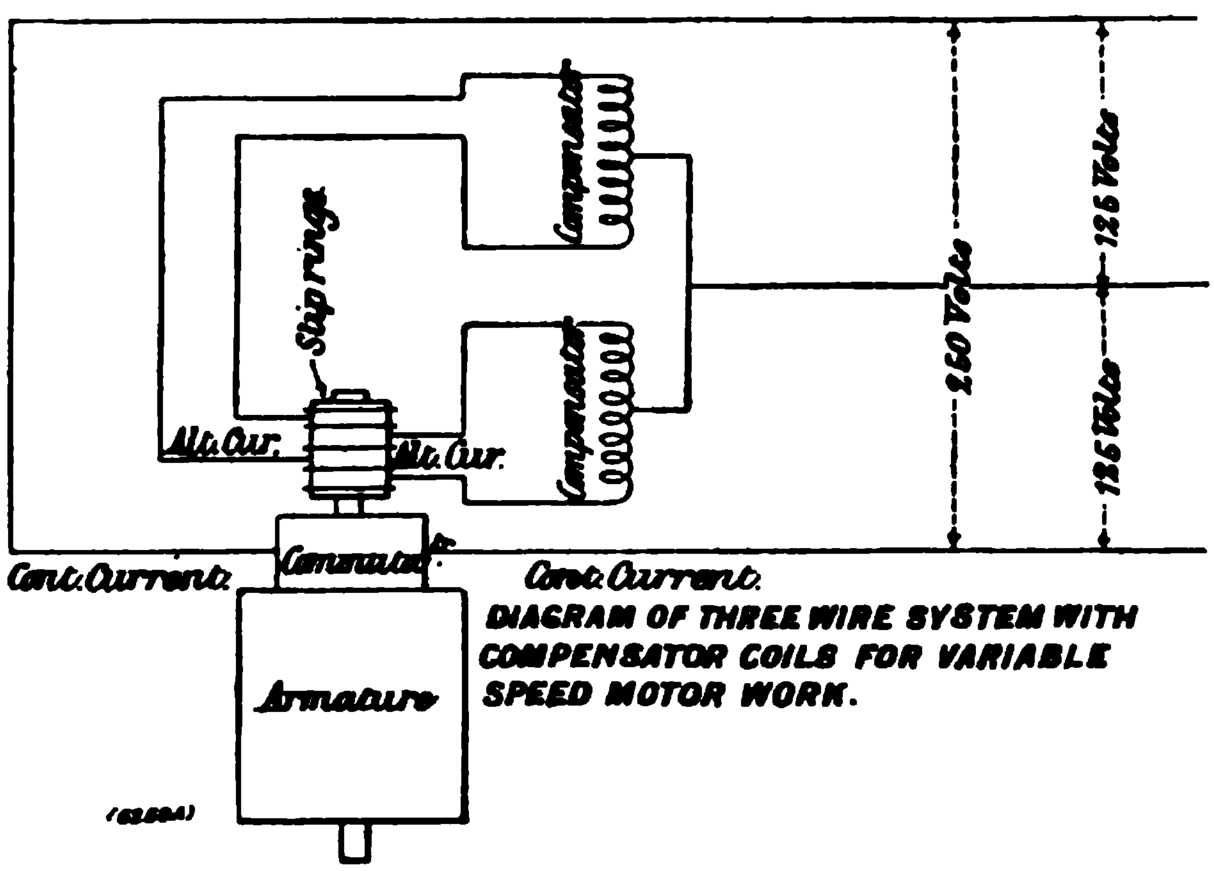


FIG. 234A.

the next method to be described for obtaining variable speed, namely, the system of multi-voltage control.

§ 3. **Multi-voltage Control of Speed.**—Several forms of this system are in wide use, especially in America. The underlying principle will be sufficiently understood by reference to an interesting three-wire system, described by Mr N. W. Storer in a paper presented to the American Institute of Electrical Engineers, November 21st, 1902, from which the diagram in Fig. 234A is taken. The 250-volt continuous current generator from which the system is operated is, in addition to a commutator, equipped also with four slip rings, across which two compensators are connected as shown.

(Of course, three slip rings and three Y-connected compensators could equally well be used, as well as various other similar plans.)

The four slip rings are tapped into the armature winding at four equidistant points per pair of poles; or, for a two-circuit winding, four points equidistantly spaced over the entire winding. The middle points of these two compensator coils are obviously at zero potential, and so long as the two sides of the three-wire system are equally loaded, no current is carried by the slip rings or compensator coils, which simply act to balance the circuit, and carry the small current corresponding to the inequality in the load on the two sides of the system. The motor is provided with a controller, arranged to throw the armature across either the 250-volt outer wires, or from one outer wire to the neutral; also to supply its shunt circuit with either 250 volts or 125 volts; and, finally, to insert more or less resistance in its shunt circuit.

Suppose first that the field is excited by 250 volts, and that the armature is supplied with 125 volts. This gives the slowest speed, which we may, for example, take as 100 revolutions per minute. Neglecting saturation, the speed would then become 200 revolutions per minute by transferring the armature to the 250-volt circuit, and 400 revolutions per minute by subsequently transferring the field to the 125-volt circuit. By then weakening the field to two-thirds of this last strength by means of the shunt resistance, a speed of 600 revolutions per minute is obtained. The intermediate speeds are also obtained by rheostatic control of the shunt circuit, hence efficiently. If the motor is built normally for operation at 400 revolutions per minute at 250 armature volts and 125 volts across the field, and if under these conditions the saturation of the magnetic circuit is low, and the commutating properties really excellent, its normal torque may be developed at any speed between 100 revolutions per minute and 600 revolutions per minute.

A double commutator motor on a two-wire system may, with series parallel control, give the same advantages as such a single commutator motor on a three-wire system.

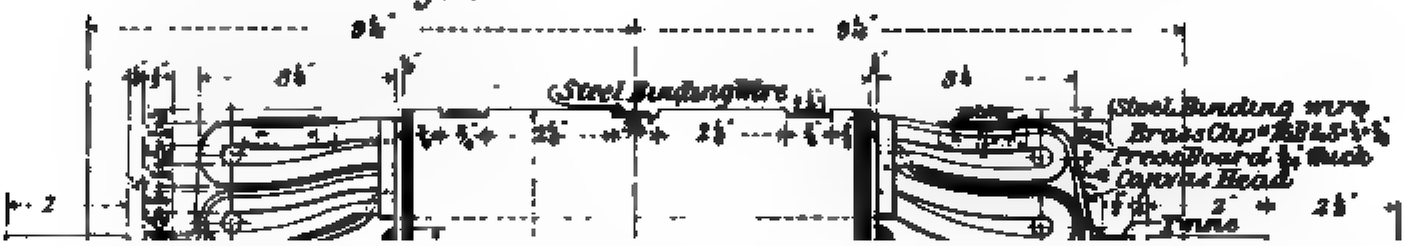
§ 4. **Series Parallel Control of Double Commutator Motors.**—A good practical example of the skilful application of these and similar principles to a system with two double commutator motors operated with series parallel control, and in which compound wound fields are employed, is afforded by the Johnson-Lundell Electric Traction Company's tramway equipment, which will now be described.

The drawings in Figs. 235 to 245 relate to the 35 horse-power,

FIGS. 235 and 236.—35 Horse-power, Compound Wound, 4-pole Johnson-Lundell Tramway Motor.

compound wound 4-pole tramway motor of the Johnson-Lundell Electric Traction Company, to whom the writer is indebted for permission to make use in these articles of the data and tests con-

Fig. 238.



Figs. 237 and 238.—Sections through Field Armature and Commutators.

tained in the following description. The tests were made by the Johnson-Lundell Company's engineer, Mr Gustaf Lang. The motor, designed by Mr Robert Lundell, presents numerous interesting features, notably the completely laminated magnetic circuit, the

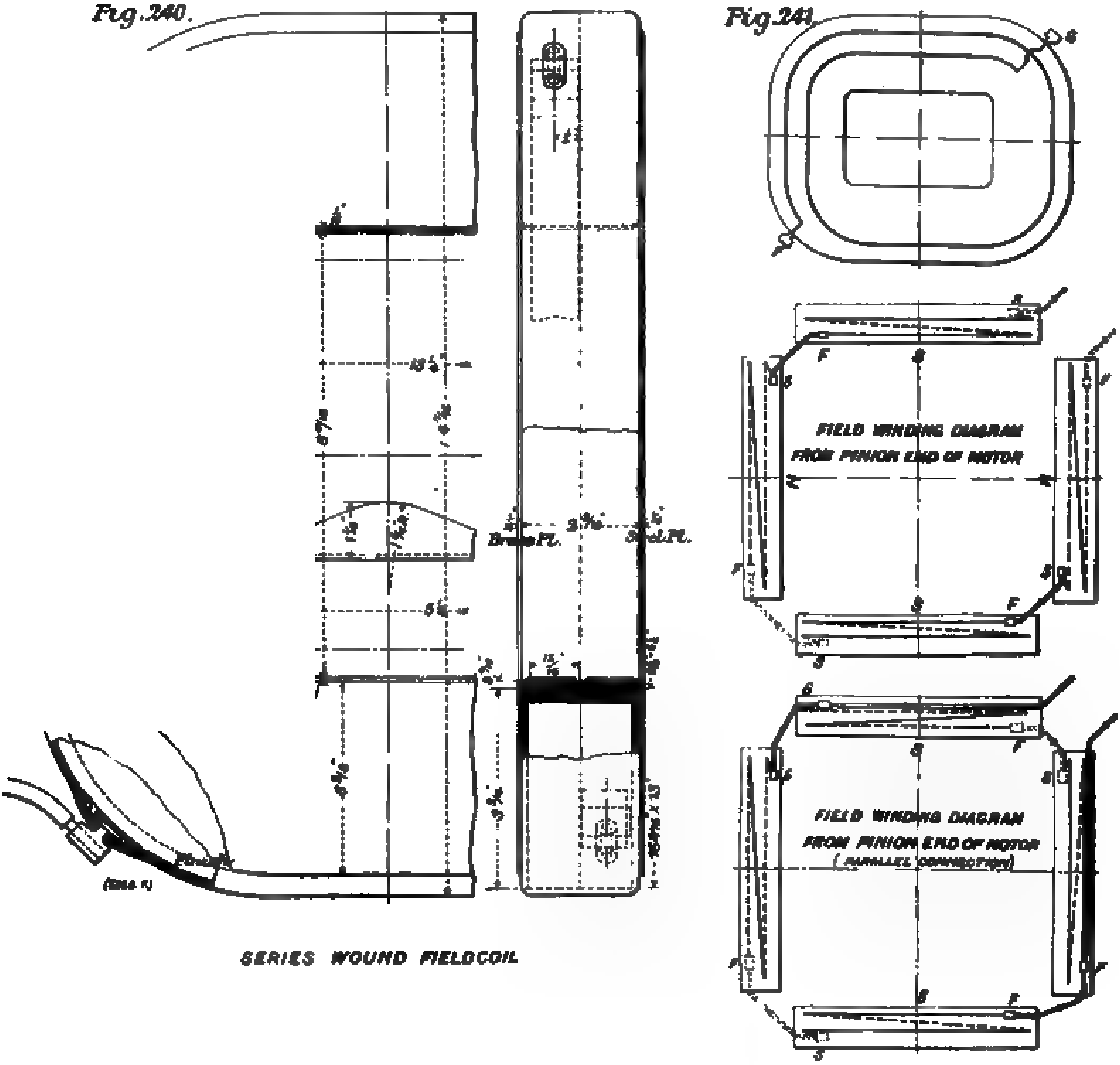
two commutators, the type of form-wound coil, and the employment of a flat strip conductor, not only for the series, but also for the shunt winding, in the interests of obtaining a high "space factor." The laminated field is employed not only as leading to an economical design, but also from the requirements of the system, one feature of which is to use the motors as generators for braking and when descending grades, a very material amount of energy being restored to the line.

Each commutator has 115 segments, five segments per slot, and three turns per segment. Each of the twenty-three armature slots contains one side of twenty-three, three turn coils, there

FIG. 239.—Brush-holder for 35 Horse-power Johnson-Lundell Motor.

thus being sixty conductors per slot, arranged five wide and twelve deep. The conductors of the winding, corresponding to the commutator at the pinion end, are located in the lower halves of the slots; but to offset their less favourable position with respect to the magnetic flux, and also to enable the coils containing them to be wound upon the same winding form as the coils of the winding corresponding to the other commutator, which latter coils are located in the upper halves of the slots, the lower coils span one more tooth than the upper coils. The motor is designed to run at 560 revolutions per minute when the commutators are connected in parallel, and at a terminal potential of 500 volts. The double armature winding has, as we have seen, sixty conduc-

tors per slot, hence $30 \times 23 = 690$ total turns, or 345 turns per commutator. The winding being of the two-circuit type, there are 172 turns in series between brushes.



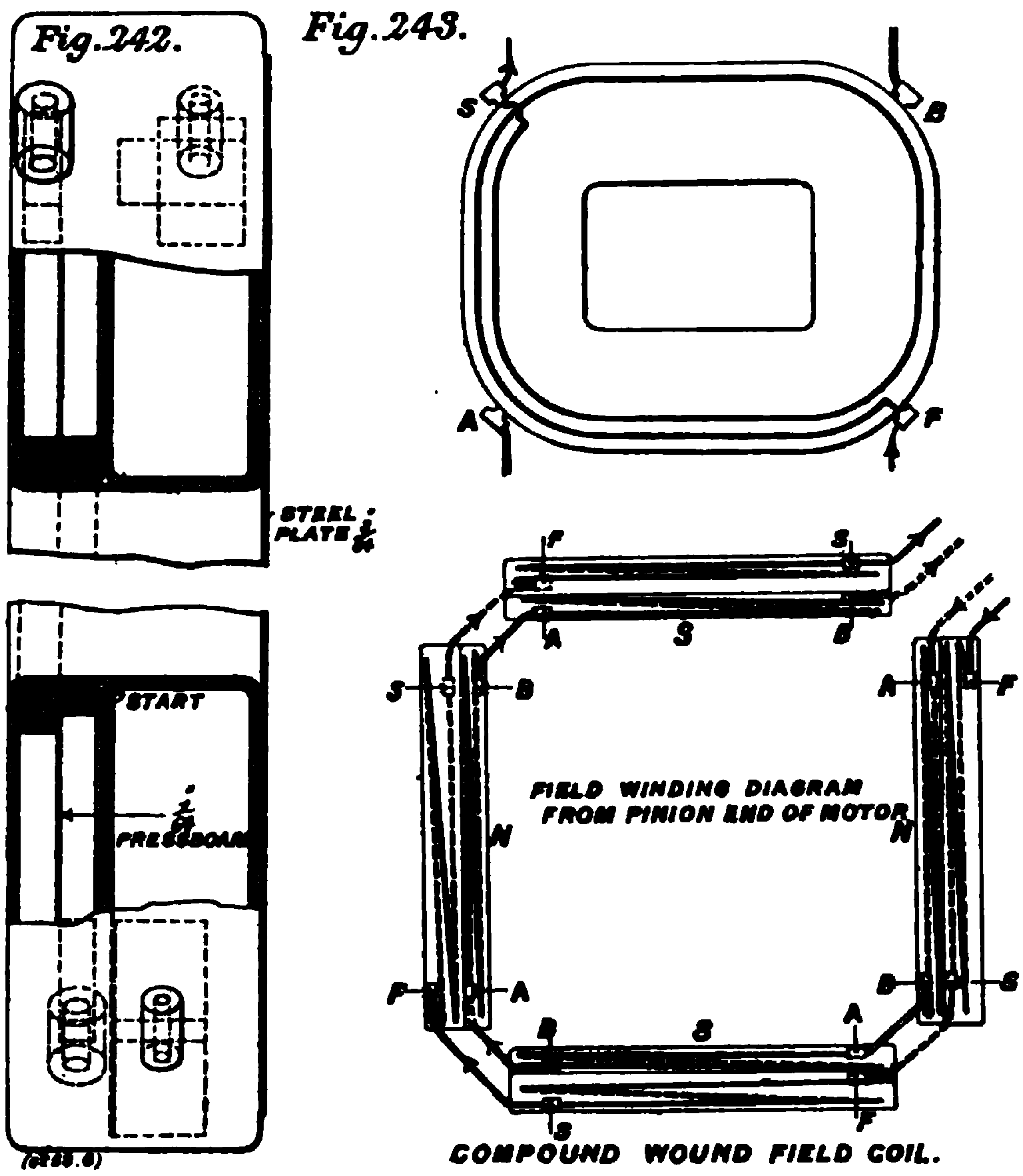
FIGS. 240 and 241.—Spool for Field Coils of 35 Horse-power Johnson-Lundell Motor.

Bare diameter of armature conductor ...	2·30 millimetres
D C C diameter	2·62 millimetres
Copper cross section of armature conductor	·0415 square centimetre

The series field winding consists of 102 turns per spool of flat strip conductor.

Dimensions of series conductor ...	... 7·0 mm. by 1·9 mm.
Cross section of series conductor ...	... 0·133 sq. cm.
Mean length of one turn	... 105 centimetres
Weight of series copper per spool	... 12·3 kilogrammes
Total weight series copper ...	... 49 „
Resistance per spool at 60° Cent....	... 0·154 ohm
„ of four spools at 60° Cent.	... 0·62 „

The shunt coils were wound with 1100 turns per spool of flat copper strip.

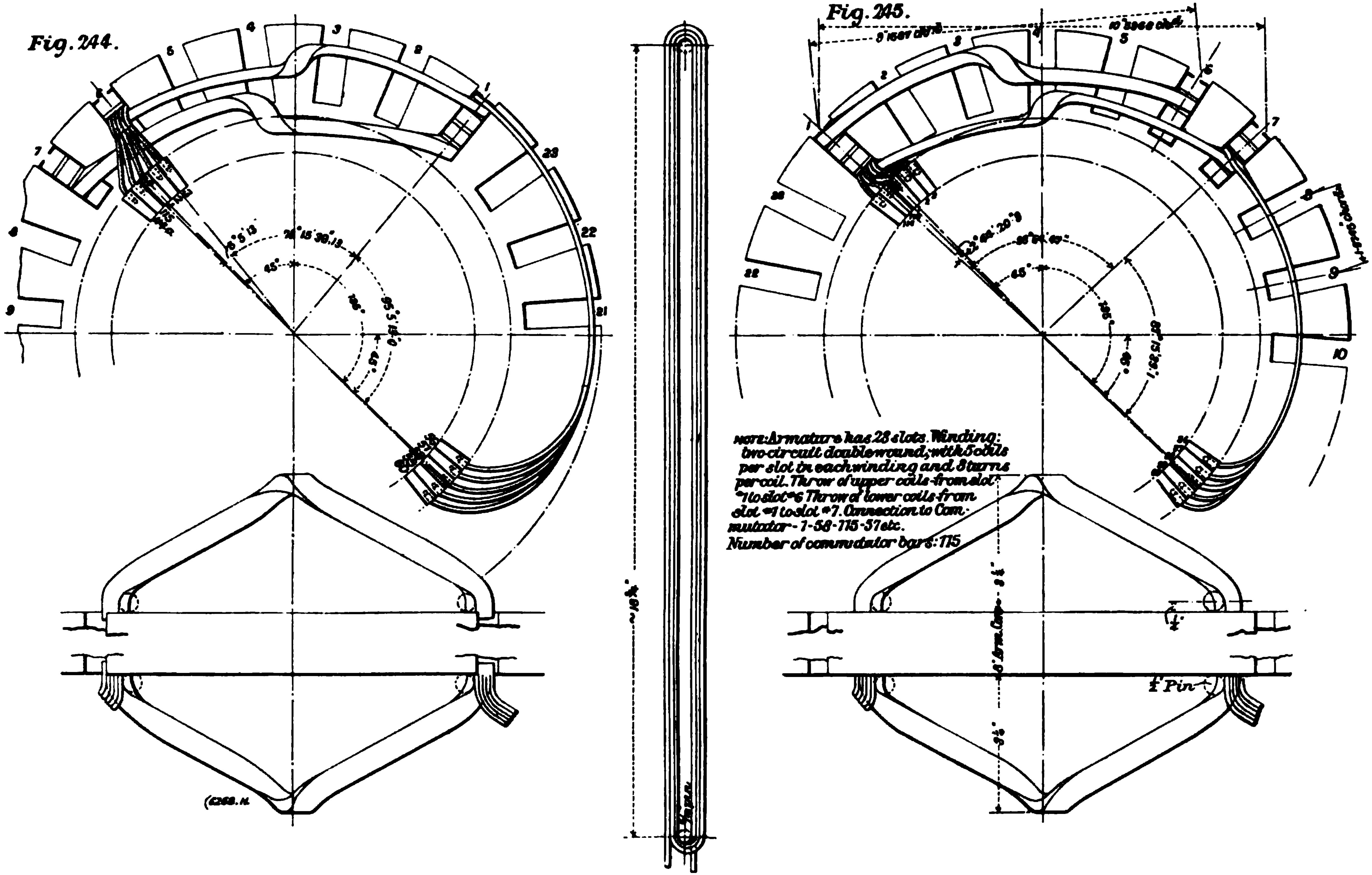


FIGS. 242 and 243.—Field Winding, 35 Horse-power Johnson-Lundell Motor.

Dimensions of shunt conductor ...	... 7·0 mm. by 0·30 mm.
Cross section of shunt conductor ...	... 0·0210 sq. cm.
Mean length of one turn	... 100 centimetres
Weight of shunt copper per spool	... 22 kilogrammes
Total weight of shunt copper ...	... 84 „
Resistance per spool at 60° Cent.	... 11 ohms
„ of four spools at 60° Cent.	... 44 „

In the case both of the series and the shunt windings, the turns of flat strip copper were insulated from one another with cotton tape 10 millimetres wide and 0·06 thick.

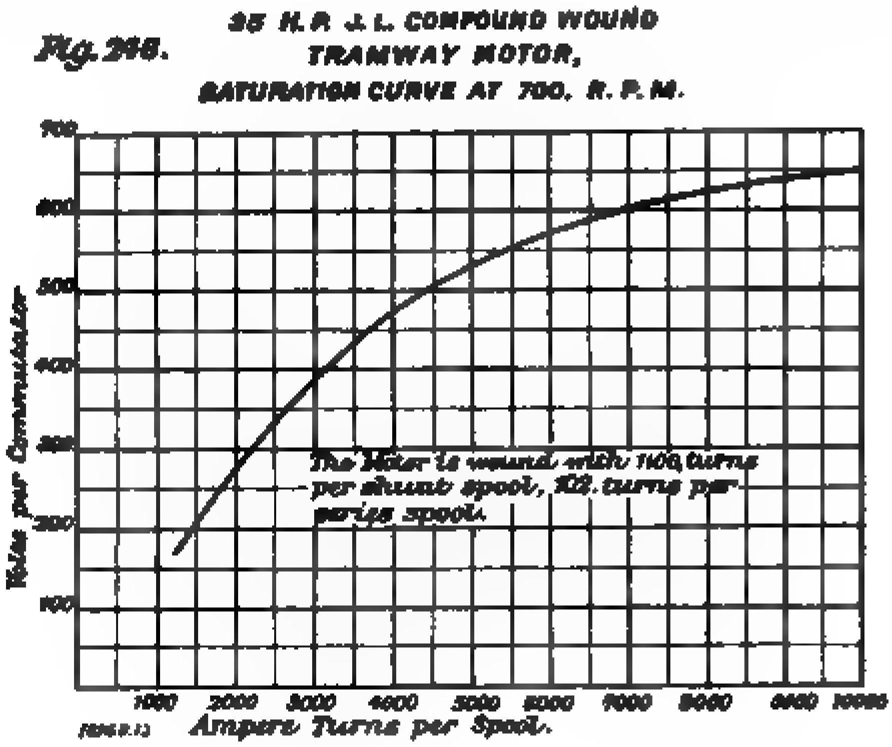
The saturation curve taken at 700 revolutions per minute is



FIGS. 244 and 245.—Armature Winding, 35 Horse-power Johnson-Lundell Motor.

given in Fig. 246, and the approximate curves in Fig. 247, for the core loss of 600 revolutions per minute and 1000 revolutions per minute have been deduced from test results.

From the mean of a number of tests, the friction of bearings, gearing, and windage is taken at 2500 watts at 700 revolutions per minute.



Figs. 246 and 247.

The contact surface per commutator brush is 13 millimetres, by 41 millimetres, or 5·3 square centimetres, and there is one positive and one negative brush per commutator; hence there are 10·6 square centimetres of brush-bearing surface per commutator. At 0·2 kilogramme brush pressure per square centimetre, the total brush pressure for the two commutators of one motor is

$2 \times 10{\cdot}6 \times 0{\cdot}2 = 4{\cdot}3$ kilogrammes. The commutator diameter is 10·25 ins. = 26·0 centimetres; its periphery is 26·11 ins. = 82 centimetres. Hence the brush friction loss at a speed of 1 revolution per minute is $\frac{0{\cdot}82 \times 4{\cdot}3}{6} = 0{\cdot}59$ watt, and at 700 revolutions per minute it is $700 \times 0{\cdot}59 = 410$ watts.

For the purposes of the subsequent calculations this is added to the other friction losses, giving a total friction loss per motor of $2500 + 410 = 2910$ watts at 700 revolutions per minute.

TABLE XXXII.—MEAN VALUES EMPLOYED FOR THE RESISTANCES IN ESTIMATING THE CHARACTERISTIC CURVES OF THE JOHNSON-LUNDELL COMPOUND WOUND, DOUBLE COMMUTATOR MOTOR.

	Resistance in Ohms.	
	From Measurements at 15·5° Cent.	Reduced to 60° Cent.
Series field winding (4 spools in series)	0·53	0·62
Shunt field winding (4 spools in series)	36·	44·
Armature winding corresponding to one commutator	0·35	0·41
Brush resistance corresponding to one commutator	0·08	0·08
Resistance of one armature winding, plus one set of brushes	0·43	0·49

The friction at other speeds has been taken directly proportional to the speed, as a rough but nevertheless sufficient approximation; but a strictly correct treatment of the friction for a variable speed geared motor should be based on much more thorough tests.

The brush resistance may be taken at 0·2 ohm per square centimetre of brush-bearing surface, hence at $\frac{0{\cdot}2}{5{\cdot}3} \times 2 = 0{\cdot}08$ ohm per commutator.

The gear ratio is 69 : 14, or 4·93, and car wheels of 33 ins. diameter are assumed to be employed.

From the basis of this test data and the controller diagrams of Fig. 248, Plate 12, the writer has calculated for a temperature of 60° Cent., the results set forth in Table XXXIII., pages 214 and 215, from which the performance of the motor on any controller point may be obtained.

The nine sets of curves in Figs. 249 to 257, Plates 13 to 15, show these characteristics of the equipment for the nine controller points for all values of current input up to 200

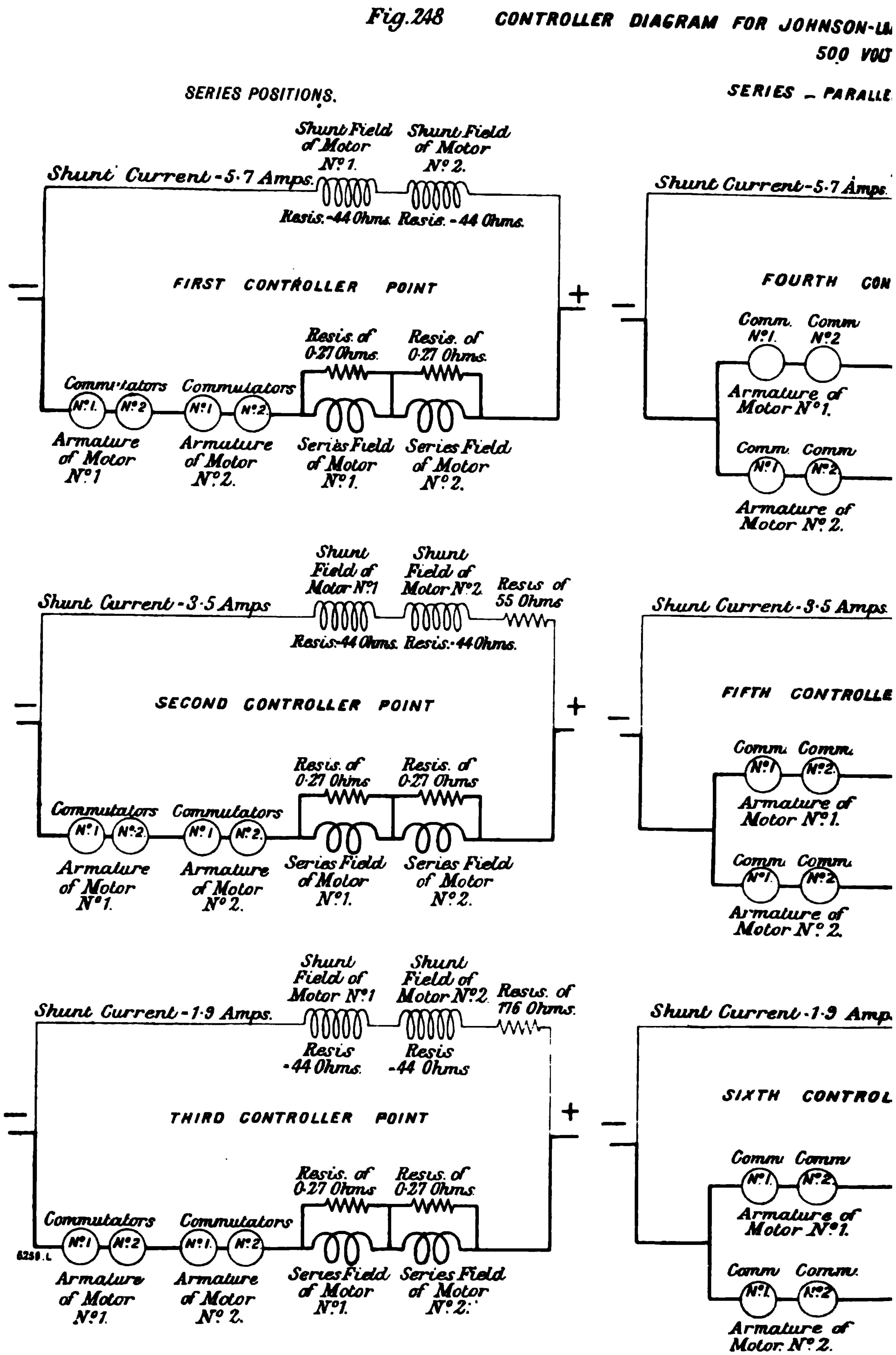
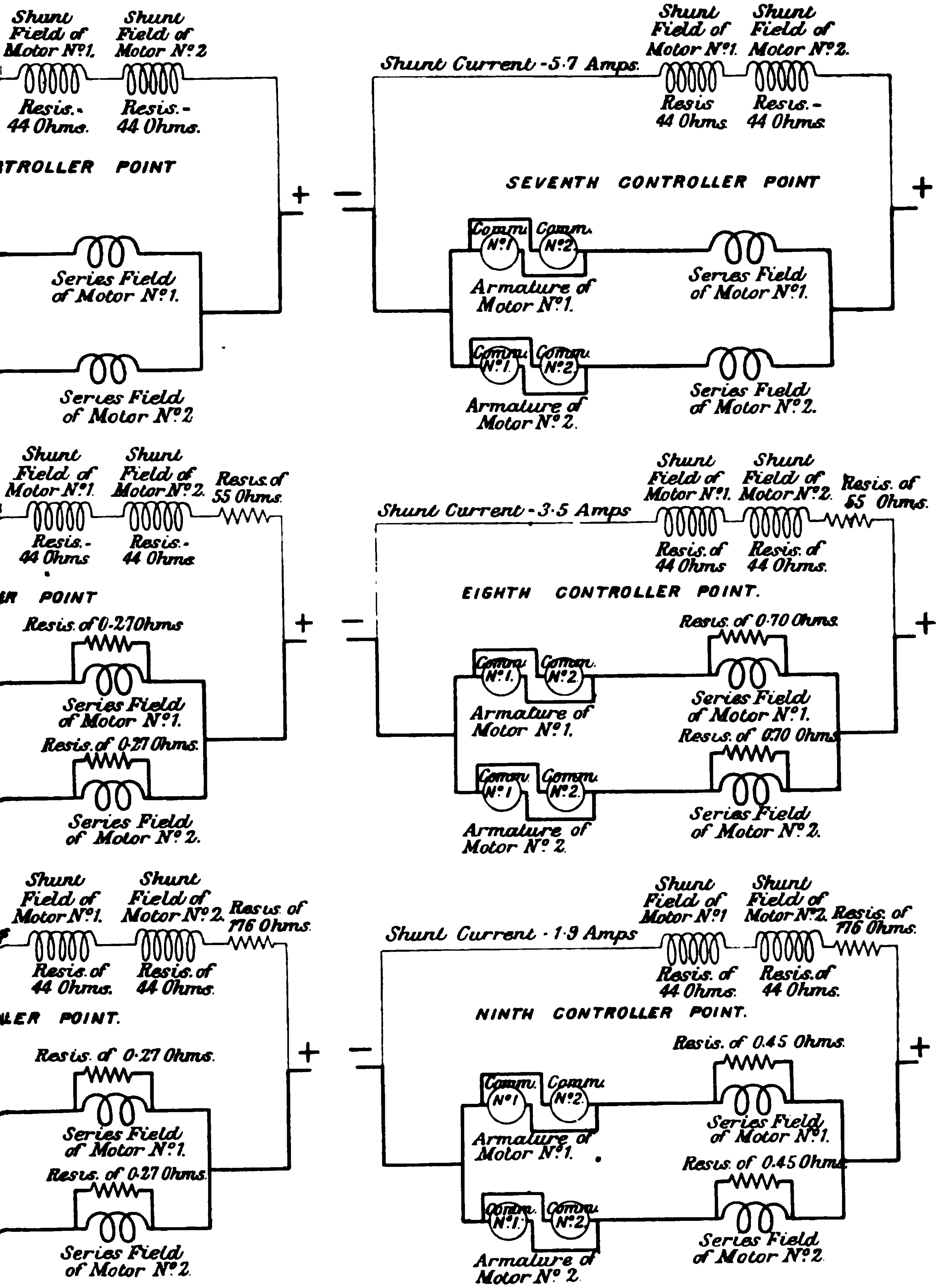


Fig. 248.—Controller Diagram for Johnson-Lundell Double Equi

...NDELL DOUBLE EQUIPMENT WITH TWO 35 HP., ..., MOTORS.

...L POSITIONS. **PARALLEL POSITIONS.**



...ipment with Two 35 Horse-power, 500-Volt Motors (see page 212).

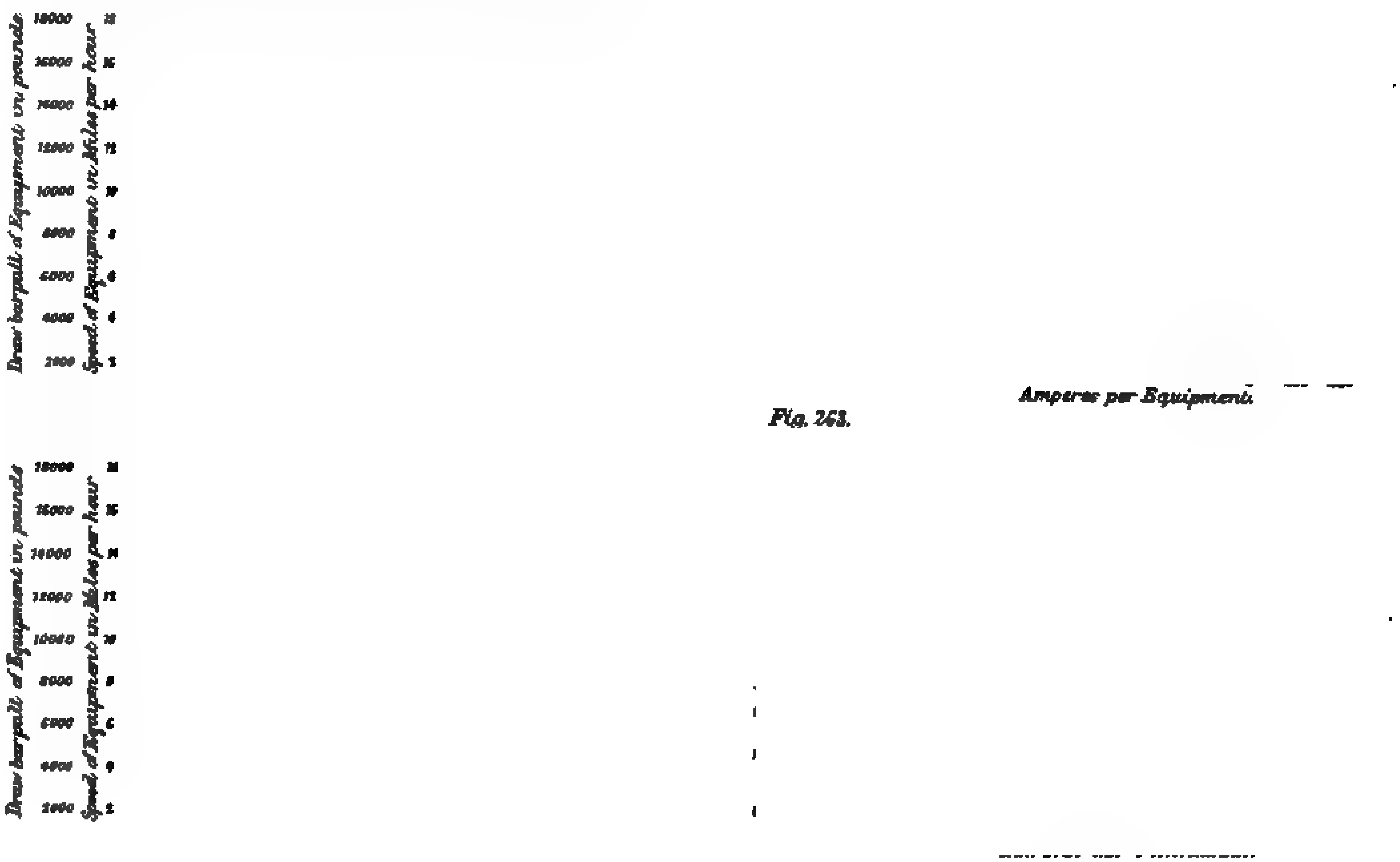
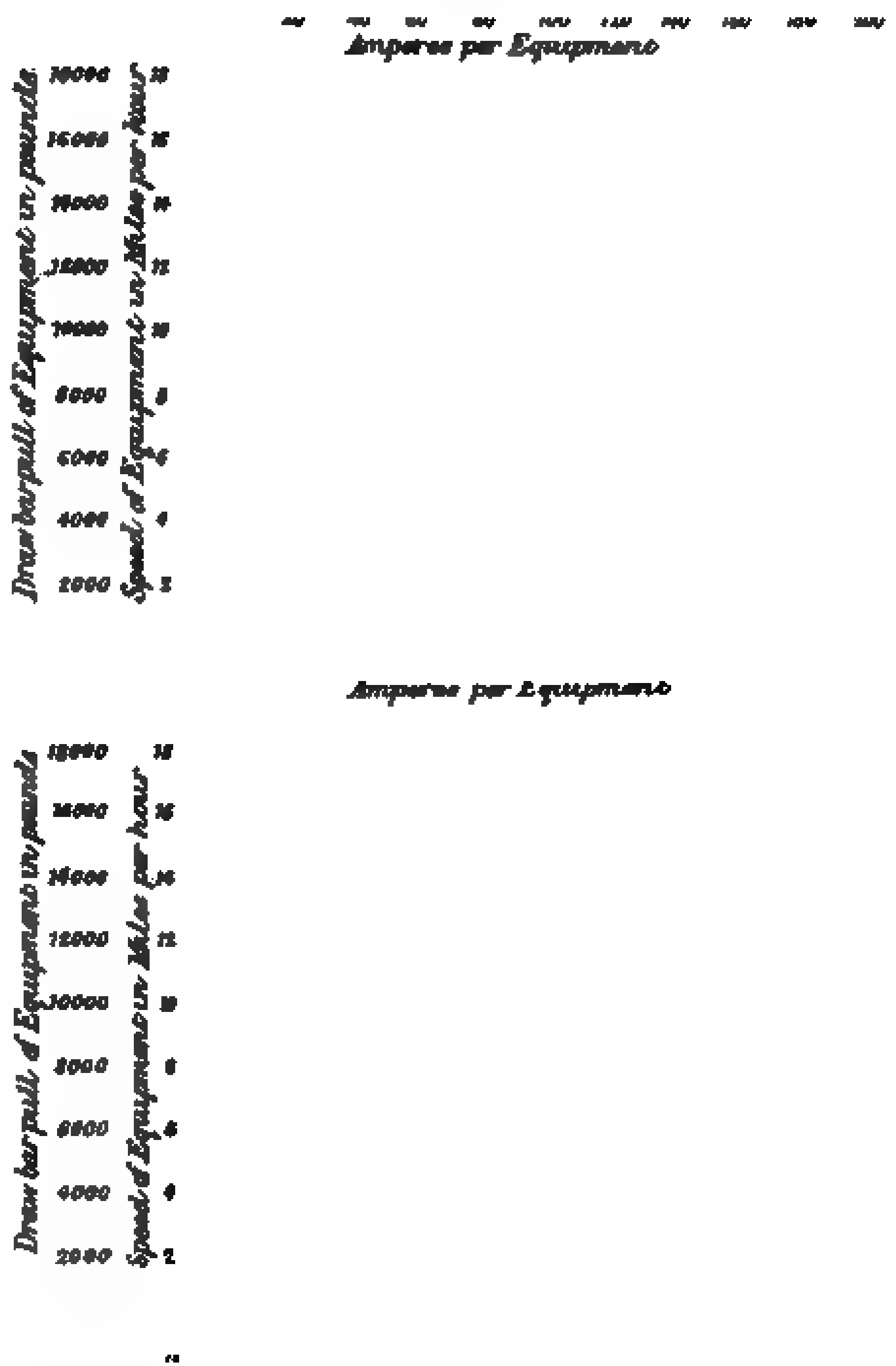

Draw barpull of Equipment in pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
Speed of Equipment in Miles per hour
Amperes per Equipment
Draw barpull of Equipment in pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
Speed of Equipment in Miles per hour

Fig. 263.



FIGS. 249–252 and 262–265.—Characteristic Curves for Johnson-Lundell and Standard Series Parallel Equipments (see pages 212–216).

Fig. 253.

Fig. 254.

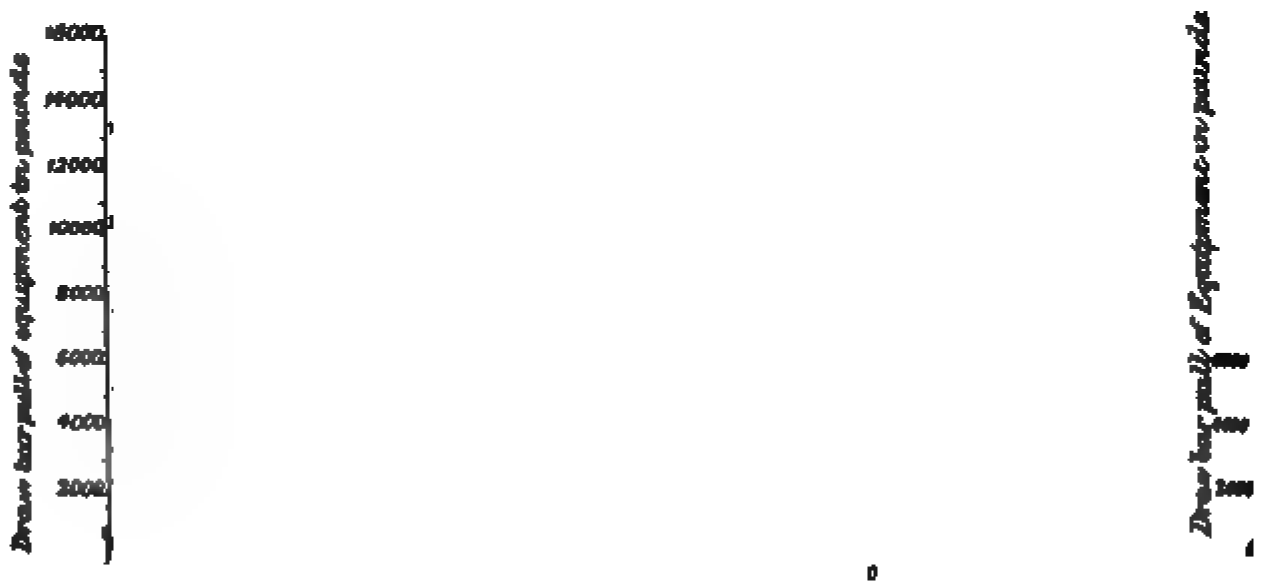


Fig.25

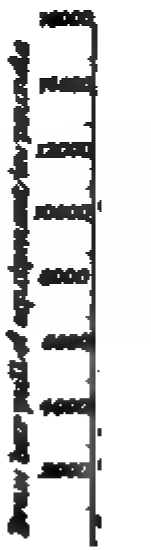
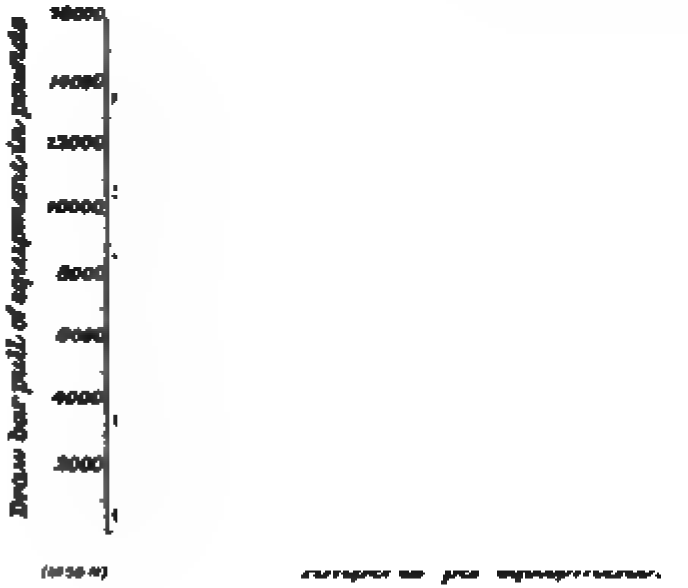


Fig.256.



FIGS. 253–256 and 266–269.—Characteristic Curves for Johnson-Lundell and Standard Series Parallel Equipments (see pages 212–216).

amperes. This series of curves brings out many points of interest, and constitutes an exceptionally thorough study of the occurrences in a compound wound motor under various conditions. It should also be noted from what a very small amount of experimental data one may, with a few reasonable assumptions, very exhaustively study the performance of such a motor. For obtaining this series of curves, all that was available was a saturation curve, some approximate core loss measurements, a few measurements of the friction loss, measurements of the resistances of the windings and a knowledge of the number of turns in the windings, and the dimensions of the motor. From these data a systematically tabulated calculation leads to a knowledge of the performance of

Amperes Input per Equipment

Fig. 261.

the motor under all conditions of service. As a matter of interest, the observed speed reading from Mr Lang's tests on some of the controller points have been plotted on these sheets of estimated curves. The agreement is very fair, but, as a matter of fact, results carefully estimated in the manner set forth in Table XXXIII. are more reliable, owing to the fact that, for really accurate measurements under load conditions, much more care, time, and expense are necessary than manufacturing companies care to incur; and owing to the satisfactoriness of curves estimated from a few simple measurements, it is hardly to be expected that this should be otherwise.

In Figs. 258 to 261, Plate 15 and page 213, curves relating to the same properties are brought together, and the results are very interesting and instructive.

TABLE XXXIII.—RESULTS GIVEN BY A JOHNSON-LUNDELL DOUBLE EQUIPMENT, WITH TWO 35 HORSE-POWER, 500-VOLT MOTORS, CALCULATED FOR A TEMPERATURE OF 60° CENT., SHOWING THE PERFORMANCE OF THE MOTOR ON ANY CONTROLLER POINT.

Controller Position.	Amperes Input per Equipment (two Motors).	Shunt Amperes.	Total Amperes in						I²R Loss in Diverter Rheostat for One Motor.	Half I²R Loss (i.e., that of
1	20	5·7	14·3	14·3	4·4	9·9	200	13	27	1
2	20	3·5	16·5	16·5	5·0	11·5	270	16	36	
3	20	1·9	18·1	18·1	5·5	12·6	320	19	43	
4	20	5·7	14·3	7·2	7·2	0	50	32	0	1.
5	20	3·5	16·5	8·3	2·5	5·8	67	39	9	
6	20	1·9	18·1	9·1	2·8	6·3	81	49	11	
7	20	5·7	14·3	3·6	7·2	0	13	32	0	1,
8	20	3·5	16·5	4·1	4·4	3·9	16	13	11	
1	30	5·7	24·3	24·3	7·4	16·9	580	34	77	1
2	30	3·5	26·5	26·5	8·1	18·4	690	41	91	
3	30	1·9	28·1	28·1	8·5	19·6	780	45	103	
4	30	5·7	24·3	12·2	12·2	0	146	92	0	1,
5	30	3·5	26·5	13·3	4·0	9·3	173	10	23	
6	30	1·9	28·1	14·1	4·3	9·8	194	12	26	
7	30	5·7	24·3	6·1	12·2	0	37	92	0	1,
8	30	3·5	26·5	6·6	7·0	6·3	43	30	28	
9	30	1·9	28·1	7·0	6·0	8·1	48	22	30	
1	40	5·7	34·3	34·3	10·4	23·9	1,150	67	164	1,
2	40	3·5	36·5	36·5	11·1	25·4	1,310	69	174	
3	40	1·9	38·1	38·1	11·6	26·5	1,420	83	190	
4	40	5·7	34·3	17·2	17·2	0	290	184	0	1,
5	40	3·5	36·5	18·3	5·6	12·7	330	20	43	
6	40	1·9	38·1	19·1	5·8	13·3	360	21	48	
7	40	5·7	34·3	8·6	17·2	0	73	184	0	1,
8	40	3·5	36·5	9·1	9·7	8·6	81	58	52	
9	40	1·9	38·1	9·5	8·0	11·0	88	40	54	
1	50	5·7	44·3	44·3	13·4	30·9	1,920	111	258	1,
2	50	3·5	46·5	46·5	14·1	32·4	2,120	123	285	
3	50	1·9	48·1	48·1	14·6	33·5	2,270	132	304	
4	50	5·7	44·3	22·2	22·2	0	480	305	0	1,
5	50	3·5	46·5	23·3	7·1	14·2	530	31	54	
6	50	1·9	48·1	24·1	7·4	14·5	570	34	57	
7	50	5·7	44·3	11·1	22·2	0	120	305	0	1,
8	50	3·5	46·5	11·6	12·4	10·3	132	95	31	
9	50	1·9	48·1	12·0	10·1	14·0	141	68	38	
1	70	5·7	64·3	64·3	19·5	44·8	4,040	235	550	1,
2	70	3·5	66·5	66·5	20·2	46·3	4,350	252	580	
3	70	1·9	68·1	68·1	20·7	47·4	4,550	265	610	
4	70	5·7	64·3	32·2	32·2	0	1,020	640	0	1,
5	70	3·5	66·5	33·3	10·1	23·2	1,090	63	146	
6	70	1·9	68·1	34·1	10·4	23·7	1,140	67	152	
7	70	5·7	64·3	16·1	32·2	0	260	640	0	1,
8	70	3·5	66·5	16·6	17·7	15·6	270	194	170	
9	70	1·9	68·1	17·0	14·4	19·7	290	123	175	
1	100	5·7	94·3	94·3	28·6	65·7	8,700	505	1,160	1,
2	100	3·5	96·5	96·5	29·3	67·2	9,100	530	1,220	
3	100	1·9	98·1	98·1	29·6	68·3	9,400	550	1,260	
4	100	5·7	94·3	47·3	47·2	0	2,200	1,880	0	1,
5	100	3·5	96·5	48·3	14·7	33·6	2,300	184	305	
6	100	1·9	98·1	49·1	14·9	34·2	2,370	187	315	
7	100	5·7	94·3	23·6	47·2	0	560	1,380	0	1,
8	100	3·5	96·5	24·1	25·6	22·7	570	405	360	
9	100	1·9	98·1	24·5	20·6	28·5	590	262	365	
1	150	5·7	144	144	44	100	20,400	1,300	2,700	1,
2	150	3·5	146	146	44	102	20,900	1,300	2,800	
3	150	1·9	148	148	45	103	21,500	1,250	2,850	
4	150	5·7	144	72	72	0	5,100	3,200	0	1,
5	150	3·5	146	73	22	51	5,200	300	700	
6	150	1·9	148	74	23	52	5,300	300	730	
7	150	5·7	144	36	72	0	1,260	3,200	0	1
8	150	3·5	146	36·5	29	34	1,300	940	810	
9	150	1·9	148	37	31	48	1,350	600	840	

TABLE XXXIII.—*continued.*

Mr Gustaf Lang has kindly furnished the writer with a series of curves representing the performance of a standard plain series wound motor of about the same capacity, and these, which are reproduced in Figs. 262 to 274, Plates 13 to 15, and page 216, afford means of comparing the characteristics of these two different types of traction motor.

Returning to the compound wound Johnson-Lundell equipment, it may be added that it is proportioned for operating a 12½-ton tramcar under the ordinary conditions of urban service. Owing to the special features of the Johnson-Lundell system of motor control, whereby rheostatic losses are minimised, all the controller points are relatively fairly efficient, as may be seen from Fig. 258,

Horse-Power Output per Equipment

Fig. 274.

and it is of much less importance than with the older and cruder methods of series parallel control, to run on any particular controller points; in other words, all the points may be regarded as fairly efficient running points, and the car may be run economically on whichever points lead to the desired speed at any particular instant, this depending on the state of the surrounding traffic, the load, the gradient, and the local regulations. Now these restrictions render it rather important, with the older types of series parallel control, to instal the particular type of motor with the particular windings adapted to give the maximum all-day economy, and these conditions are exceedingly difficult to determine, even after the road has been in operation for some time—much more so, however, for projected roads—and as a consequence the wrong motor is generally at first installed. But with the system in which these

CHARACTERISTIC CURVES FOR

Fig. 258. EFFICIENCY CURVES FOR JOHNSON-LUNDELL EQUIPMENT WITH TWO 35 HP MOTORS, 33" WHEELS, GEAR RATIO OF 4.33 & 500 TERMINAL VOLT. ON THE NINE CONTROLLER

Efficiency of Equipment in Per Cent.

Fig. 271. CURVES OF EFFICIENCY FOR STANDARD SERIES PARALLEL EQUIPMENT WITH TWO 35 HP MOTORS WITH SINGLE COMMUTATORS & PLAIN SERIES WINDING.

Efficiency of Equipment in Per Cent.

Amperes per Equipment.

Speed of Equipment in Miles per Hour

Fig. 273.

CURVES OF DRAW BAR PULL FOR STANDARD SERIES PARALLEL EQUIPMENT WITH TWO 35 HP MOTORS WITH SINGLE COMMUTATORS

Amperes Input Per Equipment.

Amperes Per Equipment.

FIGS. 257–260 and 270–273.—Characteristic Curves for Johnson-Lundell and Standard Series Parallel Equipments (see pages 212–216).

compound wound motors are employed, it suffices to instal a motor capable of giving, on the top controller notch, the maximum speed which will ever be required for the heaviest load. The equipment will then be operated most of the time on lower controller notches, but nevertheless with high economy, owing to the small amount of the resistance losses as compared with the older systems. Thus a considerably increased adaptability to urban service is obtained; the tramcars may be operated at will, and irrespective of the load, at high speeds in the outlying districts, and at very low speeds in congested centres, and with fairly good economy in all cases, as may be seen from Figs. 258 and 259, Plate 15.

The other features of this interesting system relating to regenerative braking hardly come within the scope of this series of articles, but it may be stated that these features lead to very considerable additional economies, so that, what with the savings due to dispensing with the customary series resistances during acceleration, and to restoring to the line the energy of momentum of the car during braking, and the stored potential energy during down-hill runs, a very much higher economy per ton-mile is obtained than by the ordinary type of series parallel equipment. While not specifically alluded to above, it is self-evident that the use of two commutators per motor, thus a total of four commutators, by permitting of three stages of paralleling (see Fig. 248, Plate 12)—*i.e.*, first, the series stage (corresponding to controller points 1, 2, and 3); secondly, the series parallel stage (points 4, 5, and 6); and thirdly, the full parallel stage (points 7, 8, and 9)—together with the relative adjustments of the strengths of the shunt and series excitation, are the main elements which, supplemented by the shunt winding, render it practicable to dispense with wasteful series resistances. The attainment of the very even gradations of speeds, shown in Table XXXIII. and in Fig. 259 to have been attained for successive contact points, required an exhaustive and painstaking study of the conditions to be fulfilled, and the results appear to the writer to reflect great credit on Mr Lang, to whom the preparation and execution of this part of the final design are due. It is not at a glance apparent that this should require so very much care, nor would it for any single particular value of the current input; but to arrange such an adjustment of the shunt and series windings and of the final resistance adjustments, that these shall all best co-operate under the most frequently recurring conditions, to accomplish their purpose, and give fairly even acceleration in

operating the controller from the start to the last notch, not for one particular load, but for all loads requiring to be provided for, is a problem to which a good deal of study can be devoted to advantage before the windings for such a motor are finally decided upon.

Mr Lang's test results relating to speed and amperes input per equipment are set forth in Table XXXIV.

TABLE XXXIV.—OBSERVED RESULTS FOR SPEED CURVES FROM TESTS ON A JOHNSON-LUNDELL EQUIPMENT, CONSISTING OF TWO 35 HORSE-POWER, COMPOUND-WOUND, DOUBLE COMMUTATOR TRAMWAY MOTORS.

Measurements made on 3rd, 5th, 6th, 7th, 8th, and 9th controller points. Date of tests, March 1903.

Notch 3 *on Johnson-Lundell Controller.—Commutators in series; terminal voltage,* 250 *volts per motor; shunt field,* 1·9 *amperes constant; series field shunted by* 0·27 *ohm; constant of tachometer,* 1·035.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 250 Volts per Motor.	Speed in Miles per hour on 33 in. Wheels with Gear Ratio of 4·93.
504	48·1	156	161	161	3·22
505	43·2	162	168	167	3·34
500	36·8	173	179	179	3·58
501	32·2	182	188	188	3·76
500	23·2	204	211	211	4·22
500	17·6	224	232	232	4·64

Notch 5 *on Johnson-Lundell Controller.—Commutators in series parallel; terminal voltage,* 500 *volts; shunt field,* 3·5 *amperes; series field shunted by* 0·27 *ohm; constant of tachometer,* 1·035.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 500 Volts.	Speed in Miles per Hour on 33 in. Wheels with Gear Ratio of 4·93.
505	104·5	283	293	290	5·8
505	86·8	295	306	303	6·1
505	73·7	304	315	312	6·2
503·5	67·6	310	321	319	6·4
508	57·5	317	328	383	6·5
505	41·5	330	342	338	6·7
504	32·6	336	348	346	6·9
505	23·8	345	357	253	7·1

Notch 6 on Johnson-Lundell Controller.—Commutators in series parallel terminal voltage, 500 *volts; shunt field,* 1·9 *amperes constant; series field shunted by* 0·27 *ohm; constant of tachometer,* 2·055.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 500 Volts.	Speed in Miles per Hour on 33 in. Wheels with Gear Ratio of 4·93.
503	99·9	172	354	353	7·0
503	89·0	178	366	364	7·3
503	82·6	184	378	376	7·5
505	62·6	198	407	403	8·1
505·5	55·9	206	424	419	8·4
504	49·5	214	440	436	8·7
503	44·3	220	452	450	9·0
504	34·3	234	481	478	9·6
505	15·8	268	550	545	10·9

Notch 7 *on Johnson-Lundell Controller.—Commutators in parallel; terminal voltage,* 500 *volts; shunt field,* 5·75 *amperes constant; series field not shunted; constant of tachometer,* 2·055.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 500 Volts.	Speed in Miles per hour on 33 in. Wheels with Gear Ratio of 4·93.
510	217·6	178	368	361	7·2
510	189·8	187	384	376	7·5
511·5	171·3	193	396	387	7·7
512	150·7	203	417	407	8·1
512	133·6	210	431	421	8·4
513	109·5	225	426	450	9·0
513	86·8	237	486	474	9·5
512	69·2	251	515	503	10·1
513	31·1	280	575	560	11·2
514	19·4	291	598	582	11·6

Notch 8 *on Johnson-Lundell Controller.—Commutators in parallel; terminal voltage,* 500 *volts; shunt field,* 3·5 *amperes constant; series field shunted by* 0·7 *ohm; constant of tachometer,* 2·055.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 500 Volts.	Speed in Miles per Hour on 33 in. Wheels with Gear Ratio of 4·93.
570	193·4	243	499	489	9·7
510	177·2	247	508	498	9·9
511	156·5	256	526	515	10·3
512	107·0	280	575	562	11·2
512	92·7	290	596	583	11·7
510	82·2	298	613	600	12·0
512	72·2	305	627	612	12·2
513	34·8	335	689	671	13·4

Notch 9 on Johnson-Lundell Controller.—Commutators in parallel; terminal voltage, 500 *volts; shunt field,* 1·9 *amperes constant; series field shunted with* 0·45 *ohm; constant of tachometer,* 2·055.

Volts at Terminals.	Amperes Input per Equipment.	Tachometer Reading.	Revolutions per Minute of Armature.	Revolutions per Minute Reduced to 500 Volts.	Speed in Miles per Hour on 33 in. Wheels with Gear Ratio of 4·93.
483·5	200·0	259	532	551	11·0
485	187·6	267	549	566	11·3
486·5	175·7	275	565	581	11·6
487·5	149·9	290	596	611	12·2
488·5	121·1	312	641	656	13·1
489·5	107·0	326	670	684	13·7
489·5	87·6	345	709	724	14·5
489·5	70·7	366	752	768	15·4
489·5	36·6	434	891	910	18·2

The test results of Table XXXIV. are indicated by small circles on the sheets of estimated curves in Fig. 251, and Figs. 253 to 257, Plates 13 to 15.

The following is a record of the heat tests of these motors.

A measurement of the resistances when cold gave the results in Table XXXV.

TABLE XXXV.—MEASURED RESISTANCES.

Armature:—

	Degs. Cent.	Ohms.
Opposite pinion end, resistance at	15	0·349
Pinion end, resistance at	14	0·348
" "	15	0·354
" "	14	0·353
Shunt field, resistance at	15	36·25
" "	14	36·11
Shunt field current—1·9 amperes constant		
Series field, resistance at	15	0·550
	14	0·548[1]
Shunted with 0·45 ohm nickelin resistance.		
The combined resistance measures	at 15	0·247
" " "	" 14	0·246

Two heat tests were made, the first at a load corresponding to a mean input of 65·2 amperes (35 horse-power) per motor, and with a room temperature of 15° Cent.; the results of which are set forth in Table XXXVI., and the second at a load corresponding to a mean input of 45·7 amperes (23 horse-power) per motor, and with a room temperature of 14° Cent.; giving the results set forth on page 222. For both tests the connections were equivalent to those given in Fig. 248, Plate 12, for the ninth controller point.

[1] For the increase of resistance of the nickelin shunt no corrections are made.

TABLE XXXVI.—TEMPERATURE TESTS ON A 35 HORSE-POWER JOHNSON-LUNDELL TRAMWAY MOTOR (TYPE "U") WITH AN AVERAGE LOAD OF 65·2 AMPERES (35 HORSE-POWER).

Time o'clock.	Time from Start (Mins.).	Amperes, Commutator Pinion End.	Amperes, Commutator Opposite Pinion End.	Amperes, Total Current.	Drop in Voltage across Shunted Series Field.	Resistance of Shunted Series Field.	Resistance of Series Field Alone.	Increase in Per Cent.	Temperature Rise in Degs. Cent.
9·55	...	30	34	64·0	...	0·247	0·549	...	...
10·03½	8½	30·8	33·65	64·45	16·4	0·254	0·583	6·2	15·5
10·07	12	31·2	34·9	66·1	16·9	0·256	0·594	8·2	20·5
10·10	15	30·7	34·3	65·0	...	...	...	...	...
10·12	17	30·5	33·7	64·2	16·6	0·259	0·610	11·1	27·8
10·16	21	31·3	33·8	65·1	16·8	0·259	0·610	11·1	27·8
10·17½	22½	31·4	34·1	65·5	17·0	0·260	0·616	12·2	30·5
10·22	27	31·8	33·8	65·6	17·15	0·2615	0·624	13·7	34·3
10·29	34	32·0	33·5	65·5	17·0	0·262	0·627	14·2	35·5
10·37	42	31·5	33·5	65·0	17·1	0·263	0·633	15·3	38·3
10·47	52	31·8	33·8	65·6	17·4	0·2655	0·646	17·7	44·3
10·52	57	32·0	34·0	66·0	17·6	0·267	0·656	18·9	47·3
10·55	60	...	...	...	...	...	...	...	...

TEMPERATURE BY THERMOMETER.

	Measured.	Temperature Rise. Degs. Cent.
Armature	65	50
Commutator opposite pinion end	71	56
Commutator at pinion end	83	68
Armature bearing opposite pinion end ...	50	35
Armature bearing at pinion end	54	39
Axle bearing (both sides)	30	15

HOT RESISTANCE MEASUREMENTS MADE IMMEDIATELY AFTER THE RUN.

Winding.	Volts.	Amperes.	Resistance.	Increase per Cent.	Temperature Rise in Degs. Cent.
Shunt winding...	77·7	1·9	40·9	12·8	32·0
Armature opposite pinion end	15·40	30·0	...	...	...
	15·15	30·5	...	...	...
	15·60	30·75	...	...	...
	13·55	30·20	...	...	...
	14·46	29·75	...	...	...
	13·55	30·75	...	...	...
Mean values	14·12	30·32	0·466	33·6	84·0
Armature at pinion end ...	13·52	30·75	...	...	...
	13·50	30·25	...	...	...
	13·02	30·00	...	...	...
	13·18	30·50	...	...	...
Mean values	13·305	30·375	0·439	24·0	60·0
Mean resistance, increase and temperature rise	...	...	...	28·8	72·0

TABLE XXXVII.—TEMPERATURE TESTS ON A 35 HORSE-POWER JOHNSON-LUNDELL TRAMWAY MOTOR (TYPE "U") WITH AN AVERAGE LOAD OF 45·7 AMPERES (23 HORSE-POWER).

Time o'clock.	Time from Start.	Amperes, Commutator Pinion End.	Amperes, Commutator Opposite Pinion End.	Amperes, Total Current.	Drop in Voltage across Shunted Series Field.	Resistance of Shunted Series Field.	Resistance of Series Field Alone.	Increase in per Cent.	Temperature Rise in Degs. Cent.
8·40	...	...	...	...	...	0·246	0·548	...	...
8·45	5	21·6	23·1	44·7	11·2	0·251	0·568	3·65	9·2
9·25	45	20·3	25·3	45·6	11·78	0·258	0·605	10·4	26·0
10·40	120	20·4	26·5	46·9	12·65	0·270	0·675	23·2	58·0

TEMPERATURE BY THERMOMETER.

	Measured.	Temperature Rise in Degs. Cent.
Armature...	69	55
Commutator opposite pinion end	77	63
Commutator at pinion end	81	67
Field coils	43	29

HOT RESISTANCE MEASUREMENTS MADE IMMEDIATELY AFTER THE RUN.

Winding.	Volts.	Amperes.	Resistance.	Increase per Cent.	Temperature Rise.
Shunt winding...	80	1·91	41·95	15·4	38·5
Armature opposite pinion end	13·35	28·4	...	...	...
	12·92	28·5	...	...	...
	13·2	28·1	...	...	...
	13·2	28·0	...	...	...
	13·35	28·7	...	...	...
Mean values ...	13·20	28·34	0·466	33·9	84·8
Armature at pinion end ...	12·66	27·94	...	...	...
	12·85	28·05	...	...	...
	12·52	28·32	...	...	...
	13·12	28·50	...	...	...
Mean values ...	12·79	28·20	0·454	28·6	72·5
Mean resistance, increase and temperature rise for both armature windings ...	...	...	...	31·25	78·6

§ 5. **Speed Control by Commutating Potential Regulators.**—The principal remaining methods of varying the speed of continuous current motors consist in the employment of a commutating potential regulator; this is generally a motor-driven dynamo with armature interposed in the circuit supplying the motor, the speed of which is to be varied. If the potential

regulator is driven at a constant speed, the voltage induced in its armature will be dependent upon its field strength, which is arranged to be regulated from maximum in one direction through zero to maximum in the other direction. Thus, if its range of voltage is from $+300$ volts to -300 volts, and it is interpolated in a 600-volt circuit, the potential at the armature of the driven motor may be varied from $600+300=900$ volts down to $600-300=300$ volts, and hence a wide variation of speed may be obtained with constant excitation of the driven motor, as this latter may have its field connected across the constant

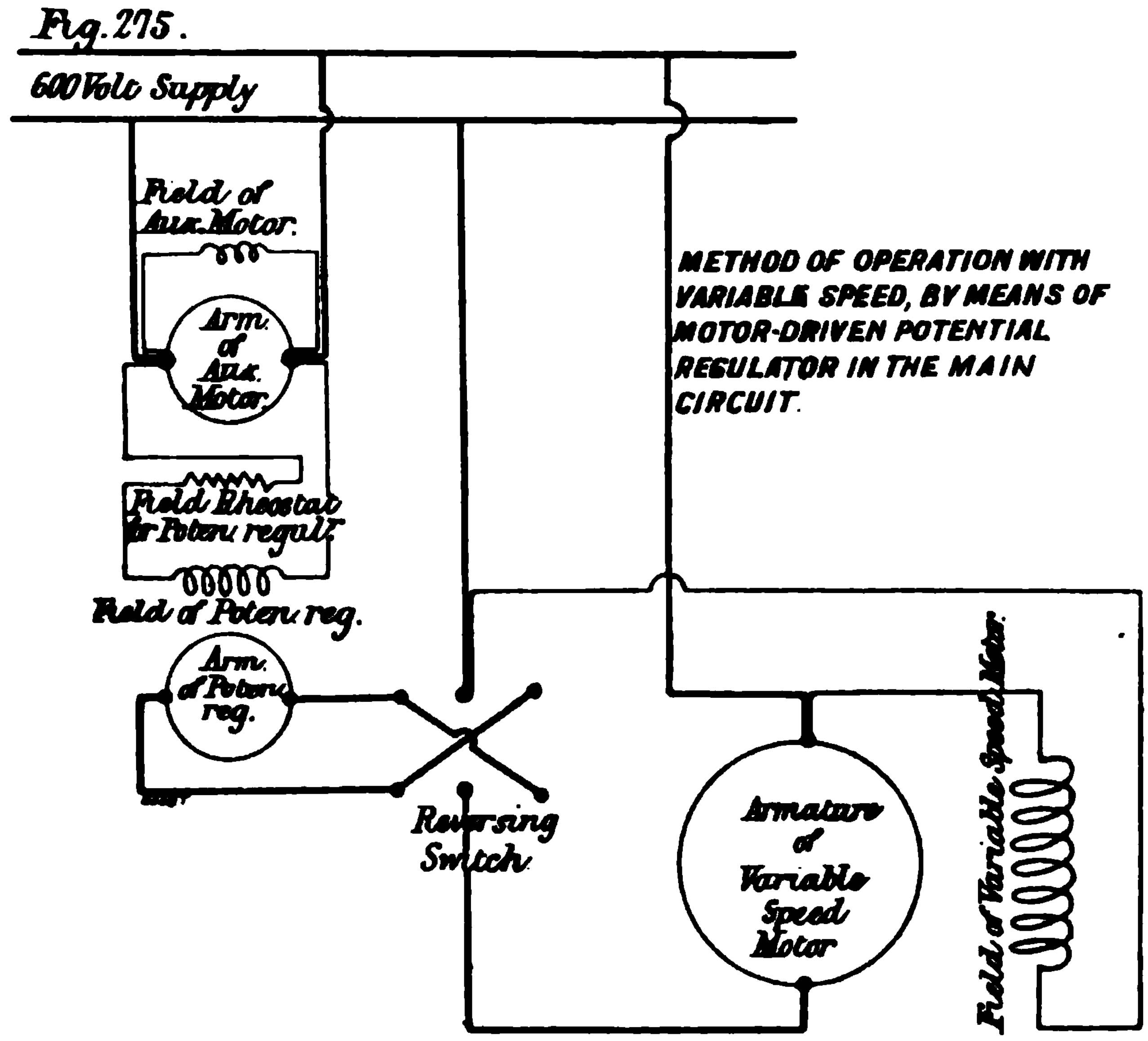


FIG. 275.

600-volt supply. This arrangement is shown diagrammatically in Fig. 275. The method is capable of numerous modifications; the speed of the motor driving the potential regulator may be varied by shunt regulation, or otherwise; a variation in the shunt field of the driven motor may be superposed upon the variation in the terminal voltage at its armature; the different machines may have double commutators and series parallel control, or they may be compound wound. By combinations of these and other principles, the continuous current motor may be arranged to operate with great economy over a wide range of speeds and loads. The application of electric driving to iron

and steel works, to the operation of large high-speed hoists in mining work, and even to the lighter requirements of machine tool driving, has led to the inception of many ingenious systems of motor control on these lines, and the choice amongst them is generally a question of balancing up the factors of first cost, in economy operation, and simplicity and reliability of control.

§ 6. **Forty-five Horse-power, 550-volt, 4-pole Geared Railway Motor.**—The drawings in Figs. 276[1] and 277, Plate 16, and the photographs of Figs. 278 and 279, Plate 17, have been supplied to the writer by Mr A. V. Clayton, and relate to a motor designed by him, for the Elektriska Aktiebolaget Magnet, of Ludvika, Sweden. These motors were in the first case specially designed for use on an already existing narrow-gauge (0·891 metre) road, and this, together with the fact that they had to be used on the old bogies, left the space at disposal very limited. The over-all axial length of the motor is therefore but 800 millimetres.

One of the chief points of novelty about the motor is the use of triple silk insulated conductors for the armature winding, it being the designer's opinion that the usual cotton covering is not sufficiently durable under the high temperatures attained in the armature winding of railway motors.[2]

In railway motors an especially heavy charge to maintenance expenses is caused by brush and commutator wear, and the frequent renewals thereby occasioned, and with the object of reducing this expenditure, especial attention has been devoted to the commutating qualities of the design (the number of segments, 165, being much more liberal than is generally the case in 4-pole tramway motors), and to the selection of a suitable grade of carbon for the brushes, with a view to the reduction of wear.

[1] The mechanical design of this motor is due to Mr Sylvander.

[2] Prior to the construction of these motors, some interesting tests as to the insulating qualities of the silk-covered wire were made, and it was found that the breakdown pressure for two conductors, laid beside each other, and either tightly screwed together, as with wires in jointing, or clamped together in a wood vice, was 1300 volts to 1500 volts alternating, with no insulating varnish. (The writer would not be of opinion that the use of silk-covered wire was justified in this case, for substantially these same insulation values are obtained with an equal thickness of cotton covering. Silk covering is more suitable for exceedingly fine wire where a higher "space factor" is obtained by its use.)

Fig. 276.

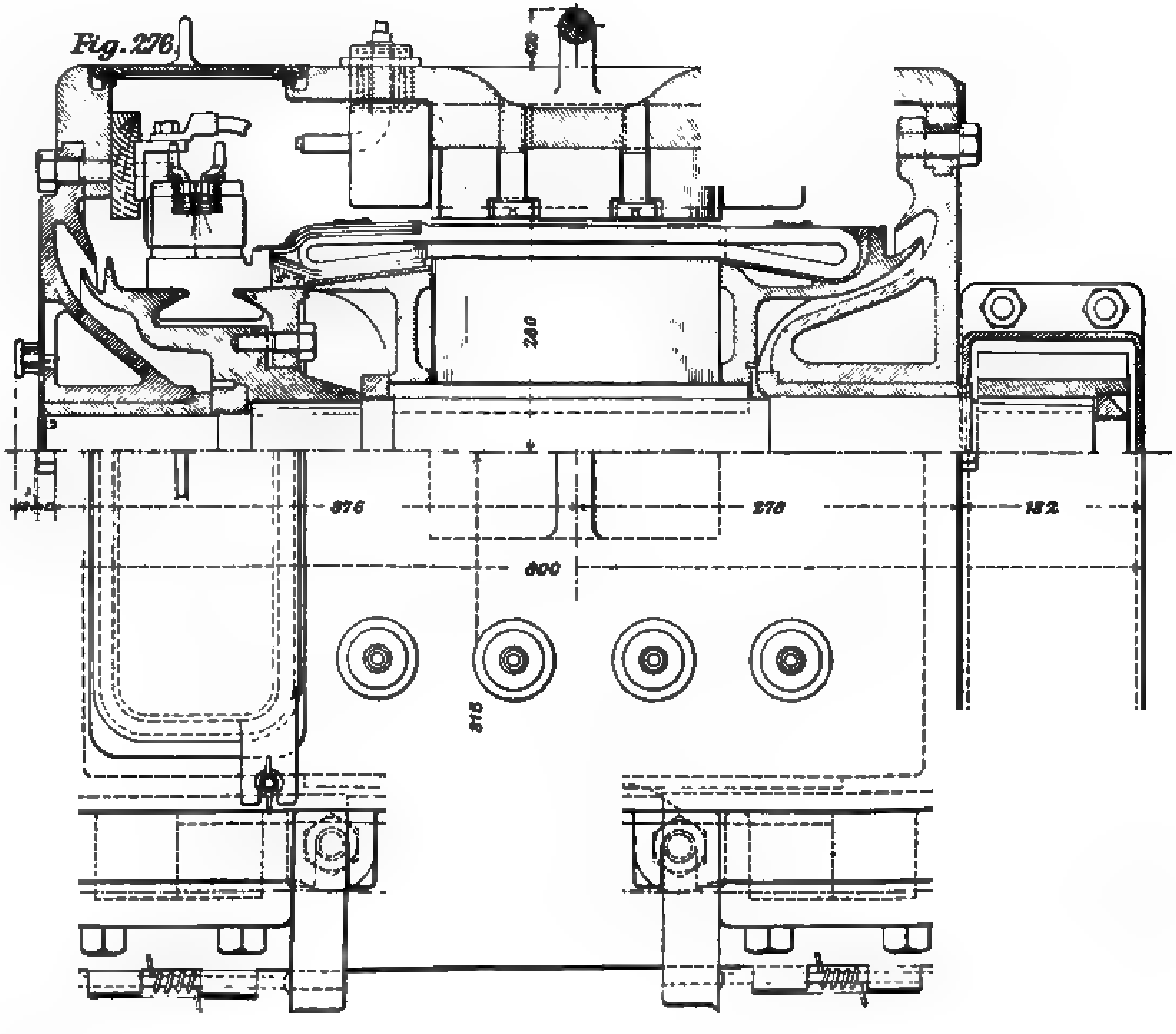
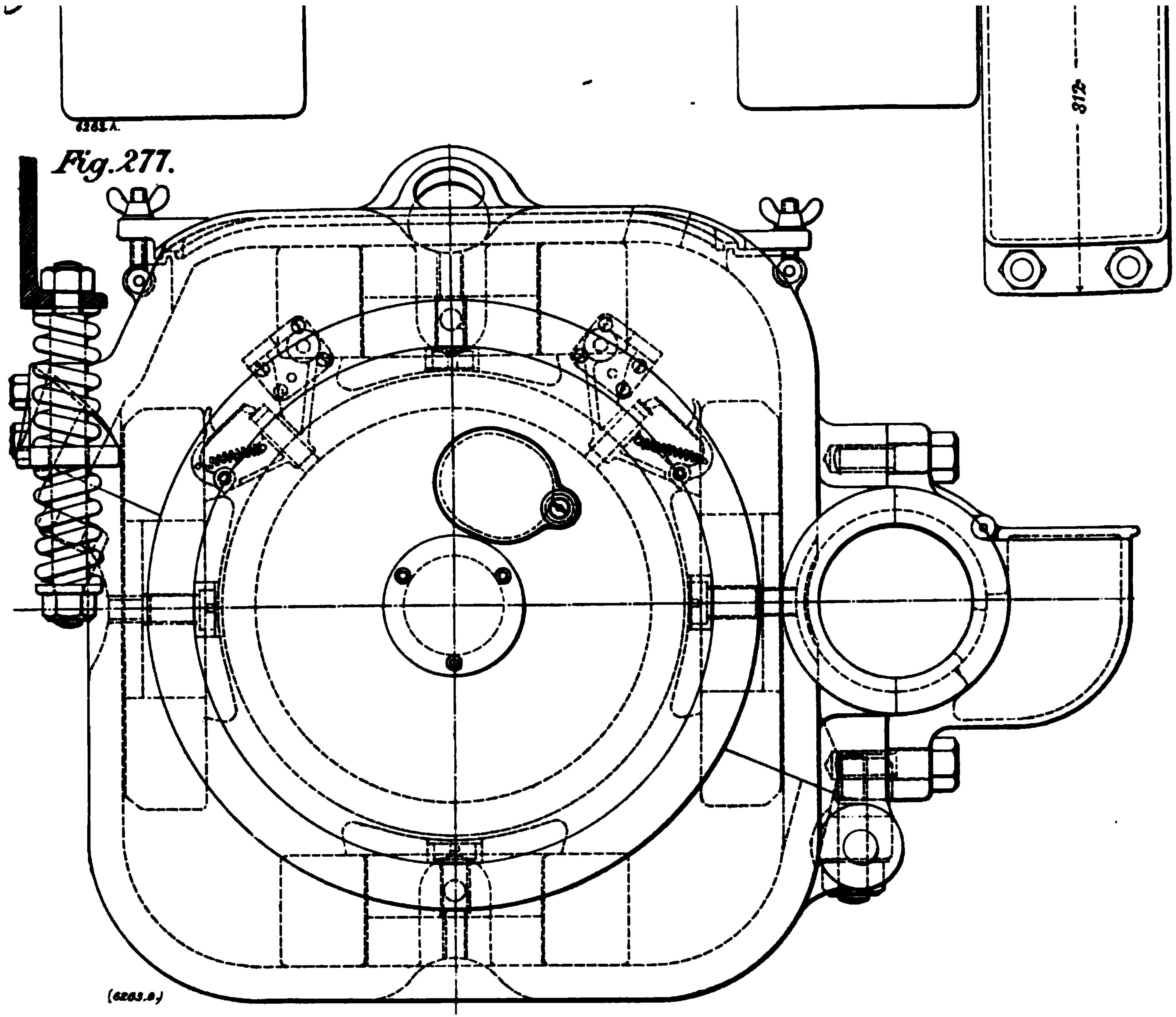


Fig. 277.



FIGS. 276 and 277.—45 Horse-power, 550-Volt, 4-Pole Geared Railway Motor of the Elektriska Aktiebolaget Magnet (see page 224).

FIGS. 278 and 279.—45 Horse-power, 550-volt, 4-pole Geared Railway M
of the Elektriska Aktiebolaget Magnet (see page 224).

TABLE XXXVIII.—DIMENSIONS AND DATA OF 45 HORSE-POWER RAILWAY MOTOR.

Armature speed, revolutions per minute ...	750
Ratio of gear reduction	4 to 1
Diameter of car wheels	800 millimetres
Speed of car, kilometres per hour	28·2
Horse-power output	45
Efficiency at full load, exclusive of gearing	88 per cent.

(The actual efficiency, exclusive of gearing, was 89½ per cent., but 88 per cent. was taken as a basis for the calculations. The gearing friction in tramway motors generally ranges from 5 per cent. to 7 per cent. of the full load rating.)

Amperes input at 550 volts	68
Armature diameter (outer)	330 millimetres
Armature diameter (inner), punchings assembled direct on axle	75
Number of slots	55
Effective conductors per slot	12
Style of winding	2-circuit single
Turns in series	165
Flux (510 volts internal)	3·1 megalines
Armature ampere-turns per pole	2800
Size of conductor, bare	2 mm. diameter, two in parallel
Size of conductor, insulated with triple silk covering	2·25 millimetres
Density in conductor, amperes per square centimetre	540
Measured resistance of winding at 20° Cent.	0·223 ohm
Size of slot	24·5 mm. by 10·1 millimetres
Width of tooth at face	8·75 millimetres
Width of tooth at root	6·0 „
Width of tooth, mean	7·35 „
Ratio width of tooth to slot	0·73
Length of pole arc	168 „
Gross length armature laminations	210 millimetres (no vent ducts)
Net length armature laminations	190
Average density in teeth	22,000
Density pole face	8,800
Density magnet core (part cast steel and part laminated)	16,000
Density magnet frame (cast steel)	16,000
Density armature core	8,100

CALCULATION OF MAGNETIC CIRCUIT.

	Length.	Density (Kilolines).	Amperes Turns.
Armature core	8·5 cm.	8·1	40
Armature teeth	2·45 cm.	20·5	1300
Air gap average	3·5 mm.	8·8	2500
Magnet core	8 cm.	16·0	500
Magnet yoke	22 cm.	16·0	1000
Sum	...	...	5340
Allowance[1]	...	...	1600
Total	...	...	6940

Spool:—

All 4 poles carry windings, two spools being small and two large.

Side spools winding space	50 mm. by 75
Size conductor bare	5·4 mm. by 5·4
Size conductor insulated with doubled cotton covering	6·0 mm. by 6·0
Turns in series	84
Top and bottom spools winding space ...	66 mm. by 75
Size conductor bare	5·4 mm. by 5·4
Size conductor, insulated with double cotton covering	6·0 mm. by 6·0
Turns in series	120
Mean turns per pole $\frac{120+84}{2}=$	102

Ampere turns per pole, full load, $68 \times 102 = 6940$

Density in conductor	2·3 amperes per millimetre
Resistance of whole field winding at 20° Cent.	0·23 ohm

Commutator:—

Diameter	280 millimetres
Number of segments	165
Effective length of segments	70 millimetres
Number of brush studs	2
Brushes per stud	2
Brush dimension	13 by 32 mm.
Density brushes, amperes per square mm. ...	$\frac{68}{2\,(1{\cdot}3 \times 3{\cdot}2)} = 8{\cdot}1$
Brush quality	medium hard carbon

[1] One of the motors was first tested with temporary field spools to ascertain the correct value for field excitation. The large discrepancy between the calculated and the actual values is probably due to the steel castings, which, owing to their being so thin, have been cast of a steel containing a high percentage of silicon, to ensure freedom from blow-holes. It is also probable that in such a confined space, and with such high densities, the leakage factor is much more than 1·28, the value used in the calculations.

Tests (Commutation).—The machine has been tested up to double normal load in both directions of rotation, and runs quite sparklessly. Different qualities of carbons have been tested, from the very hardest retort carbon to the softest vegetable carbons and also brushes of pure graphite, and, while all run sparklessly up to the overload mentioned, the softest sorts of carbon get very hot after even a few minutes' run. A fairly hard grade of retort carbon appears to be most satisfactory for tramway motors.

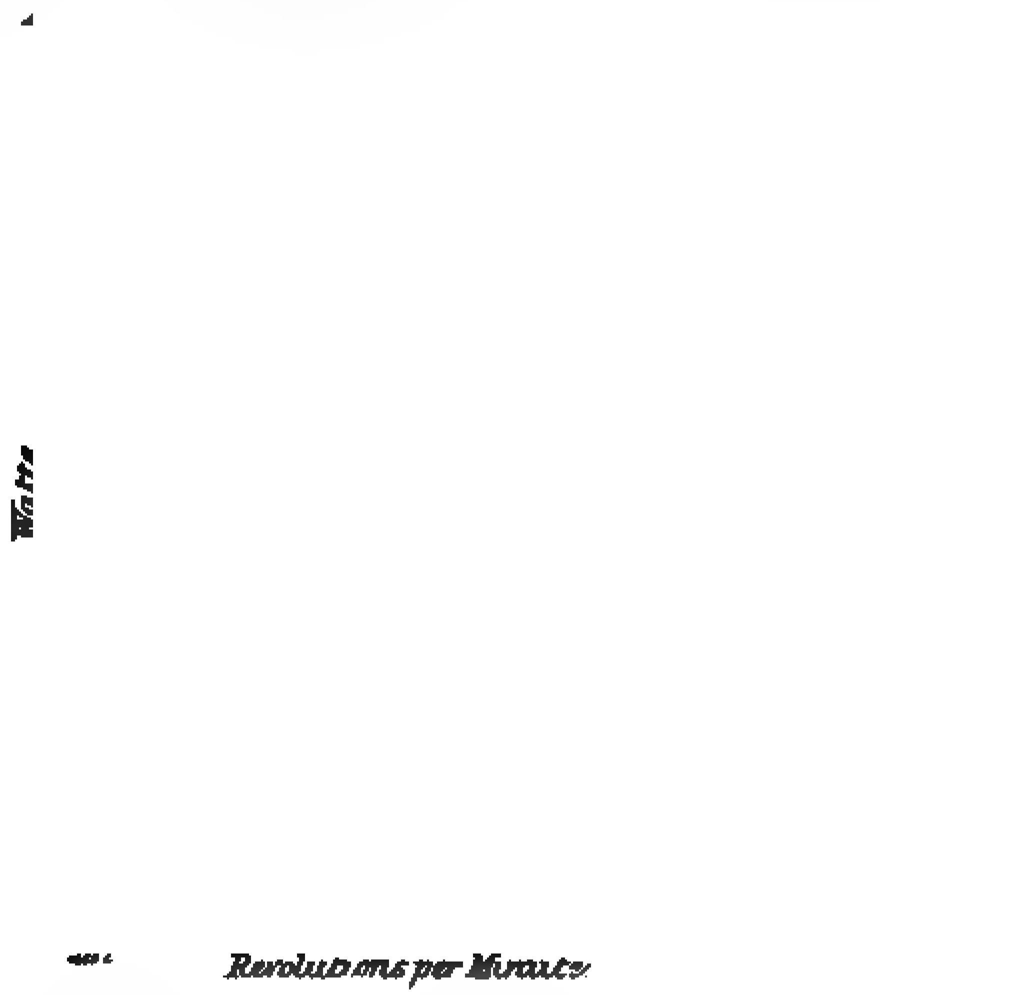


FIG. 280.

Temperature Rise:—

BY THERMOMETER.

	After 1 Hour Full Load. Degs. Cent.	After 1½ Hours Full Load. Degs. Cent.
Armature	56	78
Field spool	46	65
Commutator	53	75

BY RESISTANCE.

	After 40 Minutes Full Load.	After 1 Hour Full Load.
Armature winding	41·5	63

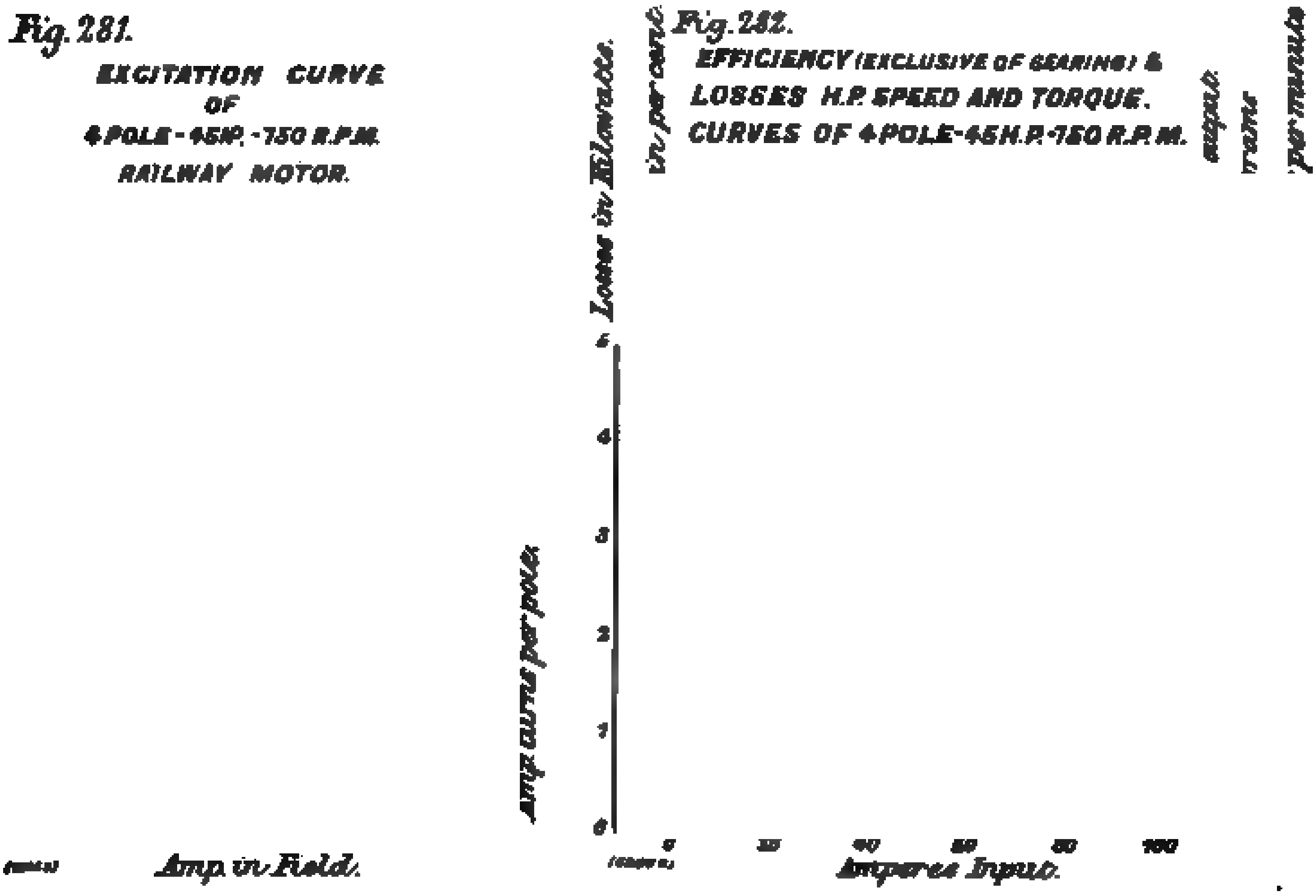
Curves of iron and friction losses, excitation, torque efficiency, etc., are given in Figs. 280, 281, and 282.

Mr Clayton's ingenious arrangement of the curves in Fig. 280 is worth especially noting. Had the gear losses been included in his curve of "characteristic losses," the results would have been still more interesting.

Efficiency :—

At full load, with motor hot, at end of one hour's full load run.

	Watts.
Iron and friction losses (measured)	1,130
Armature $I^2R = 68^2 \times 0{\cdot}28$	1,300
Field $I^2R = 68^2 \times 0{\cdot}292$	1,350
Commutator I^2R (calculated)	185
Total losses	3,965
Output 45 horse-power...	33,100
	37,065



Figs. 281 and 282.

Commercial efficiency 89·4 per cent., exclusive of gearing. No measurements could be made of loss in gearing.

Weights :—

	Kilogrammes.
Of motor complete with pinion, oil boxes, car axle bearings, etc.	864
Gear wheel	67
Gear case	43

Part II.—ALTERNATING CURRENT MOTORS

CHAPTER XII

DESIRABILITY OF USING POLYPHASE CURRENTS

§ 1. **Introductory.**—In the first article of Chapter I. on continuous current motors, the relative advantages of continuous current motors and induction motors were briefly mentioned, and the opinion expressed that the merits of the latter had in general been overrated, and that the continuous current motor would not only hold its own, but would gradually come to be recognised as, on the whole, better adapted for many classes of work, and for many conditions where it is now considered good practice to employ induction motors. These opinions are at present not generally held, and the wide interest now taken in the numerous large power distribution plants at present being constructed in England will inevitably be accompanied by a temporary increase in the use of induction motors in the districts supplied by these plants. From the present outlook there will be many very extensive areas where alternating current alone will be available. For these large power distribution plants it is, of course, quite correct to generate the electric power in the first instance in the polyphase form, since this permits of economical distribution at high tension over great distances; moreover, the polyphase generator may, under most conditions, be designed for a high output per unit of material employed in its construction, and, through the absence of a commutator, and the use of an external stationary armature, the I^2R and friction losses at the brushes are avoided, and the machine requires somewhat less attention. For very large generators these are distinct advantages. Polyphase generators may be said to have a greater advantage over continuous current generators, the higher the normal speed and the

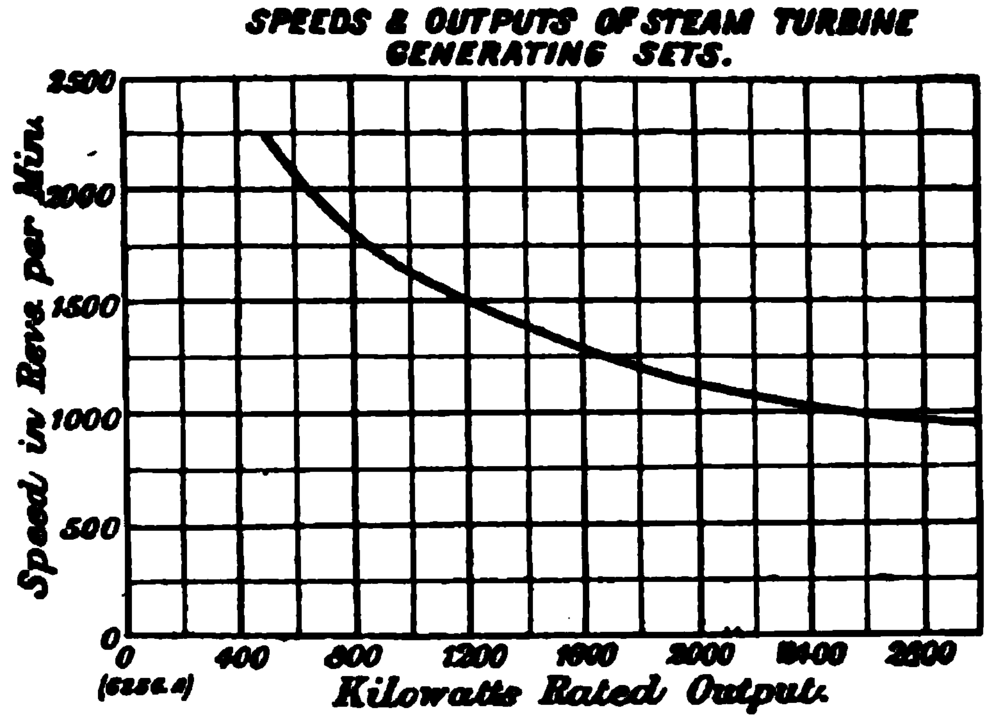
higher the rated output of the machines compared. For a high-speed continuous current generator requires as much commutator surface, for a given capacity, as a low-speed generator, and hence requires practically the same amount of active material in the commutator. For mechanical strength, the high-speed commutator requires even more material. Hence it is only in the iron of the magnetic circuit and in the copper of the windings that saving is effected in continuous current generators by an increase in rated speed. In addition to these considerations, departures from the otherwise most economical proportions have to be made in continuous current generators, out of regard for the requirements for good commutation, and these departures have to be greater the higher the rated speed, output, and voltage. Indeed, at the speeds required for direct connection to steam turbines, the design for a large continuous current generator, when not altogether out of the question in a single unit, becomes very abnormal, expensive, and generally unsatisfactory. But polyphase generators of fairly good design are practical even at steam turbine speeds.

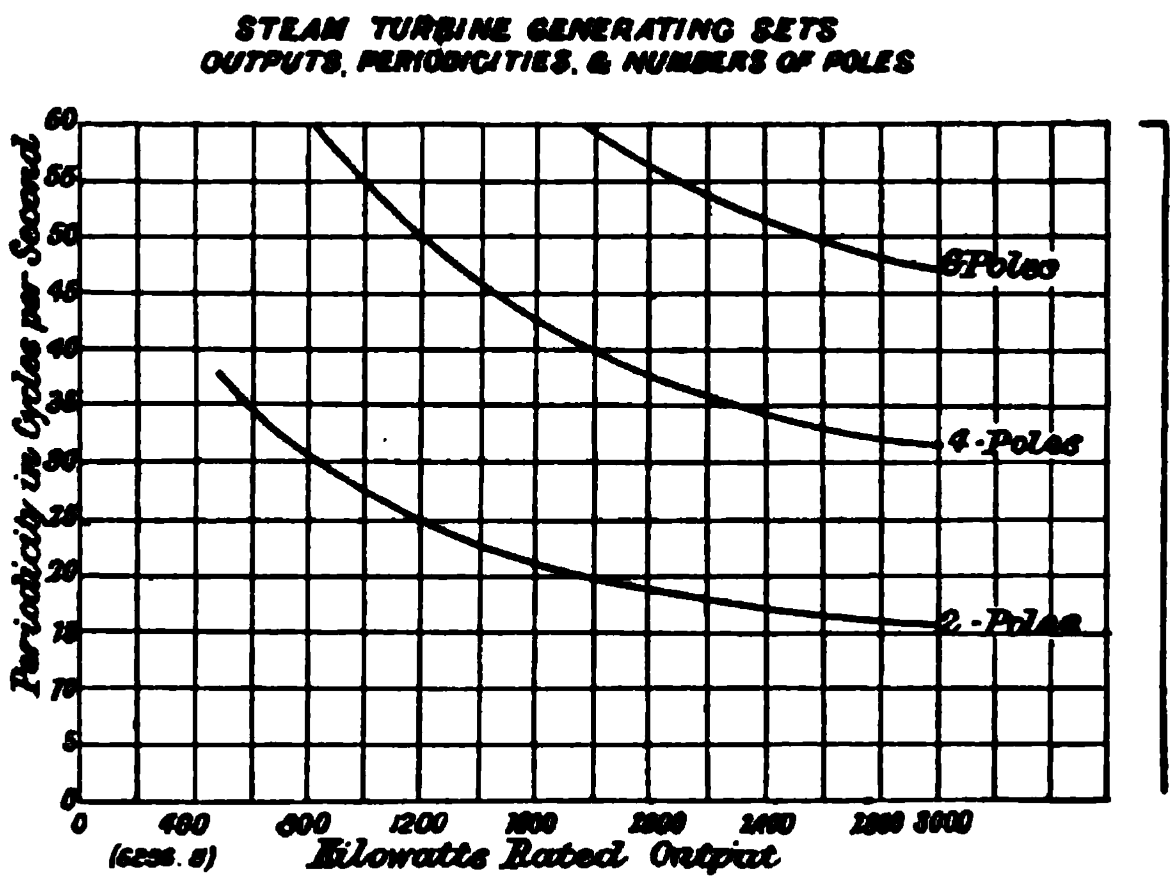
Granting the desirability of employing polyphase generators in these cases (large consumers still have the alternative of installing a motor generator, and operating continuous current motors from its secondary), and that this will lead, whether advisedly or otherwise, to a rather general employment of polyphase motors on the consumer's circuits, it becomes desirable to investigate the conditions leading to the most satisfactory design of polyphase motors. These will generally be found to consist in the adoption of low frequencies and of high speeds for the motors. Up to the speeds corresponding to direct connection to high-speed engines, the design and the subsequent operation of the polyphase *generator* are also more satisfactory the lower the periodicity, especially in large units.

§ 2. **Periodicity of Steam Turbine Alternators.**—The same cannot be said without reservation for polyphase generators for direct connection to steam turbines, where even fairly high periodicities lead to 6 or 4-pole designs for the generators; and the 2-pole polyphase generators, which become necessary at low periodicities, while by no means out of the question, are much less satisfactory designs.

Thus for 25 cycles per second, 750 revolutions per minute would have to be the corresponding speed for a 4-pole design, and even the largest turbines are generally rated at rather higher speeds.

Although turbine generating sets, as built on the lines of the Parsons type by various manufacturers, are, for a given output, to be found in operation at widely different rated speeds, the curve of Fig. 283 may be taken as fairly representative of the average practice with this, the type of large turbine at present in most extensive use. (Nor do the speeds of the recently-developed





Figs. 283 and 284.

turbines of other types fall sufficiently below the curve in Fig. 283 to modify the general conclusions here set forth. The speeds of the Curtis and Rateau types of turbine are compared with the speeds of the Parsons type of turbine in an article by the writer in the *Light Railway and Tramway Journal* for June 5th, 1903, page 377.) In Fig. 284 are given three curves of the periodicities corresponding respectively to 2, 4, and 6 poles at the speeds of the curve of Fig. 283.

When, for a given rated output, the speed most favourable to the turbine design falls, as will frequently be the case, a good deal away from that corresponding to either 2 or 4 poles, it will, from almost all standpoints except that relating to the advantages of low frequency, tend to a preference for a 4-pole design; and for a 6-pole design when the choice lies between 4 and 6 poles, and consequently a tendency toward higher frequencies. The many economies incidental to the use of steam turbine driven sets for a generating plant will thus stand considerably in the way of the introduction of lower frequencies, and even with the gradual lowering of turbine speeds for large units of the Parsons and other types, will make it very probable that frequencies lower than 25 cycles per second will rarely be employed. There is the further circumstance that 25 cycles per second is about the lowest periodicity as yet consistent with satisfactory lighting by vacuum incandescent lamps. A *precise* statement of the minimum periodicity for satisfactory operation of incandescent lamps is impracticable, since lamps of large candle-power and low voltage will give satisfaction at a periodicity much lower than would be permissible with low candle-power or high voltage lamps. The wave-shape supplied is also not without effect. It may therefore be concluded that 25 cycles is the lowest likely to be adopted as standard, and this only in the case of large central generating plants for systems designed mainly for power distribution. An additional reason for not employing very low frequencies at the low-tension distribution network in the case of lighting systems lies in the very greatly decreased life of the Nernst lamp when operated on circuits of low frequency. Mr A. J. Wurts has conducted comparative life tests on the Nernst lamp, which have shown as average results a life of 1200 hours at 133 cycles, 800 hours at 60 cycles, and 400 hours at 25 cycles. Owing to its high first cost, the life is a more important factor in the Nernst than in the vacuum incandescent lamp. The use of 133 cycles at the generator is generally altogether out of the question for many reasons, but there remains the very practical alternative of generating at low periodicity and supplying the low-tension lighting networks from the secondaries of motor generator sets.

It is, however, at last beginning to be generally recognised that low frequency is distinctly preferable, and there is rarely sufficient reason for compromising on intermediate values such as 40 or 30 cycles per second.

§ 3. **Increase in Size of Transformers.**—Of course the increased cost of stationary transformers with lowered periodicity has always been a leading factor in preventing the adoption of low frequencies, for before the great increase in size of individual units in transformers, and the introduction of artificial cooling, the cost of transformers represented a far greater percentage of the total cost of plant than it does now.

In 1890, transformers had rarely been built of a capacity of over 30 kilowatts each, and there was in operation probably no single stationary transformer of over 75 kilowatts capacity. In 1894, capacities of 300 kilowatts each were beginning to be used. In 1898, single transformers of 850 kilowatts capacity were in operation at Niagara Falls. In 1902, several single transformers of 2750 kilowatts rated capacity, and weighing about 11 tons each, had been installed; and these large artificially ventilated transformers are so much cheaper per kilowatt, and of so high efficiency (the efficiency above half load lying above 98 per cent.), as to have quite altered the circumstances, and to have rendered the item of increased cost of transformers for low periodicities of considerably less importance. A still further very substantial decrease in cost and increase in economy in transformers is resulting from the more general employment of one large three-phase transformer in the place of three single-phase transformers. This was the practice on the Continent from the earliest days of polyphase working, but is only just beginning to be generally customary in England and America.

§ 4. **Permissible Speed Variation in Alternators Designed to Run in Parallel.**—The higher the speed, and the lower the periodicity, the more satisfactory becomes the parallel running of polyphase alternators, and a less expenditure need be entered into for fly-wheels and auxiliary governing apparatus. This is for the reason that in parallel running the limiting factor depends upon the instantaneous relation to one another of the rotors of the different machines, which should never differ in their relative displacement by more than a certain percentage (say 1 per cent.), not *of the circumference*, but of that *portion* of the circumference occupied by one pair of poles, which corresponds to 360 magnetic degrees, or 1 cycle. Thus if a 60-cycle generator runs at 120 revolutions per minute, it must have 60 poles (30 *pairs* of poles), and its rotor should at no time during one revolution depart more than $\frac{1}{30}$th of 1 per cent. from perfectly uniform speed of rotation. Had the speed been 360 revolutions per minute (*i.e.* three times

as great), 20 poles only would have been required, and the same degree of electro-magnetic uniformity as before (1 per cent.) would have been secured, even when the departure from uniform speed of rotation was three times as great—that is, $\frac{1}{10}$th of 1 per cent.

There is now in many quarters a reaction toward allowing a considerably greater speed variation, as the expense of obtaining so close regulation by fly-wheels is now fully realised, and not much confidence is felt in the alternative plan of employing lighter fly-wheels and relying upon special governing devices. But for systems with sub-stations employing synchronous motors, and more especially those employing rotary converters, close speed regulation is imperative, but it is obtained most economically and satisfactorily by means of low frequencies and fairly high speeds for the generating sets.

Now if, in addition to this higher speed, the frequency had been, say, half as great—*i.e.*, 30 cycles per second, only 10 poles would have been required, and $\frac{1}{10}$th of 1 per cent. departure from uniform angular rotation would still have given 1 per cent. electro-magnetic regulation. It is obvious from the consideration just explained, that for direct connection to gas engines, low frequency is of still more vital importance, the more especially so in large slow-speed gas engines, and in such cases a lower frequency than 25 cycles per second would generally be very desirable, or even absolutely necessary, in order to secure satisfactory electro-magnetic uniformity in spite of the very uneven turning moments. It must be remembered that it is not alone the parallel running of the generators which we have to consider, but also the obtaining of satisfactory operation of synchronous apparatus at sub-stations. Suppose large slow-speed gas engines and direct connected high-tension generators for 5 or 10 cycles per second were adopted, this would in no wise detract from the efficiency or satisfactoriness of the system up to the sub-stations, at which points the voltage and periodicity most suitable for the distribution system would be obtained from the secondaries of motor generator sets. This example should sufficiently illustrate this very important advantage of high speed and low frequency so far as it relates to the generating units. It will subsequently be shown that high speed and low frequency are also very essential to obtaining good and cheap results in induction motor design. A very low-speed, high frequency induction motor is inevitably poor unless of a disproportionately expensive design, and often even in spite of being thus disproportionately expensive.

§ 5. **Relation between Good Design, High Speed, and Low Frequency in Alternating Motors.**—We have seen in Part I. that the designing of continuous current motors for high speeds is beset with difficulties, and especially for large machines of high voltage. We shall, with the induction motor, find precisely the opposite state of affairs—namely, that the lower the speeds the poorer is the design, and that these difficulties are the more accentuated the higher the frequency of the motor.

The three-phase, 150 brake horse-power, 36-pole, 68 revolutions per minute, 21-cycle, 350-volt mining pump motor described in the *Electrical Review* for June 26th, 1903 (page 1078), excellently illustrates this point. In spite of its great size (the rotor diameter is 2·92 metres, the air-gap 1·8 millimetres) and large output, the maximum efficiency is but 88 per cent., and the power factor at quarter, half, and full load respectively is but 0·60, 0·80, and 0·88. Running unloaded, the motor consumes 32 per cent. of its full load current; the no load power factor is 0·15. The motor is of good design, but, nevertheless, cannot compare, in cost or economy, with a good continuous current motor for the same rating. Had the periodicity been high, the result would have been correspondingly worse.

For large power distribution schemes it should not be at all impracticable, and would, in the writer's opinion, lead ultimately to the most economical and satisfactory results, to employ for the generating plant the periodicity most suitable according to whether the prime movers are steam turbines, water wheels, or gas engines. This periodicity will generally be as low as 25 cycles per second, and may sometimes and for large slow-speed gas engines preferably should be very much lower. Unless the distance over which it is required to transmit the power is such as to require more than, say, 12,000 volts, the generators should generally be wound for the line voltage. At the end of long transmission lines, unless an uneconomical amount of copper is employed in the cables, it is impracticable to attempt to secure any reasonably close inherent regulation, and the use of stationary step-down transformers would require to be supplemented by considerable auxiliary apparatus such as potential regulators, inductance gear for variable ratio of transformation, etc., and even then the regulation of the secondary system would be but poor—generally quite unsatisfactory for lighting. But with motor generator sets the regulation of the primary voltage is comparatively unimportant, as synchronous motors can generally be economically

designed to carry heavy overloads even at considerably reduced terminal voltage, and the speed of the primary (the motor) is held perfectly uniform by the constant periodicity of the generating station, and the secondary generator may be for continuous current or for polyphase current, and for any desired voltage and periodicity, and the regulation of the secondary circuit supplied from it, will be precisely as perfect as if supplied from an independent installation. In fact, the regulation will be better, since it will be the close regulation corresponding to a large central station engine instead of that customary with the small engines of private installations. This is of especial importance in the interests of securing satisfactory lighting. Where power is required, the decision as to whether the secondary generator at the sub-station should be a continuous current or a polyphase dynamo would depend upon the special requirements of the work. Thus, if much of the work were required to be done at varying speeds, continuous current motors would generally be used. When other considerations should not definitely determine the choice, one should by no means overlook the point already referred to—namely, that continuous current motors are far better suited for low speeds, and polyphase motors for high speeds. A clear realisation of the importance of this point, which unfortunately is seldom obtained except by the designers of the motors, would often show it to be worthy of receiving consideration in the arrangement of a distribution system. The high-speed, low frequency induction motor may economically be so designed as to be characterised by high power factor (even at considerably less than full load), low current when running light, high overload capacity, and greater economy of material and labour in construction. Here, however, there is also a limit, for 2-pole windings are distinctly inferior for induction motors, and hence for a given periodicity the speed corresponding to 4 poles is generally the highest desirable speed. For 25 cycles per second this would be 750 revolutions per minute, which, while a very moderate speed for small motors, is generally considered a rather high speed for motors of over 50 horse-power rated capacity. Where steam turbines are to be employed, the frequency chosen must correspond to either 2, 4, or 6-pole generators at the speeds the turbine manufacturers are prepared to furnish for the required capacities. This is important to keep in mind, since, the choice being between 2 and 4, and 4 and 6 —*i.e.* 100 per cent. and 50 per cent. respectively—the choice of periodicity is much more dependent upon the practicable speeds

than in the case of piston engines, where a pair of poles more or less is, for a given periodicity, associated with but a comparatively small percentage change in the required speed.

§ 6. **The Inferiority of the Single Phase Induction Motor.**—It no longer needs to be stated that, unless possibly for locomotive purposes, single-phase motors and generators are almost invariably very undesirable. Single-phase generators and motors are far less economical with respect to first cost and operation, the generators are much less satisfactory in matters of regulation and efficiency, the motors less satisfactory with respect to no-load current, power factor, overload capacity, and starting torque. The apparently hopeless inferiority of the single-phase *induction* motor is, indeed, now a matter of practical experience. None of the innumerable types which have been placed upon the market has come into really extensive use. The inferiority of the single-phase generator is strikingly illustrated in the annexed Table XXXIX., taken from page 150 of vol. xxxi. (1901) of the *Journal of the Proceedings of the Institution of Electrical Engineers* from Mr M. B. Field's paper on "The Relative Advantages of Three, Two, and Single-phase Systems for Feeding Low-tension Networks." The table gives the prices quoted by three different firms for three-phase and single-phase generators to comply with the following specification:—

Output	2500 kilowatts.
Voltage	6500 volts.
Efficiency, full load	96 per cent.
„ three-quarter load...	95 „
„ half load	93 „
Speed	75 revolutions.
Cycles	25 per second.

Fall of pressure between full load and no load, at constant speed and excitation, and power factor unity, to be not more than 7 per cent.

Generator to be supplied without outboard bearing or shaft, but with bedplate rheostat, etc.

TABLE XXXIX.—WEIGHT AND COST OF GENERATORS COMPARED.

	Three Phase.		Single Phase.	
	Weight (Tons).	Cost.	Weight (Tons).	Cost.
1	123	£6000	184	£8900
2	120	5400	140	6200
3	110	4600	125	5200

Quite independent of this data of Mr Field's, it may be said to be the practice of most experienced manufacturing companies to rate a generator of a given design and weight, when specially arranged for single-phase working, at about 70 per cent. of its rating as a polyphase generator.

§ 7. **Three-Phase System Preferable to Two-Phase.**—There remains to mention that a three-phase system is preferable to a two-phase system, not from any considerations relating to the generating units, which are practically of equal cost and weight for a specified performance, but on account of the transmission system, which for a given R.M.S. voltage between lines, and a given line loss, requires but 75 per cent. as much weight of copper as with a two-phase system, and has the further advantage of employing three wires of equal cross section instead of four, or instead of three, one of which may be of different cross section from the other two. A three-phase wound induction motor is decidedly superior to one for two phases, ultimately on account of the greater "breadth coefficient" of three-phase windings as compared with two-phase windings.

Thus, while the generators may be wound for three-phase or quarter-phase with equally good results, the transmission line should be for three phases, and three-phase induction motors should be employed on the receiving circuits.

§ 8. **Single-Phase Commutator Motors.**—With very low frequencies, however, good single-phase *commutator* motors may be designed which will have the speed-torque properties of the continuous-current series motor. In such motors the magnetic circuit must be laminated throughout, the reactance voltage per segment and the voltage induced by the primary alternating current must be maintained low. Means must be provided for minimising the sparking ensuing in consequence of secondary induced currents in the short-circuited turns at starting, and in general for reducing the inductance of the motor in order to obtain a high-power factor. Even the perfection and successful introduction of this type of motor will not mean a reversion to single-phase *generators*, as the generation and transmission will be more economically accomplished by polyphase energy, the motors being single-phase, just as incandescent lighting is provided from polyphase systems.

CHAPTER XIII

METHODS OF STARTING INDUCTION MOTORS

§ 1. **Forms of Induction Motors.**—In the modern induction motor the primary winding is almost invariably upon the external stator, and the secondary upon the internal rotor. Formerly the reverse arrangement was occasionally adopted, the internal rotor carrying the primary windings. The chief advantage of having the primary upon the rotor consisted in the reduced amount of iron subject to hysteresis and eddy current loss, for this loss is almost entirely confined to the iron of the primary member, the secondary iron only being subjected to the slow rate of reversal corresponding to the percentage "slip" of the motor, and as much less than half of the total length of the magnetic circuit is in the rotor, a motor so arranged will have lower "constant" losses, hence a higher efficiency at light loads. Recent investigations point, however, to the presence of a very considerable additional core loss in the secondary member, not dependent upon the "slip," but probably largely upon local high periodicity variations in the reluctance of the magnetic circuit as the relative positions of stator and rotor teeth change with reference to one another as the rotor revolves. See also *Elektrotechnische Zeitschrift* for March 7th, 1901, "Uebererhöhte Reibungs und Hysteresisverluste bei Drehstrommotoren," by J. Hissink. But the type required the use of collector rings in all cases, and as the primary winding is often for much higher voltage than the secondary, the type had the additional disadvantage of requiring further outlay for material and space, for insulating and securing this higher voltage winding on the rotor. Somewhat less space and much more reliable results can be obtained when the higher voltage windings are stationary. In the application of three-phase motors to railway work, the diameter is limited by the space available on the trucks, and it would at first appear of advantage to let the rotor carry the primary winding. This plan

was followed, and for the above reason, by the firm of Siemens & Halske in their first equipment for the Berlin-Zossen high-speed railway tests, as this equipment carried step-down transformers reducing the 10,000 volts at the trolleys to 1150 volts at the slip rings of the motors. But subsequently this firm supplied another equipment, in which the motors were wound directly for 10,000 volts, thus avoiding the necessity of having transformers. The primary windings of the motor were, of course, in this case, placed upon the stator. In the equipment which the Allgemeine Elektricitäts-Gesellschaft supplied for these Berlin-Zossen tests, the primary windings were placed on the stator, although the primary voltage was only 435 volts. In traction work, slip rings must be employed in any case, as starting resistances are necessary, hence this consideration does not enter into the choice of arrangement.

Fig. 285.

So nowadays the rotor generally carries the secondary winding, which sometimes consists in a so-called "squirrel cage" construction, which is merely a series of bars laid with but light insulation in the slots or holes, and soldered or otherwise secured to short circuiting rings at the ends. This is the ideal construction, but it has one great disadvantage—viz., that the squirrel cage motors have but very little starting torque, and nevertheless consume a large current at starting. One can only increase the "specific" torque (the torque per ampere) in the motor at starting, by increasing the resistance of the face conductors and end rings of the rotor, but this to a certain extent injures the good properties of the motor with respect to efficiency, overload capacity, and heating.

§ 2. **The Compensator.**—One may, however, largely relieve the line of the heavy starting current by employing a so-called compensator or "auto-transformer." In Fig. 285 is shown,

diagrammatically, a plain squirrel cage motor, arranged to be switched directly upon the line, and such a motor would, at the instant of switching on, take several times full load current from the line, and for a motor of any considerable size, the regulation of the system would be deleteriously affected. Such a construction is generally only used in comparatively small motors, but could often be employed to advantage, even with larger motors, when any considerable number of motors should be required, since they could be grouped together upon the secondary of a motor generator set devoted exclusively to them, and designed with this end in view—*i.e.* for reasonably good regulation on inductive load.

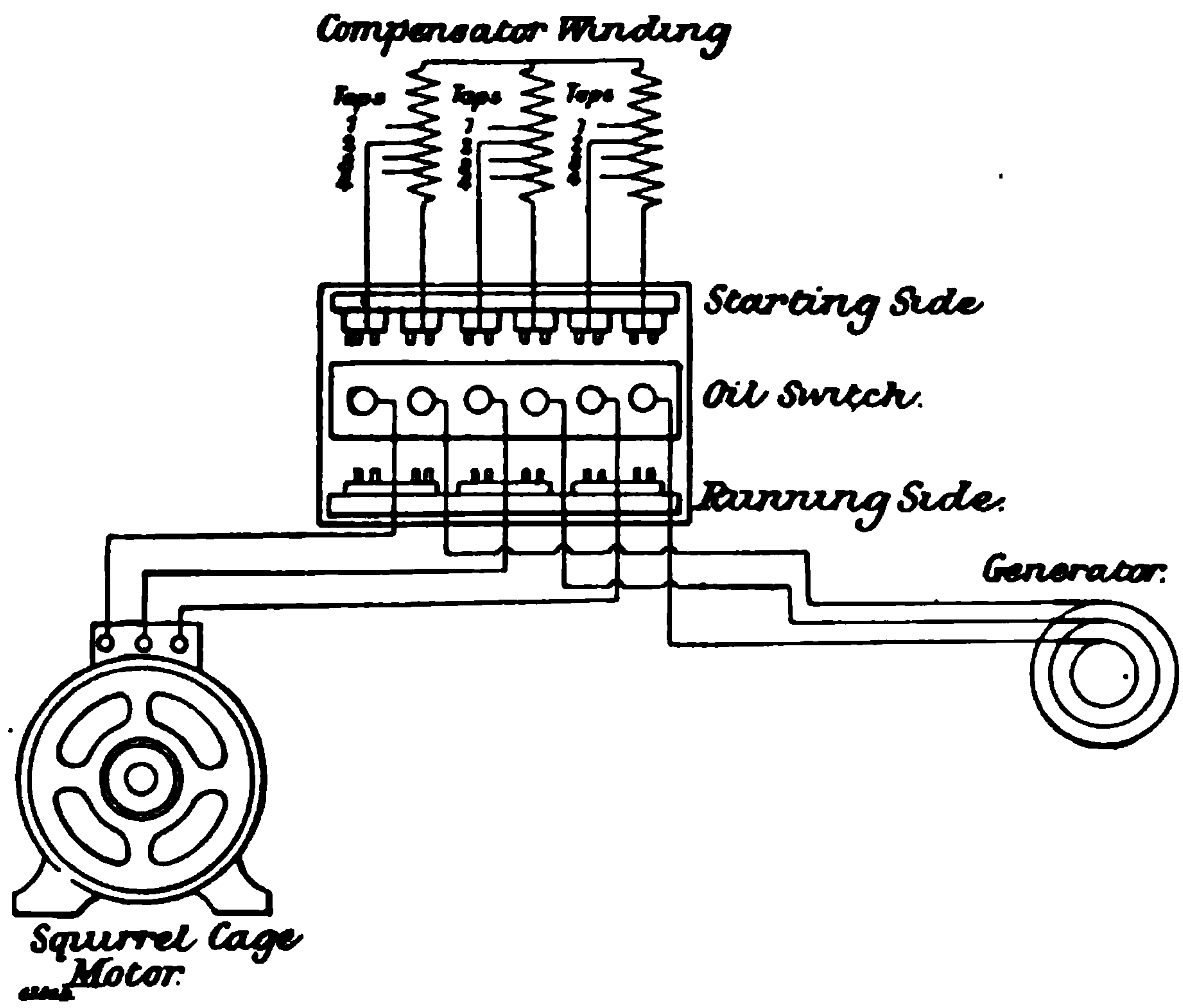


FIG. 286.

Fig. 286 gives a diagram of connections for employing a compensator for starting a squirrel cage motor. By this arrangement, provided the motor can start practically free, it will be fed from taps into the compensator at such points as to give only enough voltage to start it against the friction of its bearings. Suppose this requires but 33 per cent. of the terminal voltage, and that the motor then requires 2·0 times full load current. There will then, at starting, be drawn from the line but $0·33 \times 2·0 = 0·66$ times full load current. If, however, the motor would not start till tapped off at 50 per cent. of full load voltage, and then took 3·0 times full load current, it would, at starting, draw from the line $0·50 \times 3·0 = 1·5$ times full load current. The motor might be

required to start against still more torque, and to be fed from taps at 67 per cent. of the terminal voltage. It would then absorb, say, 4·0 times full load current, and the current from the line would be $0{\cdot}67 \times 4 = 2{\cdot}7$ times full load current. When thrown directly on the line without the intervention of a compensator, the motor used for the purposes of this explanation would take about six times full load current, and might give one and a half times full load torque, or even more. This illustrates the principle of the compensator method of connection. In practice, starting compensators are generally provided with a number of sets of taps, and that particular set is employed which corresponds to the lowest voltage, which, after the motor is installed, is found sufficient to start it under the required conditions.

For modern squirrel cage motors, the values given in Table XL. (from page 77 of Oudin's *Standard Polyphase Apparatus and Systems*, third edition, 1902. Sampson Low, Marston & Co., London) are representative of the current in motor and in line, torque of motor and percentages of full load voltage corresponding to the various sets of compensator taps.

TABLE XL.—VALUES FOR SQUIRREL CAGE MOTORS.

Voltage at Motor in Per Cent. of Line Voltage.	Line Starting Current in Per Cent. of Full Load Current.	Motor Starting Current in Per Cent. of Full Load Current.	Starting Torque of Motor in Per Cent. of Full Load Running Torque.
40	112	280	32
60	250	420	72
80	450	560	128
100	700	700	200

Motors intended to be operated from compensators should be designed with fairly low reactance in order that they may take the current required for starting with the necessary torque with the smallest practicable percentage of line voltage, *i.e.* when connected to the lowest compensator taps, since, for a given current taken by the motor, the line current will be less the lower the voltage at the motor terminals.

In Figs. 287 to 290 are illustrated several types of squirrel cage motors which have been built by different firms.

In Fig. 291, page 244, is seen another type of squirrel cage construction, namely, that of the Allmanna Svenska Elektriska Aktiebolaget, of Westeras, Sweden. As shown in Fig. 291 and in Fig. 292, the rotor conductors are inter-connected at their ends

Fig. 288.—Squirrel Cage Rotor.

Fig. 289.—Squirrel Cage Rotor (British Thomson-Houston Company).

by means of a series of high resistance wires systematically interwoven. This construction permits of securing, when required, a high rotor resistance, and, as we shall see later, a higher specific starting torque. At the same time it avoids the possibility of

Fig. 290.—Squirrel Cage Rotor.

undue heating, since although the rate of heat generation in the end connections is high at starting, so is also the radiating surface; hence no considerable rise of temperature will occur. The individual wires constituting the end connections are of such small

Fig. 291.—Squirrel Cage Rotor (Almanna Svenska Elektriska Aktiebolaget).

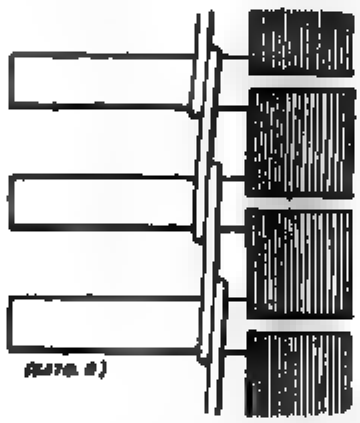


Fig. 292.—Detail of Squirrel Cage Rotor, Fig. 291.

cross section that, although they are sometimes proportioned for a current density of over 2500 amperes per square centimetre, there is no undue heating. The method of winding is sketched in Fig. 292.

In Figs. 293 and 294 are shown types of compensator provided for starting squirrel cage induction motors by the method

illustrated diagrammatically in Fig. 286. The compensator shown in Fig. 293 is that employed by the British Thomson-Houston Company; it is an auto-transformer with oil-break switch, and is for a three-phase induction motor; the compensator shown in Fig. 294 is made by the British Westinghouse Company. As the compensator is only in circuit during the fraction of a minute necessary to start the motor, its core and windings may be run at high densities, and hence it may be made very compact and

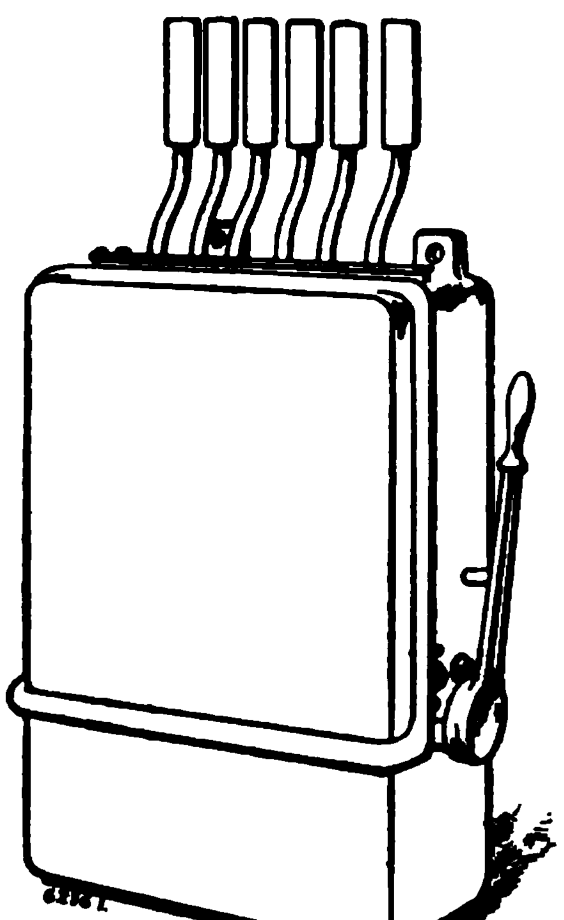


FIG. 293.—Starting Compensator (British Thomson-Houston Company).

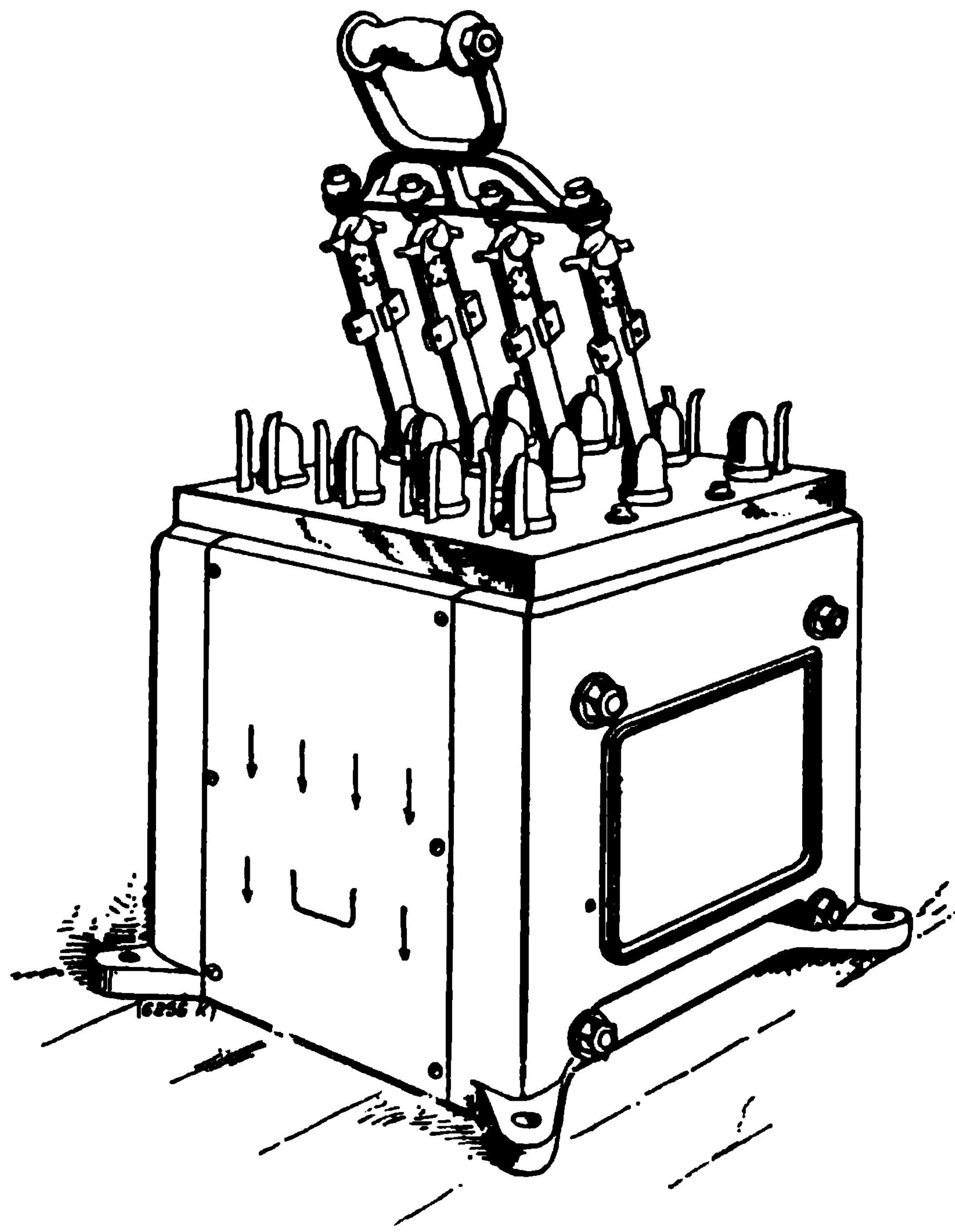


FIG. 294.—Starting Compensator (British Westinghouse Company).

inexpensive. Owing to the superposition and partial neutralisation of the primary and secondary currents in the coil of a compensator, it is in any case materially cheaper than the equivalent transformer, with its independent primary and secondary windings, especially when only required in circuit for a fraction of a minute.

The low specific torque at starting has hitherto generally limited the use of squirrel cage designs to comparatively small motors. But it is the writer's opinion that their superiority in

all other respects, and their more satisfactory and cheaper construction, justify arranging for their use whenever the conditions can economically be arranged to permit. This is probably much oftener than would be inferred from present practice.

§ 3. **Devices to Enable Motors to Start Light.**—Special friction clutch devices could often be employed, enabling the motor to start light, and only take up its load after having acquired a suitable speed.

FIG. 295.—Hydraulic Coupling for Induction Motors.

A hydraulic coupling for this purpose, as employed by the Schuckert Company for use with squirrel cage induction motors, is illustrated in Fig 295. By means of this hydraulic coupling the motor is started up light and the load thrown on automatically when full speed is attained, and is thrown off automatically also when the speed has fallen to a very low value or to zero. The starting up thus becomes a quick and simple operation, and the motor is also freed when it pulls up through the application of too heavy an overload. The device consists essentially of a friction clutch automatically operated by the centrifugal force of a heavy whirling liquid acting on a flexible diaphragm. The sectional

sketch in Fig. 295 shows the design of the coupling as arranged for driving a belt pulley. The coupling consists essentially of a casting *a* keyed fast to the shaft *w*, and carrying the friction plate *d*, which has a small end play. The disc *f*, loose on the shaft, is rigidly coupled or cast in one piece with the pulley *n*, and has no end play.

Two concentric channels, *b* and *c*, are cast in *a*. The channel *b* is filled with glycerine, and is connected with *c* by a series of holes fitted with stop screws, as at *k*, by which the rate of outward flow may be accurately regulated.

The sliding disc *d* padded with a leather ring *m* is held up against *a* by a series of adjustable spiral springs, as at *i*. The disc *f* is usually cast in one piece with the pulley, and is bronze bushed. The concentric pinching rings, *p* and *q*, tighten the flexible diaphragm against *a*, thus preventing leakage.

On switching in the motor, *a* begins to revolve, the centrifugal force drives the glycerine past *k* into *c*; the pressure on the diaphragm gradually increases, bulging it and pressing *d* up against *f*. By adjusting the rate of flow at *k* the disc *d* may be made to couple up only when a certain desired speed is attained.

Should the motor become overloaded or be shut down, the speed falls, the centrifugal force decreases, and at a certain point the springs *i* are able to overcome the pressure in *c*, and force the glycerine back into *b* through spring traps, as at *l*, the action instantly uncoupling the pulley, etc., and freeing the motor.

The use of this or any other form of automatic coupling secures the advantages incident to starting a motor light, in which case the heavy current consumption necessary for starting against load, with the attendant and undesirable reaction on the generator and prime mover, and the undue drop of volts in the line, are obviated. Couplings of this type are especially beneficial when the motor is difficult of access.

Arrangements which allow the rotor to make a complete revolution before taking up the load, relieve the line of any very excessive rush of current. Mr Roslyn Holiday has introduced an arrangement of this kind, in applying induction motors to drive coal-cutters. For this work, 2 : 1 gearing is used, in order to obtain the required lower speed at the cutter, and the rotor may make nearly two complete revolutions before taking up any load beyond its own bearing friction. A motor may sometimes, especially if of large capacity, have its own step-down transformer or group of transformers, in which case, instead of having a

compensator in addition, taps may be taken out at suitable points of the secondary windings. Such an arrangement is illustrated diagrammatically in Fig. 296. An interesting system of connections for this purpose is that devised by Mr H. S. Meyer ("Some Notes on Induction Motors," London *Electrician*, June 13th, 1902, page 308), and illustrated in Fig. 297. This diagram shows the advantage of the "delta" connection in such arrangements, since a double-pole, double-throw switch suffices to enable half voltage to be obtained at the motor. Not only 50 per cent. but any other desired proportion of normal voltage may be provided by this arrangement. With a Y connection, a triple-pole, double-throw switch would have been required.

FIG. 296.—Diagram of Induction Motor Starter (Holiday).

An exceedingly simple and satisfactory method of operating squirrel cage motors without incurring heavy current at starting, and which avoids the necessity of a compensator or of taps from a transformer, consists in designing the motors for operation under running conditions with delta-connected stator windings, these windings being, at starting, temporarily switched over to Y connection. This reduces the starting current drawn from the line by a given motor to one-third only of what it would be were it switched directly upon the line with its permanent "delta" connection.

The Schuckert Company, who employ this method with their squirrel cage motors, find that a starting current of from one to one and a half times full-load current suffices to start up fairly large motors without load, and, in the case of small motors, to obtain starting torques up to as much as three-quarters of full

load running torque. The connections employed by the Schuckert Company for starting squirrel cage motors by this "star mesh" method are shown in Fig. 298, where the two ends of each of the three windings of the stator are brought to the terminals p_1p_1, p_2p_2, and p_3p_3. Before throwing the motor on the line by means of the triple-pole, single-throw switch A, the triple-pole, double-throw switch B must be thrown over into the left-hand position, thus connecting the stator windings in "star," so that the voltage per phase is only $\frac{1}{\sqrt{3}}$, or 58 per cent., of the

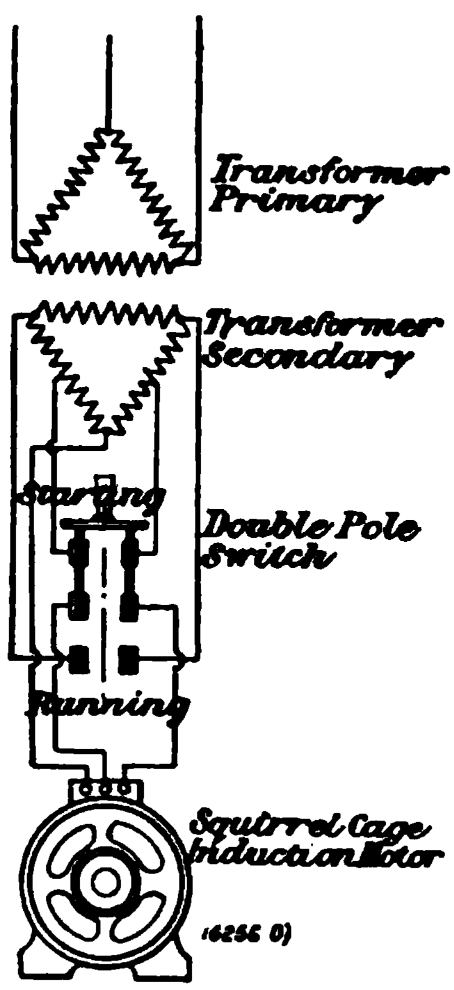


Fig. 297.—Diagram of Induction Motor Starter (Meyer).

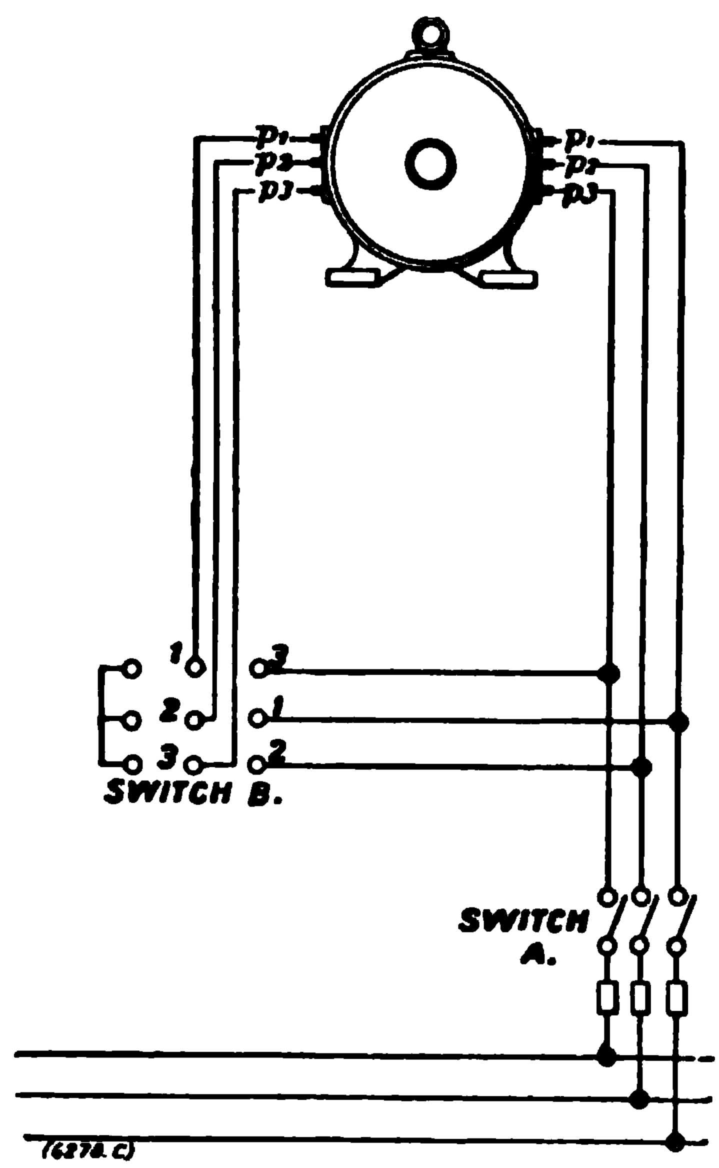


Fig. 298.—Diagram of Induction Motor Starter (Schuckert).

voltage between lines, and the current drawn from the line at starting is correspondingly reduced. On closing the switch A, the motor starts with a low torque. After it has acquired speed, the switch B is thrown quickly over to the right-hand position, connecting the windings in "mesh" so that the windings of each phase have the full load voltage.

It is frequently desirable to instal a motor large enough not only for immediate, but for prospective requirements. A disadvantage in so doing is that the motor must meanwhile operate at a small percentage of full load, and the power factor and efficiency are thus low. But by installing the motor with the

connections shown in Fig. 298 it may be comparatively economically operated at light loads with the switch B in the left-hand position, not only for starting but during normal running. When later justified by the heavier requirements, the right-hand position may be used for normal running. One objection of installing motors of too large capacity for the load at first required is thus overcome.

Cases arise where a single large motor is driven from an independent generator, in which case motor and generator may be run up to speed together. Motors even of the squirrel cage type require but little current when thus started. When large motors have to be started but once in a day, running thereafter without interruption, it can often be arranged to employ the squirrel cage type, the starting occurring at stipulated times, when a minimum

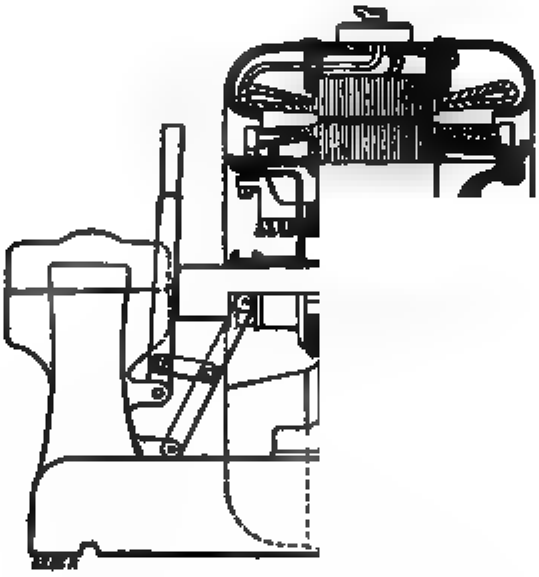


Fig. 299.—Wound Rotor without Slip Rings.

of inconvenience will be thereby occasioned. The disfavour with which squirrel cage motors have until recently been regarded is largely due to a lack of clear appreciation of their failings and of the means for overcoming them, and to their use in cases for which they are not suited. Some corporation engineers now demand squirrel cage motors even in the largest sizes, only employing wound rotors for special cases.

§ 4. **Wound Rotors without Slip Rings.**—Some makers have used rotors without collector rings, but with distinct windings, internally short circuited, instead of squirrel cages. This arrangement is distinctly inferior to squirrel cage rotors; the construction is less simple and robust, the I^2R loss greater, the power factor lower, and the efficiency and overload capacity are less. Such wound rotors without collector rings must, however, be resorted to for the arrangement illustrated in Fig. 299,

page 250, where a starting resistance is located within the rotor spider for the purpose of increasing the specific starting torque. This resistance is subsequently cut out by operating the lever

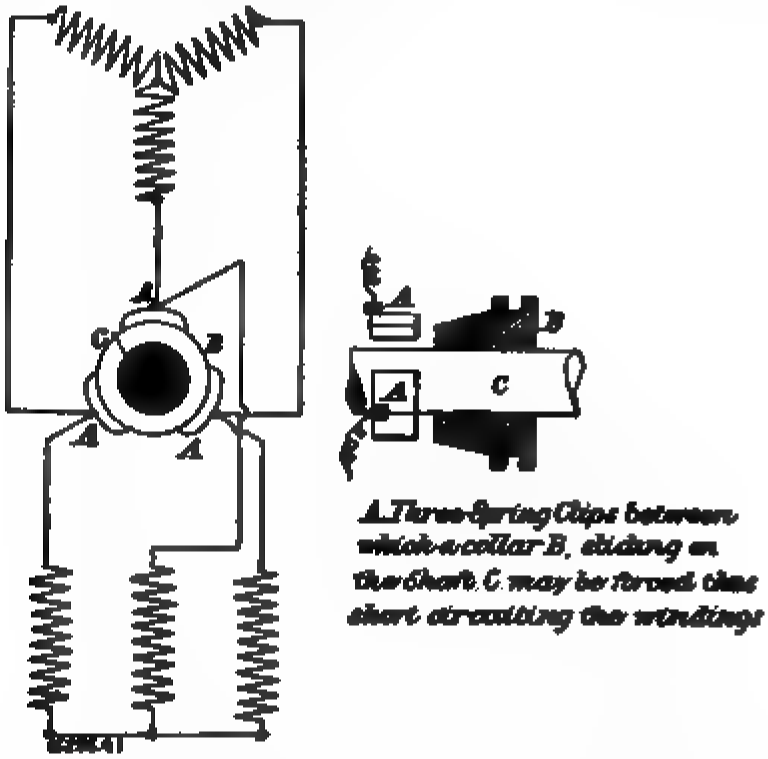


Fig. 300.—Diagram of Connections for Fig. 299.

shown in the figure, which engages with a collar free to slip longitudinally on the shaft. The collar is forced by the motion of the lever into contact with three spring clips connected at the

Fig. 301.—Three-phase Motor with Switch in Rotor Spider.

point of junction of the windings and resistances. The arrangement is shown diagrammatically in Fig. 300. Fig. 301 shows a rotor with still another arrangement for the same purpose. In this case, in order to close the switch in the armature spider, a knob, within which the end of a rod is free to turn, is pressed by

hand when the rotor has attained sufficient speed, and thereby causes the rod, which has a free longitudinal movement through a hole in the shaft, to transmit the movement by suitable levers to the switch within the rotor spider.

§ 5. **Rotors with Slip Rings.**—The plan of arranging starting resistances within the rotor spider was much more customary in the earlier days. It is now generally regarded as an undesirable method, and motors requiring starting resistances are preferably provided with slip rings, thus enabling the starting resistance box to be external to the motor, as in Fig. 302. In the earlier days water rheostats were sometimes employed, but nowadays cast-iron grids and other modern constructions are almost invariably used, just as for continuous current motors. Even when slip rings and external resistances at starting are

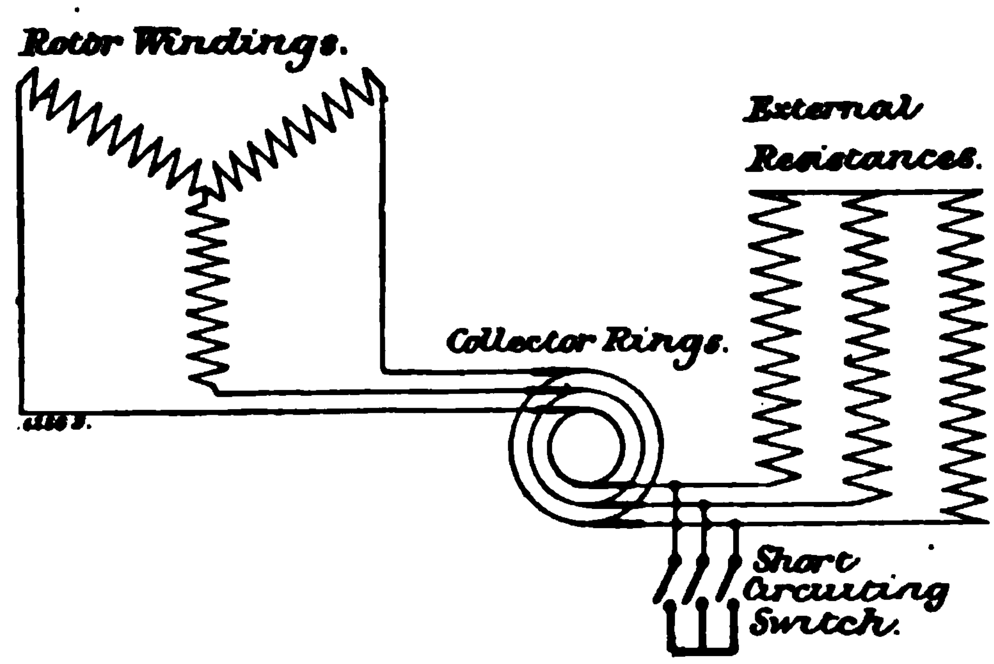


FIG. 302.—Diagram of Connections for starting Slip Ring Motor.

used, it is at present quite general practice, at any rate on large motors, to apply in addition a device equivalent, so far as concerns the result to be accomplished, to that indicated in Fig. 299, or that in Fig. 301, to short circuit the slip rings internally when the motor has acquired speed, and thus avoid the I^2R loss which would occur at the brush contacts were the short circuiting switch located externally to the motor, as shown diagrammatically in Fig. 302. Besides thus internally short circuiting the rings, motors are often provided with devices for afterwards lifting the brushes from the slip rings, thus avoiding the useless friction loss which would occur were they allowed to remain in rubbing contact, and also avoiding the wear of brushes and rings. While these devices impose additional complications, they are the inevitable consequence, when due regard is paid to economy in energy consumption, of resorting to other than squirrel cage rotors. Their

complete avoidance is another advantage of the squirrel cage rotor. It would seem just as good practice, in a large number of cases where squirrel cage motors are at present *not* employed, to incur some expense in adopting suitable methods for permitting of starting the motor without load, and throwing on the load after starting, as to introduce these expensive, complicated, and troublesome devices for obtaining economical performance in a motor capable of starting efficiently against load. Moreover, the squirrel cage motor is in every respect, except in its low starting torque per ampere (its "specific" starting torque), superior to the slip ring motor. When slip rings are used, the necessity for the devices just described is the greater because the practical conclusion has now been generally arrived at that carbon brushes are as indispensable for slip rings as for commutators. (Slip rings are practically as troublesome as good commutators.) This is the generally-accepted result of experience. Brushes on slip rings, whether of copper, brass, or iron, have been found to lead to a rapid deterioration both of slip ring and brush, and have been quite generally abandoned for carbon brushes, which, of course, require a much greater contact surface, hence larger slip rings and greater friction losses, and there is even then a much higher I^2R loss at the brush contacts.

Figs. 303 and 304, page 254 (from Prof. Dr Klingenberg's *Elektromechanische Konstruktionselemente*, Sixth Part, sheets 58 and 59. Julius Springer: Berlin, 1902) show an instance of the complications which have been resorted to in order to arrange means for internally short circuiting the slip rings, and subsequently raising the brush gear. Figs. 305 and 306, page 255 (from Mr Eborall's paper, entitled, "Some Notes on Polyphase Machinery," read before the Manchester Section of the Institution of Electrical Engineers, 1902), illustrate a somewhat less complicated device for accomplishing the same purpose, and suitable for a 50 brake horse-power motor.

In Figs. 307 and 308, page 256 (also from Prof. Dr Klingenberg), is shown a plan by the Lahmeyer Company for short circuiting the rings and raising the brushes.

Figs. 309 and 310, page 257, illustrate a design for a brush holder, slip rings, and short circuiting device, by Mr A. P. Zani, as fitted to Messrs Dick Kerr & Co.'s 5 horse-power induction motor. Where slip ring motors must be used, it is desirable that they should carry a fairly large current, since otherwise the rotor is less efficiently designed, and also more expensive, because of the

FIGS. 303 and 304.—Device for Internally Short Circuiting Slip Rings and Raising Brushes.

FIGS. 305 and 306.—A Simpler Device for Internally Short Circuiting Slip Rings and Raising Brushes.

FIGS. 307 and 308.—Lehmeyer's Device for Short Circuiting Rings and Raising Brushes.

FIGS. 309 and 310.—Zani's Device for Brush Holder, Slip Rings, and Short Circuiting.

great number of smaller individual conductors for which space must be provided. Mr Zani has found 200 amperes to be often desirable even in fairly small motors, and as this would make an efficient low resistance contact rather difficult to obtain in continuous running, this short circuiting device is employed, thus enabling comparatively small slip rings to be used, as they are only in circuit for the short time required for starting. In this design it has not been considered worth while to combine the short circuiting of the collector rings with the lifting of the brushes, as it was considered that this would only add a complication without any actual advantage, since, on account of the small diameter of the slip rings and the high density at which the carbon brushes may be temporarily worked, the friction loss is in any case small, and there is no harm done if the attendant forgets to lift the brushes when the rings are short circuited. Moreover, the lubricating effect of carbon brushes prevents any considerable wear of the slip rings when the tension on the brushes is carefully adjusted. The main features of this short circuiting device are standard for all sizes of motors, the design shown in Figs. 309 and 310 corresponding to sizes up to 30 horse-power. The lifting of the brushes is done by means of the handle shown, which is arranged to lock the brushes in the two positions by means of a pin. The brushes are made of special low resistance carbon, and are capable of carrying for a short time, and without appreciable heating, current up to 20 amperes per square centimetre.

The Schuckert Company (D.R.P. 114,828 of November 6th, 1900, also D.R.P. 116,267 of December 21st, 1900) has employed an arrangement by which it is not alone rendered impossible to raise the brushes before the rings are short circuited, but by which, in the event of the motor stopping, the brushes are automatically replaced and the resistances inserted. One type of this device is shown in Fig. 311, in which, of the two halves *a* and *b* of a coupling, *a* is keyed to the shaft, and *b* is free to move in the direction of rotation. The contact blocks *c* are mounted opposite one another on *a* and *b*. The springs *d* tend to bring *a* and *b* into such relative positions as to break the contact at *c*. When, after starting up, it is desired to short circuit the secondary windings, this is accomplished by applying the brake lever *e*, thus holding back *b* with respect to *a*, and thus bringing the contact blocks together. In this position *a* and *b* are held together by means of a latch *f*, which, thrown outward by the centrifugal force of the weight *g*, engages with a corresponding lock. The weight *g* also

carries a stud h, which now projects from b, and in turning, releases a lever k, carried by an extension of the brush holder i, out of its engagement through the stud n and the recess l. The brushes may now be raised by a downward movement of the brake lever e, and against the tension of the spiral spring m on the

FIG. 311.—The Schuckert Company's Arrangement for Raising Brushes.

extension of the brush-holder. The stud n of the lever k springs into the recess o, and the brush-holder is thereby held in the raised position. This position of the coupling, the contacts, and the brushes corresponds with normal running. Should a sufficient fall in speed occur, the spiral spring p draws the weight g inward, and the stud q, which revolves normally in a surface of smaller radius than the stud h, projects through b, and, by lifting the lever

k, releases the brush-holder, which, under the influence of the spiral spring *m*, brings the brushes to bear on the slip rings, the brake lever *e* being also brought from the running to the starting position. The weight *g* falls still further back, withdrawing the latch *f* from engagement with *a*, and permitting the spiral spring *d* to draw the contact blocks *c* apart, thus cutting in the resistances ready for starting the motor again.

§ 6. **Görges' Method for Starting Induction Motors.**—A most interesting method of starting induction motors has been devised by Görges (British Patent 21,141 of 1894, Siemens Bros. & Co., German Patent No. 82,016, of 1894, Siemens & Halske). The fundamental idea consists, not in limiting the secondary currents at starting by resistances in series with them, but in so arranging the secondary windings that the electro-motive force in some of them shall oppose that in others in the same series circuit, thus leaving a residual electro-motive force only sufficient to send a suitably small current through the total resistance per winding. As soon as the motor has attained sufficient speed, the separate windings are so connected that the full electro-motive force of each winding is utilised for producing current. As actually carried out in a 30 horse-power Siemens & Halske motor, very fully described, with drawings, by Kapp, in *Elektromechanische Konstruktionen*, second edition, 1902, page 261, this basic idea is combined with the introduction of resistances to secure still smoother starting; but as this non-essential modification only serves to complicate the explanation, it will not here be introduced.

In its simplest form a rotor arranged by Görges' method has two separate three-phase windings, A and B of Fig. 312, page 261. The three component windings, *a*, *b*, and *c*, of the three-phase winding A are located respectively in the same position (or in equivalent positions) on the rotor as a^1, b^1, c^1, so that the electro-motive forces induced in *a* and a^1 are in the same phase, as also are those in *b* and b^1, and *c* and c^1. But a^1, b^1, and c^1 have, say, twice as many conductors as *a*, *b*, and *c*. The connections are at first arranged as shown in Fig. 313, page 261, or at least *equivalent* to the connection there shown diagrammatically. Obviously the electro-motive force in *a* is opposed to that in a^1, *b* to b^1, etc., and the resultant electro-motive force is but one-third of the sum of the two electro-motive forces in *a* a^1, *b* b^1, and *c* c^1. Thus the current is sufficiently small to prevent undesirable rotor reaction upon the magnetic field, and the motor starts up with suitable torque. When sufficient speed is attained, a connection is made

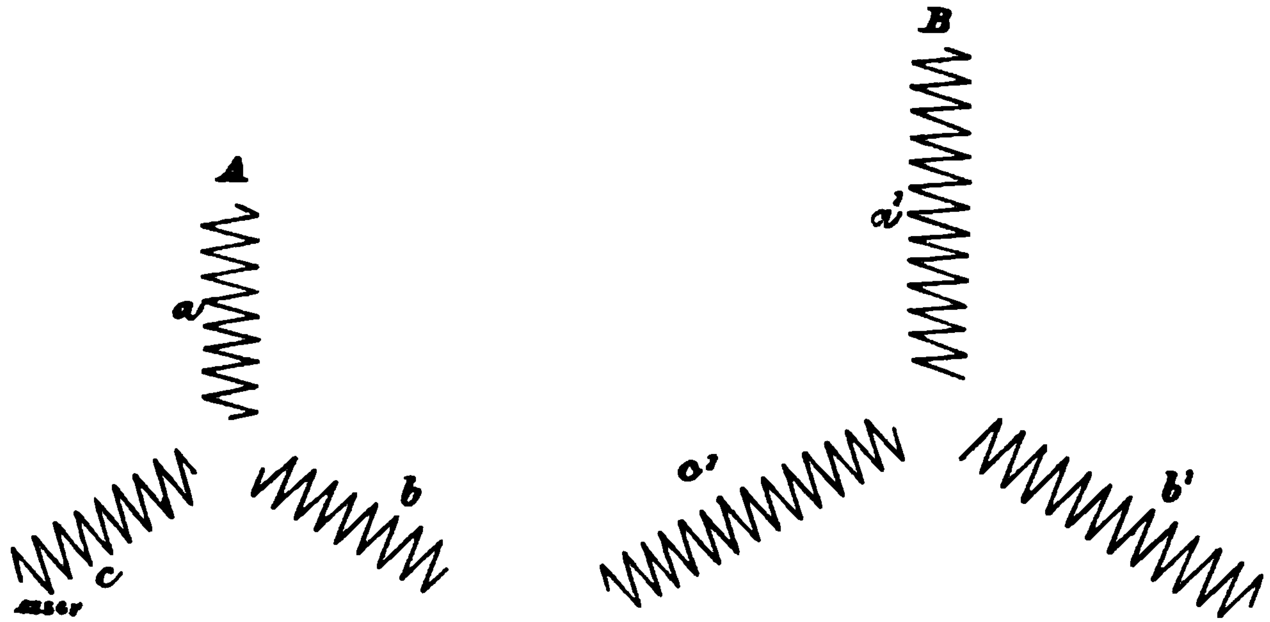


FIG. 312.

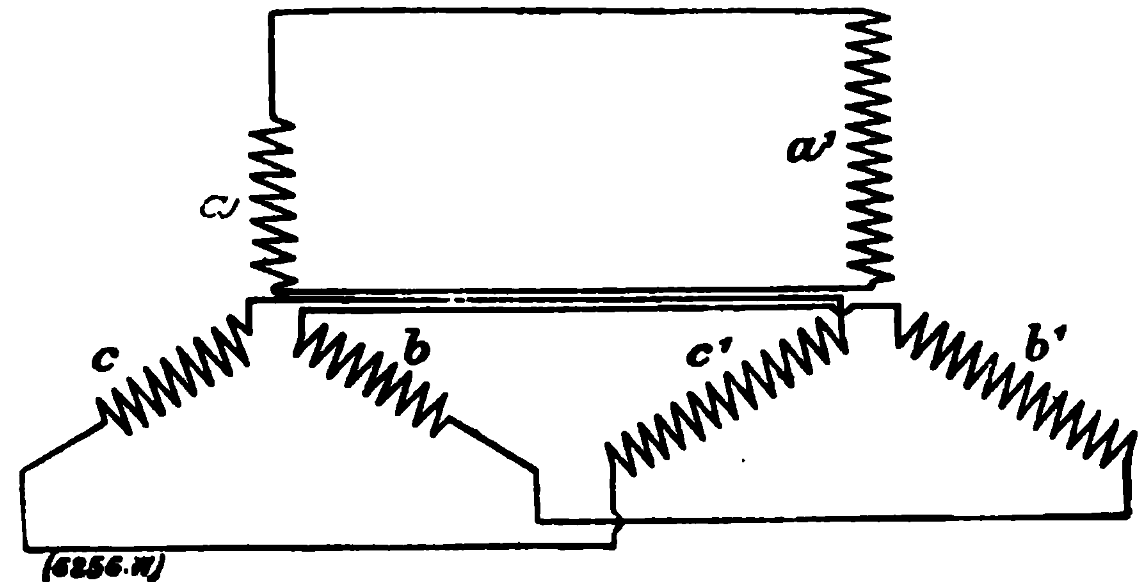


FIG. 313.

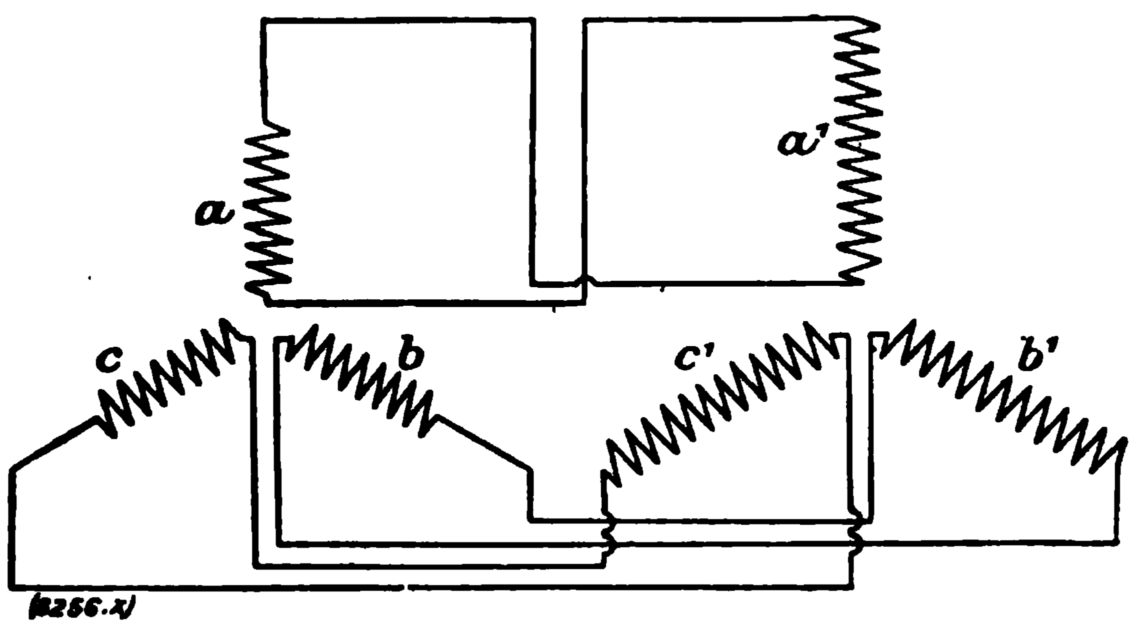


FIG. 314.

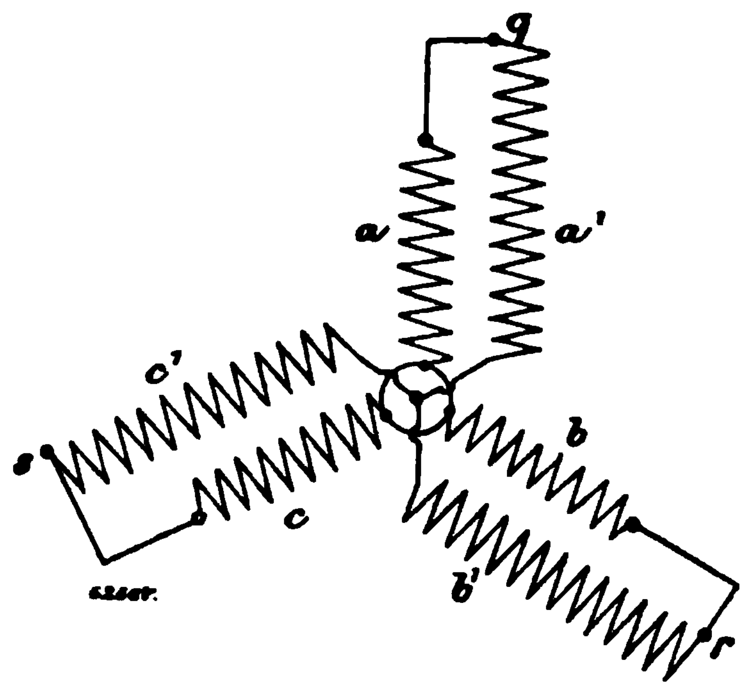


FIG. 315.

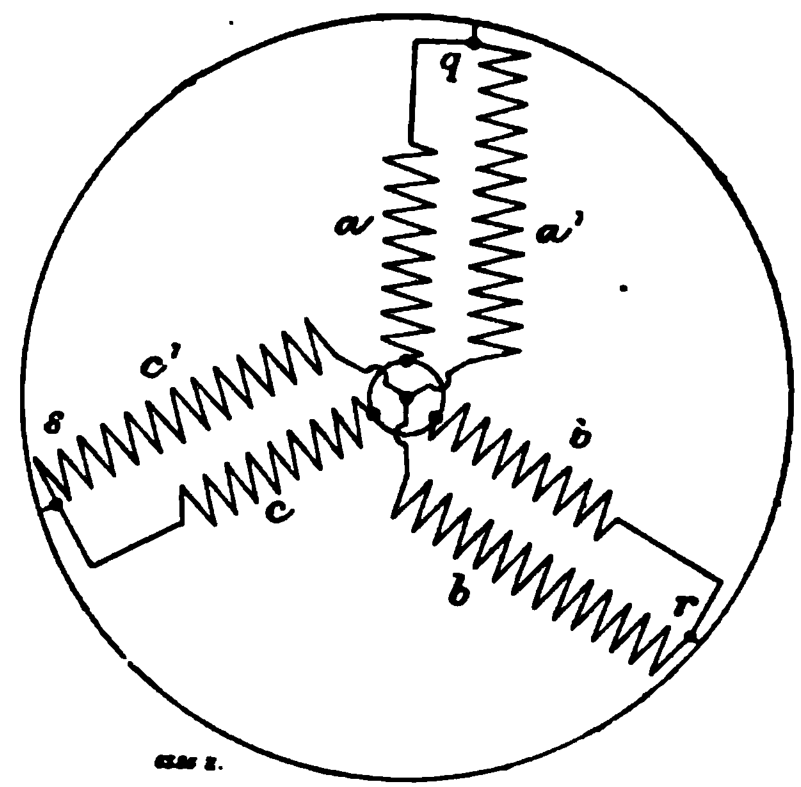


FIG. 316.

The Görges Method of Rotor Winding for Starting Induction Motor.

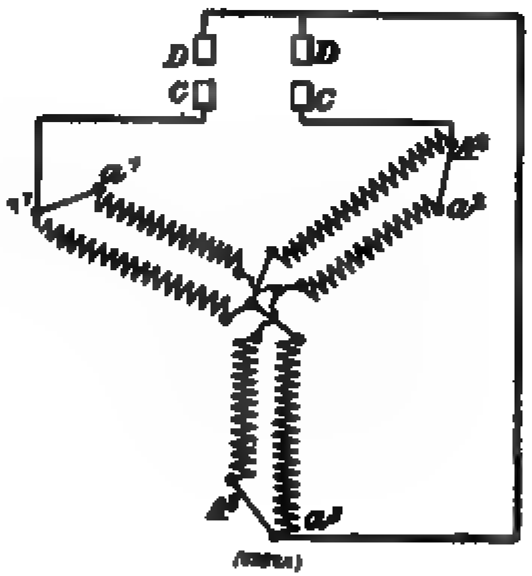
which in its result is *equivalent* to re-connecting the windings, as shown in Fig. 314, page 261, so that the electro-motive forces of the two sections co-operate with one another, and the motor operates normally.

The ingenuity of the scheme is largely due to the simplicity of the change actually made in the connections.

This is shown in Figs. 316 and 317, pages 261 and 263, the former of which shows the connection suitable for starting, the latter corresponding to the final connection. The two diagrams are evidently the equivalent of Figs. 31 and 32, except that in these latter the neutral points of the star connection were left unconnected for the sake of clearness.

It is immaterial whether the windings are arranged with two independent neutral points, as in the diagrams of Figs. 315 and 316, or whether these are connected together as one neutral point. The one short circuiting connection at the other end of the two sets of windings also suffices. Nothing would be gained by providing a separate short circuiting connection for each of the two sets of windings, for when they are short circuited, corresponding to the permanent running condition (Fig. 316), the current per conductor is determined simply by the volts and reactance per conductor. Hence the single alteration required for obtaining from the starting connections the connections suitable for permanent running is to connect together the three points q, r, s of the diagram, as in Fig. 316. In the Siemens & Halske motor, above referred to, this connection is automatically effected by a centrifugal governor. It could, when desired, be done by a hand-actuated switch of the type illustrated in Fig. 299, or that of Fig. 301, for the purpose of cutting out internal resistances after starting (see page 251).

The problem of centrifugal governors for circuit changing purposes in the rotors of induction motors has received considerable attention. Two centrifugal devices, as applied to the rotors of induction motors, but employed respectively for taking up the load after starting and for short circuiting the slip rings after starting, have been illustrated in Figs. 295 and 311, on pages 246 and 259. The arrangement devised by the Siemens & Halske Company (German Patent No. 91,135 of 1896, subsidiary to No. 82,016 of 1894), in connection with the application of the Görges starting method, is set forth in British Patent No. 21,668 of 1896, from which Figs. 317 and 318, page 263, and the following description have been taken.



FIGS. 317 and 318.—The Siemens & Halske Application of the Görges Method, and Centrifugal Governor (see page 262).

By this arrangement, instead of connecting together the points *q*, *r*, and *s* of Fig. 315, as shown in Fig. 316, the corresponding points in Fig. 317, denoted by A^1, A^2, and A^3, are connected respectively, A^1 and A^2 to separate contact plates, C C, and the point A^3 to two other contact plates, D D, arranged opposite C C, and a centrifugal governor is made to close the contacts by movable contact pieces as soon as the motor has attained the required speed.

Fig. 318 shows a plan of the centrifugal governor with the cover removed, and a cross section on line X Y.

The operative parts are enclosed in a cylindrical casing T, which is fixed on the shaft of the motor so as to rotate therewith. C C and D D are the contact plates above referred to. The necessary four supply leads to these pass from the armature of the motor through the back of the casing T, two of them being led to the contact plates C C, and the other two to the connecting blocks K K. As the two sets of these parts are identical with each other, they are indicated by the same letters of reference, and in the following description only those belonging to one pair of contacts will be referred to.

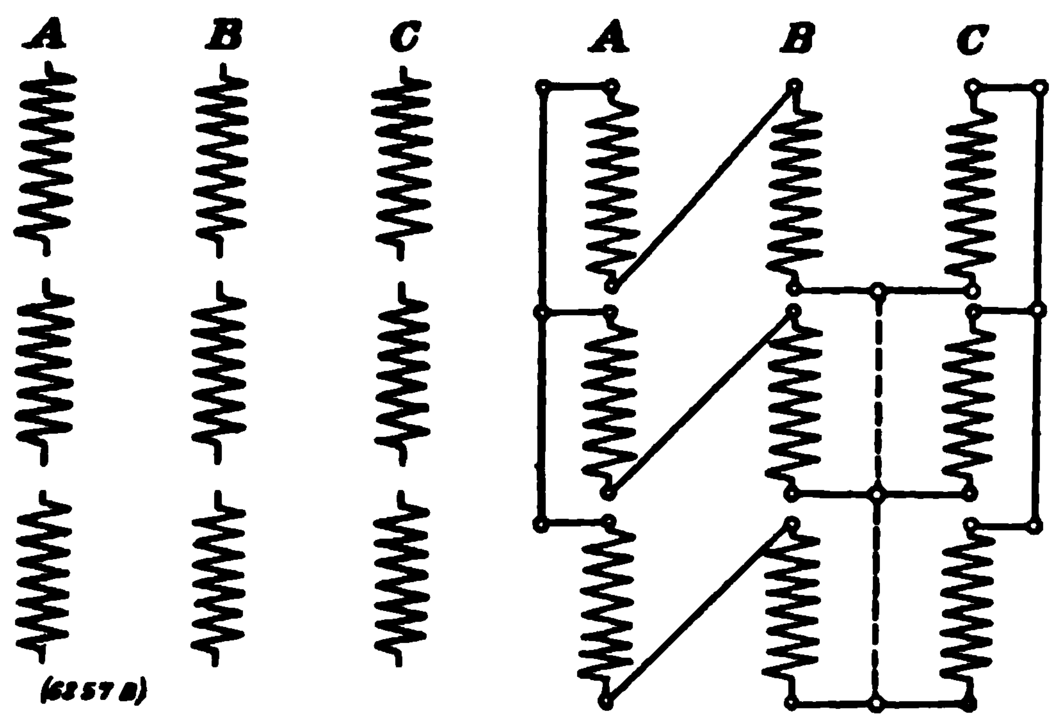
From K the flexible conductor B passes to the contact plate D, which is secured to, and insulated from, the centrifugal body M pivoted at Z to the casing. As this body is unsymmetrical relatively to the radius of the motor shaft passing through Z, on the rotation of the motor shaft the resulting centrifugal action will produce a turning moment in M around Z. A coiled spring F acts in opposition to this moment; it is connected at the one end by a bar P to a pin S on the body M, while the other end is connected by the screwed adjustable rod H to the casing. The bar P passing through the spring serves to maintain this in a straight line when the centrifugal force tends to bend it transversely outwards.

The pull of the spring is so adjusted that its moment holds the moment of the centrifugal body M in equilibrium until the speed is attained at which the contacts are required to be closed. On this speed being attained the body M will consequently move outward from its original position.

As the elasticity of the spring is so proportioned, by giving it a sufficient number of convolutions, that the moment of the spring power increases more slowly than the centrifugal force of the mass M, which increases proportionately to the increase of the radius, the body M after once leaving the original position will move

outward with increasing acceleration so as to effect the contact between C and D with suddenness and with a surplus of energy.

The two contacts are required to close simultaneously. This could, however, not always be ensured if the two contact devices were quite independent of each other, even with great accuracy of adjustment of the two springs, on account of possible unequal variations in the friction of the parts. On the other hand, the pairs of contacts must arrive independently of each other at their end position in order to ensure a reliable contact in both. In order to fulfil both conditions there is provided loose on the central boss of the casing a disc W having radial slots RR with which are engaged, with a certain amount of play, pins *s* fixed to the bodies M.



FIGS. 319 and 320.—Suggested Winding for the Görges Method of Starting.

If now the one body starts from its original position sooner than the other one, it will, after moving to the extent of the play of *s* in the slots, transmit its motion to the other body M, and as it will then have already attained an excess of energy, any difference between the tension of the two springs will thereby be prevented from detrimentally affecting the simultaneous closing of the two contacts, while, on the other hand, the play of the pins *s* in the slots R will render the contact positions of the plates independent of each other.

It has occurred to the writer that a symmetrical two-circuit triply re-entrant triple winding would adapt itself excellently to the requirements of the Görges method. One would break up into three sections each of the three independent windings, A, B, and C (see Fig. 319), and regroup them as indicated diagrammatically in Fig. 320, where the full lines represent the

connections for starting. After attaining speed, the only change requiring to be made is to add the connections indicated by the dotted lines. A triple winding, giving a reduction of electromotive force at starting to that of one-third of the total number of conductors, has been taken as representing the simplest case. Other ratios could be obtained by other windings. The Görges patent also describes arrangements for obtaining more than two steps, and thus preventing sudden and considerable changes.

§ 7. **Boucherot's Methods for Starting Induction Motors.**—M. Boucherot was one of the earliest workers in the devising of methods of starting induction motors. Of the three types into which his motors may be divided, none can afford to be overlooked, as the underlying principles may well be kept in mind in induction motor work. The earliest types were devised in 1894 and 1895, and had for their object the securing of motors with good specific starting torque and without slip rings. The first idea was to employ an ordinary stator as primary, and to provide the rotor (secondary) with several independent closed circuits of graded resistances and inductances, ranging from low resistance and high inductance to high resistance and minimum inductance. The latter only are traversed by any considerable secondary currents at the instant of throwing the motor on the circuit, since the rotor having the full primary periodicity, the secondary circuits of high inductance also have high reactance, the reactance being equal to $2\pi \times$ periodicity $\times$ inductance. But as the rotor gathers speed the periodicity of the currents in its windings decreases, and hence also the reactance, and at full speed, when the periodicity is very small indeed, all the circuits, especially that of the highest inductance (because it is the circuit of lowest resistance), carry considerable proportions of the total current. Hence the motor is more efficient at normal running than would be the case were its rotor merely provided with a single winding of sufficiently high resistance to secure a satisfactory specific torque at starting, for such a motor has high slip, low efficiency, and high heating.

As this conception developed, it was embodied by M. Boucherot in a rotor furnished with a series of squirrel cages, an end view of the core for which is illustrated diagramatically in Fig. 321. The outer squirrel cage, having fairly high resistance and minimum inductance, is most effective at starting, when the secondary periodicity is high. The next inner, after some speed has been attained, then the third, and ultimately, at normal

running conditions, all the windings contribute to furnish the torque, the much lower resistance of the most remote winding making up for its higher inductance, which higher inductance, in virtue of the low secondary periodicity at normal speed, is accompanied by a moderate reactance only.

In practice, two component squirrel cages are generally found to be sufficient. Fig. 322, page 268, shows a longitudinal section and an end view of such a rotor. In these figures T_1 represents a series of slots at the periphery of the laminations, as in the ordinary motor. Lower down is a second series of slots, T_2, connected with the first by radial openings, designed to increase the magnetic

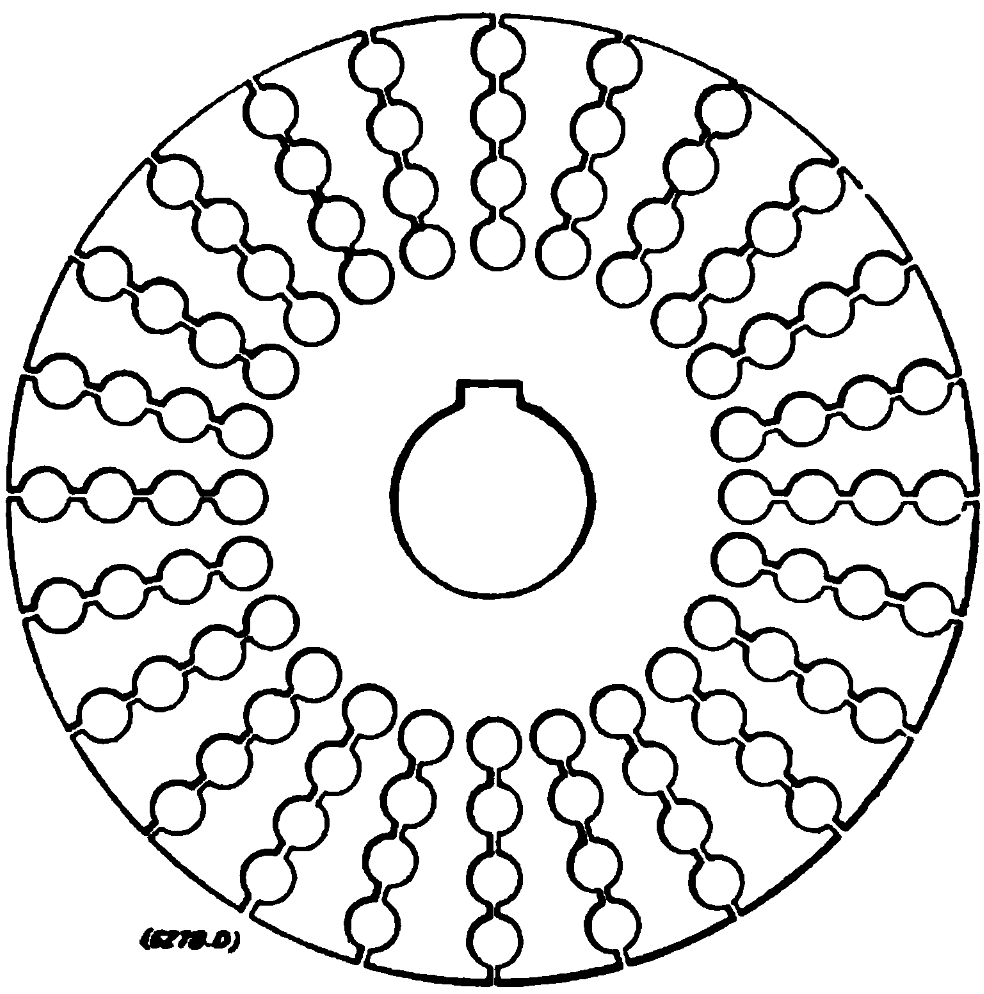


Fig. 321.—Core Disc of Boucherot Rotor.

reluctance of the annular space between the two series of slots, and thus to diminish the magnetic leakage of the inner squirrel cage.

In the outside series of slots copper bars are placed, and these are soldered to end rings C_1 C_1 of high resistance, the end rings being usually constructed of iron, German silver, or other high resistance material. In the second series of slots are placed copper bars generally of larger cross section than those in the outer series, and these are soldered at their ends to the copper rings C_2 C_2 of low resistance. A view of the parts of a 5 horse-power motor of this type is shown in Fig. 323, Plate 18. A 3 horse-power motor assembled is shown in Fig. 324, Plate 18.

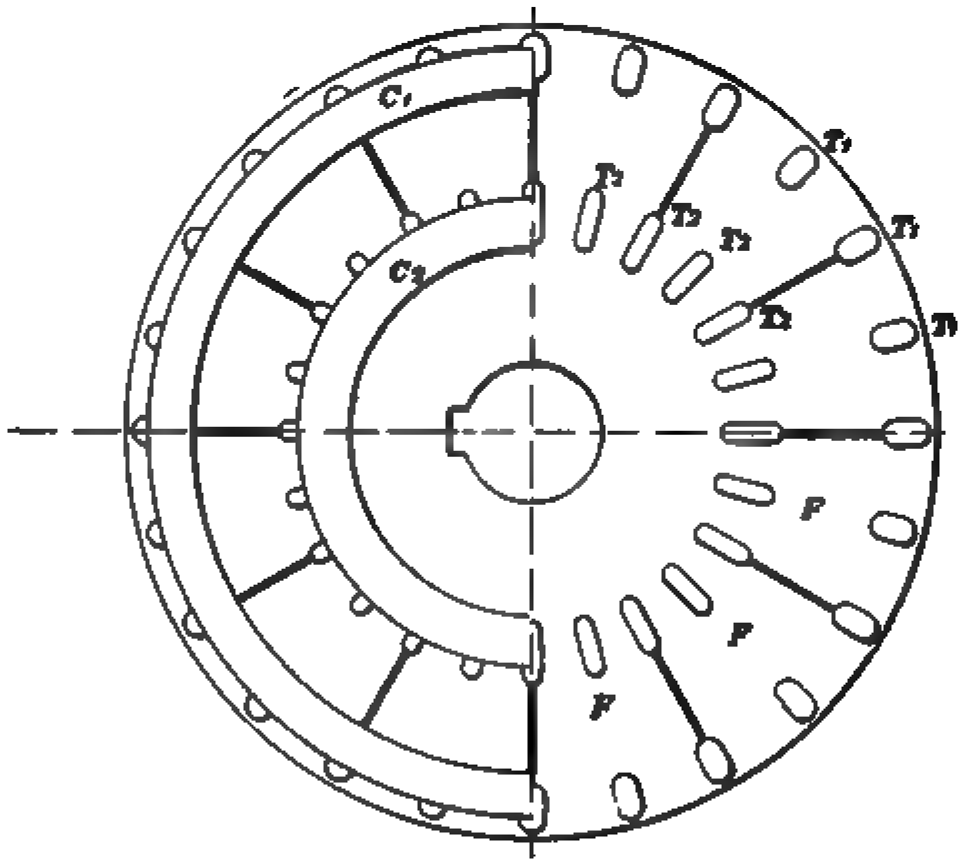


FIG. 322.—Longitudinal Section and End View of Rotor, with Two Component Squirrel Cages (see page 267).

FIG. 323.—Component Parts of 5 Horse-power Boucherot Motor (see page 267).

FIG. 325.—8 Horse-power Boucherot Motor (see page 270).

FIG. 324.—3 Horse-power Boucherot Motor, Assembled (see page 267).

FIG. 326.—20 Horse-power Boucherot Motor (see page 270).

FIG. 327. –Rotor of 30 Horse-power Boucherot Motor (see page 270).

FIG. 329.—350 Horse-power Boucherot Motor (see page 272).

FIG. 330.—Rotor of a 200 Horse-power Boucherot Motor (see page 272)

The action is clearly illustrated by the curves in Fig. 328. On using in the ordinary way a high resistance, low inductance, squirrel cage rotor winding, like $C_1 C_1$ of Fig. 322, one may obtain the curve of torque and speed marked I. In this case, double full load torque is obtained at starting, and full load torque corresponds to about 75 per cent. of synchronous speed, hence the "slip" at full load is 25 per cent., and such a motor is very inefficient. Were the rotor provided, instead of with $C_1 C_1$, with the winding $C_2 C_2$ of low resistance and high inductance, curve II of Fig. 328

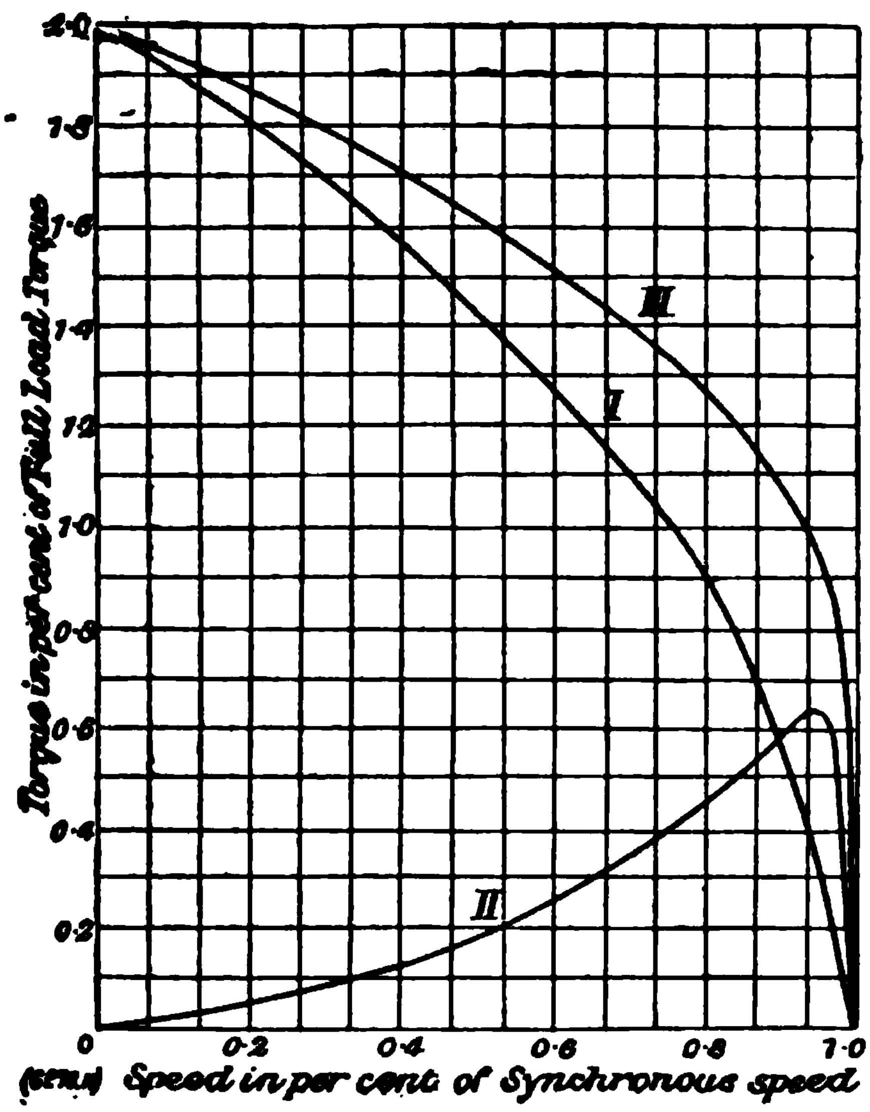


FIG. 328.—Characteristic Curves of Motor with Rotor Winding on the Boucherot Method.

would represent the relation of torque and speed. There is practically no starting torque, and the maximum load is but 62 per cent. of the normal load of the motor. But at this load the slip is but 6 per cent., and the motor then operates at a much higher efficiency. Now, in M. Boucherot's motor the windings $C_1 C_1$ and $C_2 C_2$ co-operate to give curve III, which, compared with curve I, shows the slip at full load to be reduced from 25 per cent. to 6 per cent., while a starting torque of double the full load torque is obtained, thus combining high efficiency with effective starting. M. Boucherot summarises its good features as follows:—

(1) For starting and stopping, the mere closing and opening of the main switch suffices.

(2) When pulled up by a heavy load it only takes a current equal to that consumed at starting, and as soon as the overload is removed the motor starts up again of itself. It is thus peculiarly fitted for operation at a distance, such as for motors in mines and for operating swing-bridges and draw-bridges of various types. The addition of variable resistances in the primary allows the speed to be varied according to the requirements.

M. Boucherot reports of such a motor of 8 horse-power rated output that it starts with twice full load torque, requiring two and a half times full load current. The only objection would be that its full load power factor is some 10 per cent. lower. (See also *Bulletin de la Société Internationale des Electriciens* for February 1898.)

M. Boucherot next devised the type of motor of which the 8 horse-power and 20 horse-power sizes are illustrated in Figs. 325 and 326, Plate 18. In this type the stator consists of two identical parts, the one fixed immovably to the base of the machine, the other capable of a certain annular movement, either with reference to the first, by a lever in the smaller sizes, as in Fig. 325, or by a hand wheel, as in Fig. 326, for the larger sizes. The rotor, although also a double structure, has a single squirrel cage winding, the face conductors of which extend straight through the two halves. These conductors are short circuited at their ends by two low resistance copper rings, and at the middle, between the two halves of the rotor, by a high resistance ring of German silver, or of other high resistance material. Such a rotor is shown in Fig. 327, Plate 19, this being for a motor of 30 horse-power.

The movable half of the stator is, for starting, so placed with reference to the immovable half that electro-motive forces in opposite directions are induced in the two halves of each rotor bar; hence the resulting currents must complete their circuit through the intermediate high resistance ring. Consequently the motor starts with a high specific torque, as in an ordinary induction motor with a high resistance squirrel cage. As the motor acquires speed, the stators are gradually brought into the normal running position, where they induce electro-motive forces in the same direction in the two halves of the rotor bars, and rotor currents no longer traverse the high resistance intermediate ring, but complete their circuit by the two low resistance copper rings at their extreme ends. The advantage of this motor is that, by the correct adjustment of the angular position of the movable half of the stator, it is possible to limit the starting current to the value required for the particular load and acceleration. In an ordinary squirrel cage motor, a current of a given strength flows

at the instant of closing the switch, irrespective of the load on the motor. If the load is small the motor accelerates quickly; if great, it accelerates slowly, but the starting current is the same. With the Boucherot motor, however, the starting current will be great or small, according to the relative angular positions at which the component stators are set, and during normal running the motor has the good properties of a low resistance squirrel cage motor. M. Boucherot also mentions that rearrangements of the windings by changes in the connections are sometimes preferable to the mechanical adjustment.

M. Boucherot's third type of motor (British Patent No. 9534 of 1900) is just like the second type, the one last described, except that both elements of the stator remain fixed; but the system requires at the switch-board, or at the point from which the motor

FIG. 331.—Diagram of 350 Horse-power Boucherot Motor.

is controlled, a "phase transformer," by means of which a high specific starting torque is obtained, together with a high efficiency at normal running. The motor is illustrated diagrammatically in Fig. 331, in which S S¹ are the two stators, R and R¹ the rotors, M the high resistance intermediate ring, and C C¹ the low resistance end rings, the face conductors K extending the full length of both rotors, being soldered to these three rings. If the currents sent into the windings on the two stators S S¹ have a phase difference of 180° from one another, the currents induced in the two halves of the rotor, being opposed to one another, must complete their circuits through the high resistance ring M; the motor is then equivalent to a high resistance squirrel cage motor, and has a high starting torque, but is badly suited for permanent running. When currents of the same phase are sent into the two stator windings the motor operates like a low resistance squirrel

cage motor. Hence one stator is supplied directly from the line and the other from the secondary of a "phase transformer" from which currents differing by 180° in phase from the primary currents are obtained for starting, this phase difference being gradually reduced as the motor acquires speed, until at full speed the phase difference is zero.

By means of the "phase transformer" any intermediate phase difference may be obtained at starting according to the load against which the motor must start and the ratio of acceleration desired. A 350 horse-power motor of this type is shown in Fig. 329, Plate 19. The rotor of a 200 horse-power motor is shown in Fig. 330, Plate 19. In Fig. 332 this mode of operation is indicated as applied to three motors. These may be supposed to be located at

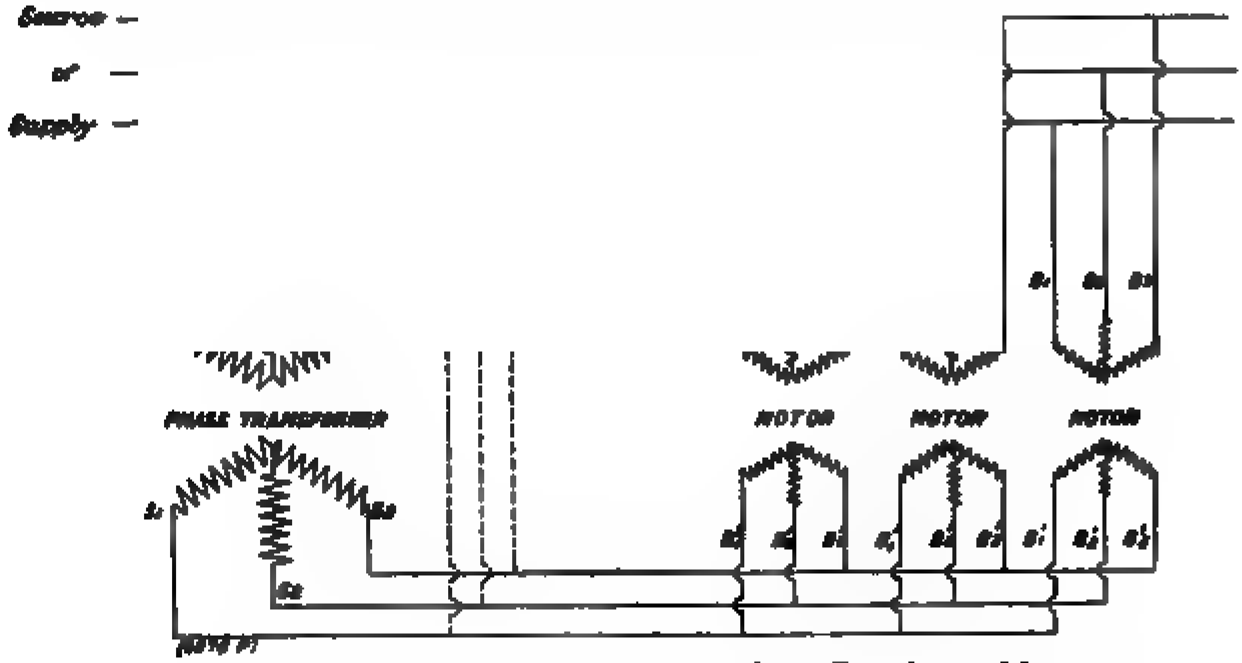


FIG. 332.—Diagram of Connections of Three Boucherot Motors.

some point difficult of access, and have to work, therefore, without slip rings or other moving contacts. They must yet be capable of a high starting torque. Each has two stators carrying the windings B_1, B_2, B_3, and B^1_1, B^1_2, B^1_3. These two sets of windings are permanently connected respectively to a line fed directly from the source and to a secondary line fed from the secondary windings S_1, S_2, S_3, of the "phase transformer," situated at the distant point from which the motors have to be controlled. For normal running, it is permissible to switch out the phase transformer, and supply both sets of stator windings from the primary source, as indicated by the three dotted lines *p*, *q*, *r*. The construction of the phase transformer may be explained by reference to Fig. 333. It is in principle practically an ordinary polyphase motor, but it is only required to be arranged for a small angular

Source of Supply

To Windings
$B_1' B_2' B_3'$
of the Distant
Motors

FIG. 333.—Diagram of Construction of Boucherot Phase Transformer.

movement of the internal secondary S, with respect to the external primary P. The primary carries the windings, W_p, which are connected to the source of supply. From the secondary windings, W_s, the motor windings B_1^1, B_2^1, B_3^1 of Fig. 332 are supplied over the secondary line. When the windings of the phase transformer occupy the relative position to one another shown in Fig. 333, the phase difference between primary and secondary currents is zero; by turning the secondary member S, until point *m* comes opposite point *n* of the primary member P, the phase difference is 180°, and this latter is the position required to obtain maximum torque at starting. This method was proposed by M. Boúcherot for operating the moving sidewalk at the Paris Exposition, but was not adopted, as continuous current motors were finally resorted to.

Many motors, some of fairly large capacity, built on these various principles, are in operation in France, and it would appear that the increased expense of production is often justified by the results which they give in special cases, where otherwise the induction motor would be unsatisfactory.

In the *Bulletin de la Société Internationale des Electriciens* for June 4th, 1902, M. Boucherot describes installations in which owing to the necessity for very frequent starting and stopping, and for quick acceleration of apparatus of great momentum, it has been found economical to have supplies of current at several frequencies, the induction motor during acceleration being gradually switched upon circuits of higher frequency.

On slowing down, instead of wasting the very considerable energy of momentum of the driven apparatus, this is restored gradually to the supply of circuits of decreasing frequencies, and thus transferred to the supply of other apparatus which may at the time be requiring to be accelerated, the induction motor thus acting as a generator. M. Boucherot has calculated the saving possible by the use of one, two, three, and four different frequencies to be as set forth in Table XLI.

TABLE XLI.—ENERGY SAVED BY BOUCHEROT'S METHOD OF REGENERATIVE CONTROL OF INDUCTION MOTORS.

Number of Frequencies Provided.	Power Required during Accelerating.	Power Restored on Slowing down.	Net Consumption of Power during Starting and Stopping.
1	1	0	1
2	0·75	0·19	0·56
3	0·67	0·29	0·38
4	0·62	0·35	0·27

It is evident that for certain special processes, when frequent starting and stopping is required, the saving set forth by this table would justify the increased expense of the installation.

§ 8. **Hobart's Method of Starting Induction Motors.**—A method which, electrically, is rather similar to the Görges method for starting induction motors has been suggested by the writer (British Patent No. 25,744 of 1897). Two component windings, A and B, Fig. 334, are wound on the secondary element (in this case the rotor R) in a double spiral, so that they may be considered to be practically superposed so far as relates to their position in

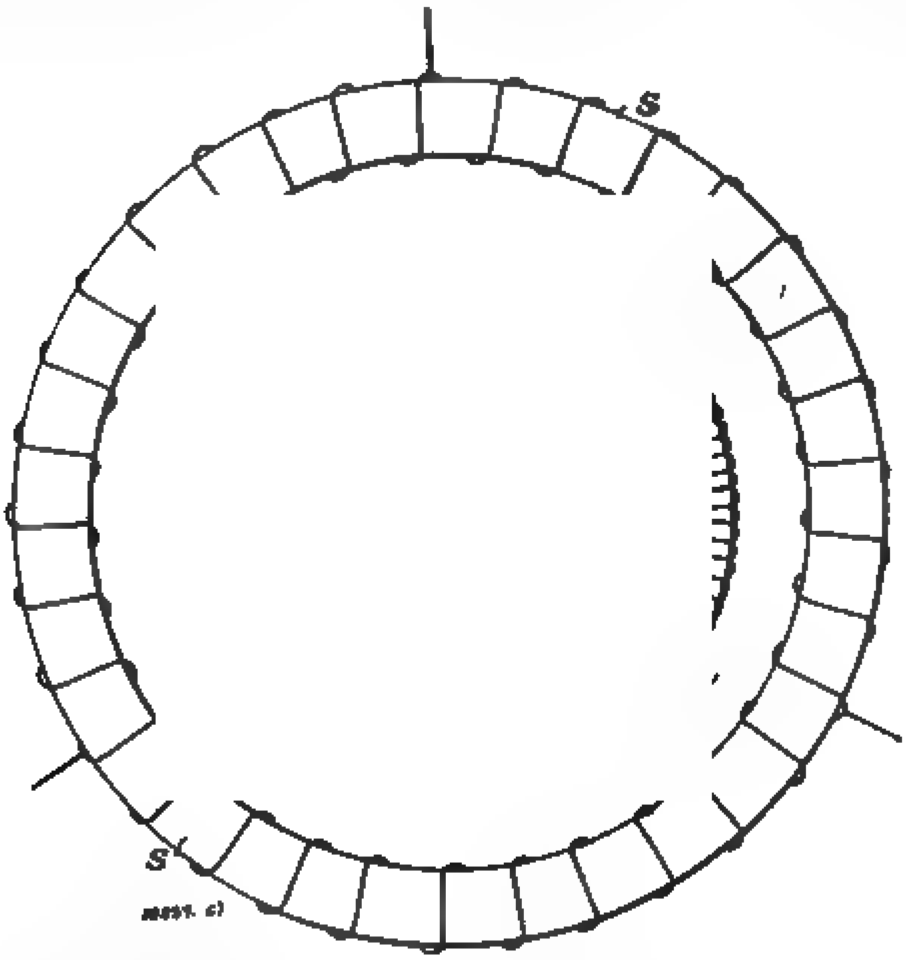


FIG. 334.—Suggested Method of Winding for Starting Induction Motors.

regard to the stator S. These two component windings are represented diagrammatically side by side. The winding A is permanently tapped off to a common connection from three points *a*, *a*, *a* (the diagram is for 2 poles; for larger numbers of poles there would be *three taps per pair of poles*). These connections, *a*, *a*, *a* for A, are the same at starting as for running. The independent winding B is tapped off at points *b*, *b*, *b*, corresponding with the points *a*, *a*, *a* of A, but has also a second set of taps from points *c*, *c*, *c*, at a suitable angular distance away from *b*, *b*, *b*, this angular distance being chosen to suit the conditions of starting torque required for each motor. The windings are shown as

of the gramme ring type merely for clearness in explaining the principle of the method. They would, of course, in practice generally be constructed as drum windings. The principle involved is, that when the rotor R is at rest, and the current is switched on to the primary P of the stator S, currents will be induced in the two component secondary windings A and B of the rotor R.

If these two component windings, A and B, were independently short circuited from corresponding points as *a*, *a*, *a* and *b*, *b*, *b*, the two sets of current-carrying conductors composing the two windings, A and B, would co-operate electro-magnetically in reacting on the system, and in the majority of cases the magnitude of this demagnetising reaction would be greater than that best adapted to secure the desired starting torque (in other cases there would be no need to have recourse to this device), but by making the common connection from the points *c*, *c*, *c* of the winding B, the points *c*, *c*, *c* are at the proper angular distance from the connections *a*, *a*, *a* of winding A, and therefore the resultant secondary reaction (of the two windings A and B) is decreased to the value most suitable for obtaining the desired electro-magnetic condition at starting. When sufficient speed has been acquired, the connections may, by any suitable device, be rearranged by opening the connection of the three leads *c*, *c*, *c*, and connecting together the leads *b*, *b*, *b*, and the motor will then be properly arranged for running.

Mr. C. S. Bradley has proposed for small motors to use an external field that may be shifted along its base so as to embrace either one of two armatures, or part of both. One of these armatures has a low resistance winding, and is suitable for normal conditions—namely, where small drop in speed is desirable; the other for starting—*i.e.* where considerable armature resistance is generally necessary.

Another way of accomplishing this result would be to have upon a single rotor a double, triple, quadruple, or higher order of multiple winding. One of these windings would be permanently short circuited, the others being short circuited one by one as occasion should demand.

Thus, take the case of a quadruple winding arranged with one conductor of each winding in each slot. This arrangement is indicated in Fig. 335, where the conductors of the four windings are denoted respectively by I, II, III, and IV. Suppose that at starting the complete component winding represented by the

conductors I should be short circuited, the others being on open circuit, then the armature resistance would be four times as great as when, after attaining speed, all four systems are short circuited. It is, therefore, equivalent to having an external resistance three times as great as the armature resistance. By this plan the method of starting would be to successively short circuit the armature windings, analogous to the way in which internally mounted resistance (see Figs. 299 and 300, pages 250 and 251) has been arranged to be gradually cut out of circuit. The concentration of the resistance in the conductors themselves overcomes the mechanical difficulties associated with suitably arranging resistances inside the spider of the rotor. Where only one step is required at starting, all the other component windings would be coupled permanently in series, or parallel, ready to be short circuited at the suitable speed. Were the conductors of that winding which is always in circuit made of some material of higher resistance

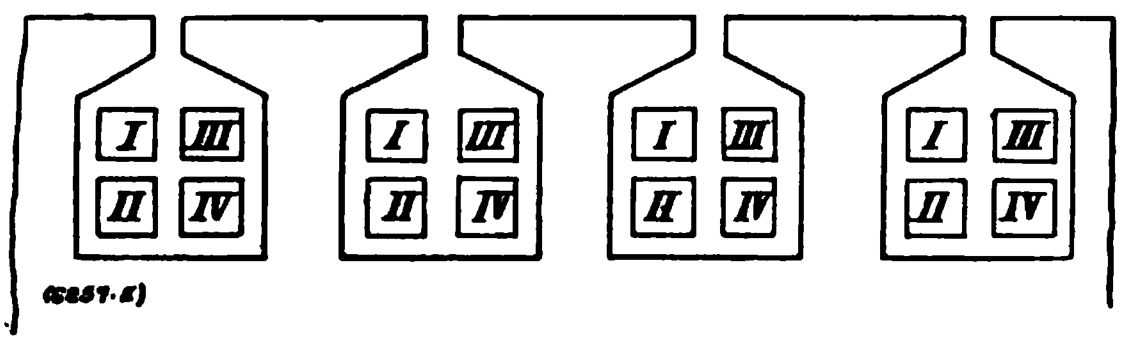


FIG. 335.—Multiple Winding on Rotor.

than copper, or of smaller cross section, the increased specific rate of generation of heat would only occur during starting, for when the other windings were connected in for permanent running, the current would be shared in proportion to the conductivity of the conductors in parallel. The heat thus generated during starting is as easily, or more easily, dissipated from the conductors at the periphery as from internally located auxiliary rheostats.

§ 9. **Utilisation of "Skin Effect"[1] in Conductors for Automatically Starting Induction Motors.**—The periodicity in the rotor is, at the moment of starting, equal to the full periodicity of the circuit, falling, as speed is acquired, to the low

[1] The following references relate to "skin effect" in conductors:—Gray, *Absolute Measurements in Electricity and Magnetism*, vol. ii., page 329; *Proceedings of the Institute of Electrical Engineers*, 1889, vol. xviii., No. 77, page 16; *Electrical World*, 1893, vol. xxi., page 300; *Transactions of the American Institute of Electrical Engineers*, 1893, April 18th; *Weidemann's Annalen*, vol. liii., page 1353; London *Electrician*, October 12th, 1900, page 920.

value corresponding to the slip. The writer has proposed (British Patent No. 8476 of 1900) to utilise this property for obtaining the requisite starting torque by means of the skin effect in conductors (more especially those of magnetic material) of fairly large diameter. In virtue of the high periodicity at starting, these conductors offer much higher resistance to the flow of the induced current than after the normal speed has been attained, the resistance then not being appreciably in excess of the true ohmic resistance. For squirrel cage motors, the end rings could be of massive wrought iron. Even for wound motors with slip rings and externally located wrought-iron starting resistance there

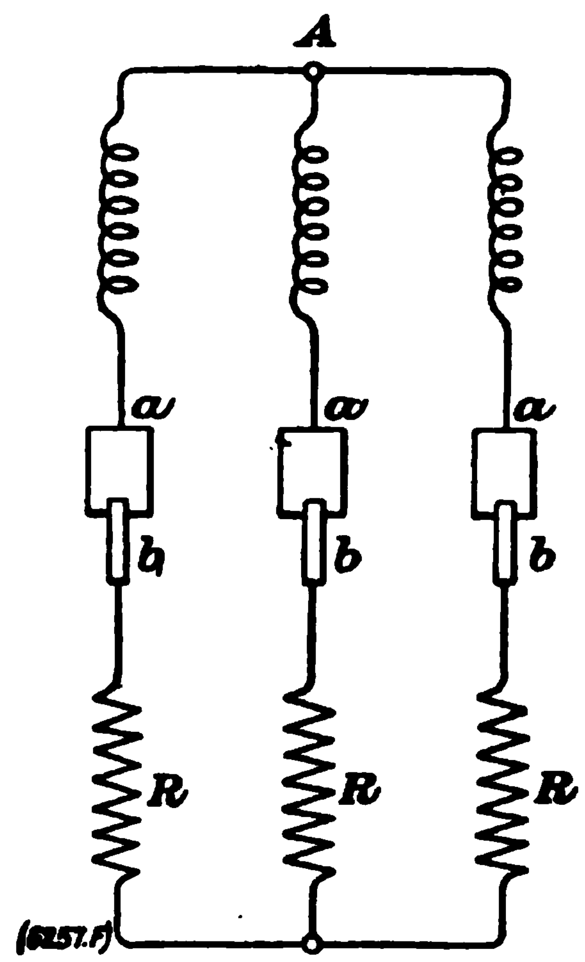


FIG. 336.—Suggested Method of Connection for Utilising "Skin Effect."

should sometimes be an advantage in the case of motors located at a distance and not conveniently accessible to attendance, as the apparent resistance itself decreases to a very small amount as the motor increases in speed, and if suitably proportioned, the resistance, at full speed, may be arranged to be sufficiently low not to be required to be cut out. When the *stator* is the secondary, and sliding contacts for the secondary currents are thus avoided, the winding should be for low voltage and large amperage in order to obtain the greatest advantage from this method.

The plan is diagrammatically illustrated in Fig. 336, in which A is the rotor of an induction motor which is provided with any suitable winding, that illustrated being a Y-connected three-phase winding, the terminals being connected to the three

collector rings *a*. The secondary currents induced in the rotor are led by means of the three slip rings and the three brushes *b* bearing thereon, to three resistances, R, these resistances being brought to one common connection, as shown. Wrought iron is an especially suitable material for these resistances.

When connection is made from the supply circuit to the primary windings of the induction motor, there are induced in the secondary circuits currents of the same periodicity as in the primary circuit, and by suitably proportioning the cross section of the conductors containing the resistances R, and by making these of suitable material, these resistances will, at the impressed periodicity, oppose to the flow of these secondary currents an apparent resistance much greater than the resistance that they would oppose to the flow of low periodicity currents; hence the rotor has sufficient torque.

In proportion as the motor gathers speed the periodicity of these induced secondary currents decreases, and at normal speed is very low, and the resistance opposed to the flow of these secondary currents is not appreciably in excess of the true ohmic resistance, and if the conductors R are of suitably large cross section, they practically short circuit the rotor windings, which is the most efficient condition for normal running. This, it will be observed, has been accomplished without any change whatever in the circuit connections, and constitutes one of the most important features of the method.

This seems to the writer to offer a means, altogether free from any mechanism whatsoever, of automatically starting a motor with a high "specific" torque, *i.e.* a high torque per primary ampere. The chief objection is the bulkiness of a rheostat constructed on this principle, but it is merely a question of cubical dimensions; there is nothing expensive about its construction. A moderate amount of the same effect could be obtained with a squirrel cage rotor with the end rings or the radial connections (see Figs. 287 to 290, pages 243 and 244), or, in certain cases, the conductors themselves, proportioned with regard to the principles above described.

It may occasion surprise that the "skin effect" in conductors is enough to be effective in this connection. The following results of tests carried out under the writer's direction by Mr B. Hopps in 1899 are, however, fairly conclusive on this point, and afford useful data of this alternating current phenomenon, which is capable of numerous useful applications.

The tests were made upon straight lengths of iron wires and rods of ordinary value, and, to ensure a fair average, special care was taken to obtain them from various manufacturers. A known current was passed through the rod, the volts drop due to a known length being measured by means of a low-reading hot-wire voltmeter. The results are set forth in Table XLII., and are plotted in Figs. 337 to 339.

TABLE XLII.—RESULTS OF TESTS OF IRON CONDUCTORS.

Diameter of Rod.	Length on which Measurements were made.	Amperes.	Volts.	Apparent Resistance in Ohms.	True Resistance in Ohms.	Cycles per Second.	Michroms per Cubic Inch.
In.	Ins.						
0·975	156	91·2	1·15	0·0126	...	45	60·25
		96	1·39	0·0145	...	67	69·4
		91·4	1·69	0·0185	...	102	88·5
		14·3	0·0163	...	0·00114	0	5·45
0·77	120	96	1·09	0·0114	...	44	44·2
		97·8	1·30	0·0132	...	64	51·2
		96·9	1·56	0·0161	...	100	62·4
		23·6	0·0325	...	0·00138	0	5·45
0·505	120	70·5	1·12	0·0159	...	45	26·5
		76·4	1·40	0·0183	...	60	30·5
		63·5	1·51	0·0238	...	100	39·7
		20·6	0·0775	...	0·00376	0	6·27
0·390	132	63·5	1·39	0·0219	...	45	19·8
		51·8	1·46	0·0282	...	66	25·5
		55·5	1·88	0·0339	...	100	30·7
		17·4	0·116	...	0·00667	0	6·03
0·278	144	39·5	1·41	0·0357	...	47	15
		36·6	1·57	0·0429	...	65	18·1
		40·8	2·07	0·0507	...	99	21·4
		7·7	0·104	...	0·0135	0	5·69
0·157	96	31·8	1·24	0·0389	...	45	7·84
		26·8	1·27	0·0474	...	66	9·56
		27·2	1·54	0·0566	...	100	11·4
		4·55	0·119	...	0·0262	0	5·28
0·105	120	15·6	1·47	0·0943	...	47	6·8
		15·75	1·63	0·01033	...	67	7·47
		21·8	2·31	0·01060	...	100	7·65
		19·8	1·5	...	0·0758	0	5·47

These tests were followed by comparative tests on a three-phase induction motor, using as starting resistance in the rotor winding:—

(1) An "iron grid" resistance as supplied with the motor.
(2) Wrought-iron rods of suitable length and diameter.

The induction motor was rated at 5 horse-power, at 50 cycles, 1500 revolutions per minute, 220 volts, and 15 amperes.

The rotor winding terminated at three slip rings, from which connections were made to the starting devices.

The resistance of the "rotor" winding between slip rings was 0·0181 ohm and of the stator between terminals 0·0611 ohm.

Figs. 337 and 338.—Curves of Tests on Iron Conductors.

The starting resistance supplied with the motor was of the "cast-iron grid" type. It consisted of three equal grid resistances,

* The two dotted curves in Fig. 338 are drawn in to indicate the possibility that the increase in the "skin effect," with increasing frequency with iron rods, while at first rapid, is ultimately at a much lesser rate. The tests were much too crude to lead to any estimate of the law of variation of the "skin effect" with the frequency.

connected in star by a triangular switch arm, and had ten steps, of which the following are the resistances per phase:—

Step Number.	Resistance at 20° Cent.	Step Number.	Resistance at 20° Cent.
Short Circuit	...	6	0·12
1	0·0213	7	0·134
2	0·0317	8	0·162
3	0·0462	9	0·194
4	0·060	10	0·249
5	0·072		

Resistance measurements were also made at 40 cycles, and were found to practically correspond to the above figures, the inductance thus being negligible.

This resistance constituted the first starting device.

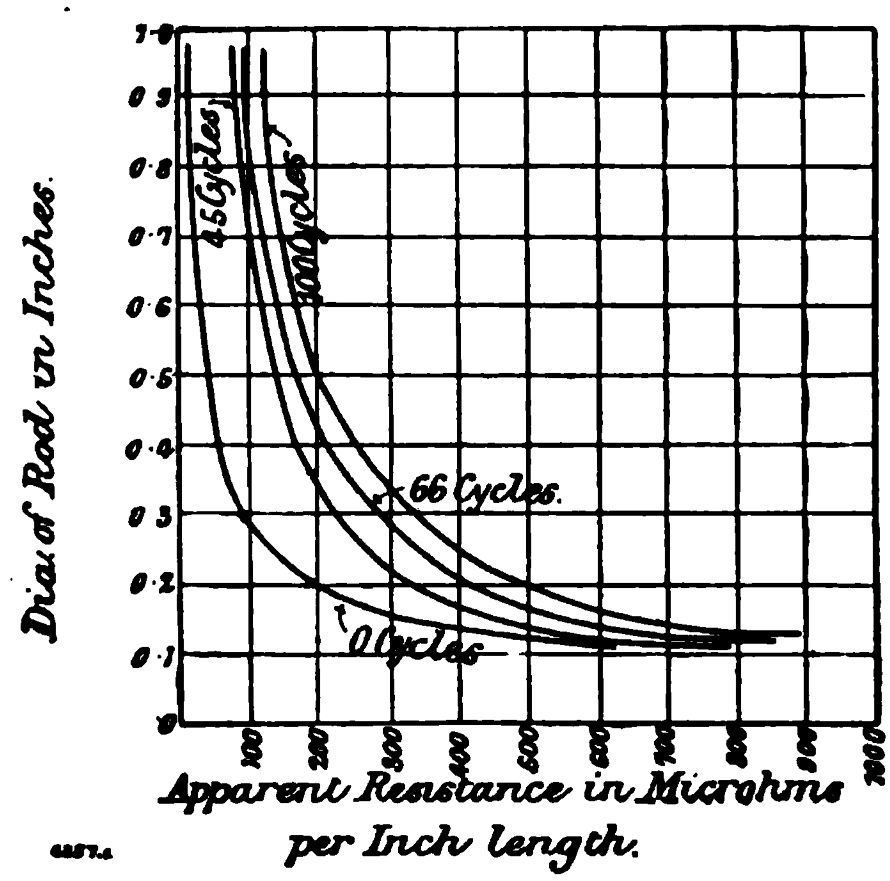


Fig. 339.—Curves of Tests on Iron Conductors.

The second starting device was composed of nine 14 ft. lengths of commercial wrought-iron rod 1 in. diameter, divided up into three equal lengths and connected star. Connections between the rods were made by soldering on their ends copper connection lugs of 400 amperes capacity, and bolting the contact surfaces of these lugs together, the resistance of such a joint being approximately 0·00008 ohm.

Tests were made, using three, two, and one length of rod per phase.

A test was made on one length of the iron without a joint, from which the specific resistance was computed to be 5·45 michroms per cubic inch. A specimen of iron was cut from a length, the permeability of which, as measured by Ewing's permeability bridge, was:—

H.	B.	μ
1·414	2,400	1,700
1·79	4,450	2,490
2·2	6,100	2,770
2·80	7,950	2,840
3·93	10,100	2,580
4·73	11,000	2,330
7·12	12,520	1,760
8·5	13,000	1,530
12·3	13,950	1,130
23·4	14,920	624
40·6	15,701	387
85·5	16,850	197
113	17,300	153
139·5	17,740	127

The following measurements were made of the apparent and real resistance of the various rods per phase:—

THREE RODS PER PHASE (Length = 42 ft.).

	Ohm.
Real resistance at 0 cycles of rods, contacts and leads	0·00479
Apparent „ 40 „ „ „	0·0436

TWO RODS PER PHASE (Length = 28 ft.).

Real resistance at 0 cycles of rods, contacts and leads	0·00365
Apparent „ 40 „ „ „	0·0303

ONE ROD PER PHASE (Length = 14 ft.).

Real resistance at 0 cycles of rods, contacts and leads	0·00234
Apparent „ 40 „ „ „	0·0158

Three-phase currents were supplied by a small experimental generator at about 80 volts on the terminals of the motor. With this low voltage, it was safe for the supply machine, in spite of its small capacity, to start the motor on short circuited slip rings.

The torque was measured at the periphery of the pulley by means of a spring balance. The torque of the rotor was not uniform during one revolution; measurements were therefore made with the rotor always in the same position relative to the stator. At the position of the rotor selected, the line amperes and volts were balanced in the three phases.

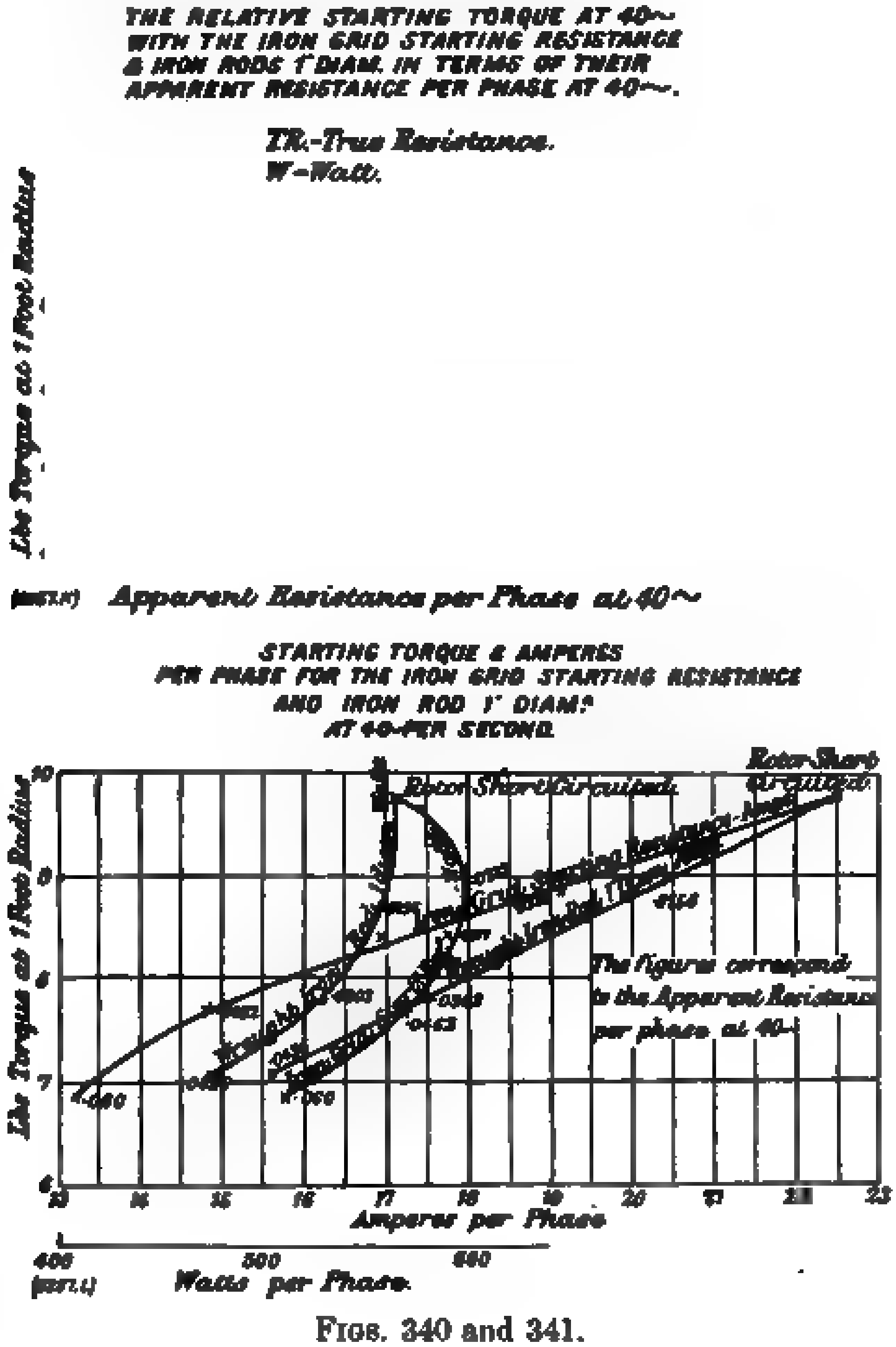
The measurements set forth in Table XLIII., page 284, were made at 40 cycles per second, this being the highest periodicity to be conveniently obtained from the experimental generator.

TABLE XLIII.—RESULTS OF STARTING TESTS WITH IRON ROD STARTING RHEOSTAT.

$\frac{W_1}{W_2}$				Lbs. per Hektowatt at 1 ft. Radius.	
0·523	33·8	9·78	0·435	1·73	Short circuited slip rings.
0·690	24·5	7·06	0·454	1·49	42 ft. of 1 in. rod per phase. Apparent resistance = ·0436 ; true resistance = ·00479.
0·673	27	7·80	0·446	1·47	28 ft. of 1 in. rod per phase. Apparent resistance = ·0303 ; true resistance = ·00365.
0·617	30·2	8·75	0·433	1·56	14 ft. of 1 in. rod per phase. Apparent resistance = ·0156 ; true resistance = ·00234.
0·692	30·06	8·85	0·463	1·48	Step 1 iron grid rheostat. True resistance = ·0213.
0·754	29·16	8·42	0·496	1·43	Step 2 iron grid rheostat. True resistance = ·0317.
0·825	26·66	7·7	0·519	1·35	Step 3 iron grid rheostat. True resistance = ·0462.
0·818	23·73	6·86	0·520	1·34	Step 4 iron grid rheostat. True resistance = ·060.

The curves of Fig. 340 show the relative starting torque at 40 cycles, with the iron grid starting resistance and iron rods 1 in. diameter, in terms of their apparent resistance per phase at 40 cycles.

The curves of Fig. 341 show the starting torque and amperes



Figs. 340 and 341.

per phase for the iron grid starting resistance and iron rod 1 in. diameter, at 40 cycles.

The curves of Fig. 342 show the relation of watts and volts × amperes per phase for the iron grid starting resistance and iron rod 1 in. diameter, at 40 cycles.

The tests were made in considerable haste with crude arrangements, but the results indicate that the method is sound, and

could in certain cases be used to decided advantage, on account of its inherently automatic nature. With the exception of the Fischer-Hinnen device, subsequently to be described, all others require some moving part, often centrifugally actuated. This method probably gives a higher specific starting torque, and, with normal running, a better power factor and overload capacity than that of Fischer-Hinnen's, though the test results are too crude to satisfactorily decide this point.

Lindstrom, in a Swedish patent, No. 10,484 of 1899 (date of application June 6th, 1899, granted December 23rd), connects reactance spools permanently in the secondary circuits of an induction motor for the purpose of decreasing the current at starting, through the increased reactance of the secondary, which at the

THE RELATION OF WATTS & VOLTS X
AMPERES PER PHASE FOR THE IRON GRID
STARTING RESISTANCE & IRON ROD 1"DIAM.
AT 40~ PER SECOND.

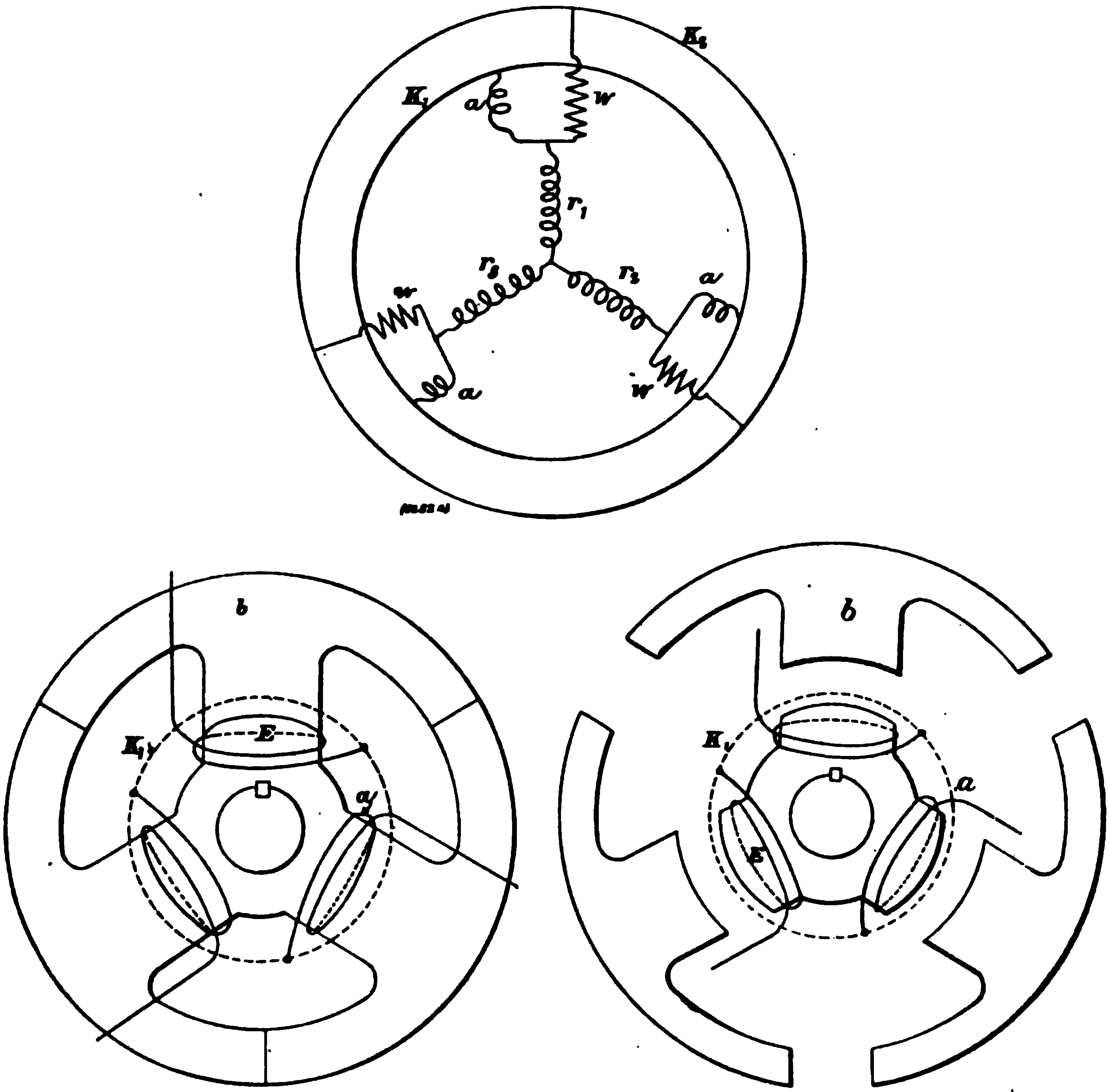
FIG. 342.

moment of starting is exposed to the periodicity of the line. Fischer-Hinnen, in Austria, obtained a patent in 1900 for starting induction motors by means of reactance coils in the secondary circuits (see Fig. 353, page 292), to be described later.

§ 10. **Zani's Use of Inductance Coils to Limit the Starting Current.**—A patent to Mr A. P. Zani, D.R.P. No. 105,986 of 1899, describes a method where inductance coils limit the starting current, and which provides for subsequently automatically decreasing the reactance of these auxiliary coils by increasing the reluctance of the magnetic circuit on which these coils are wound.

Mr Zani's interesting and very ingenious method was devised with the view of producing an automatic starting arrangement for induction motors adapted for a fairly large starting torque, and

which shall not involve an undue inferiority to a squirrel-cage motor as regards simplicity, and which shall nevertheless be far superior to the latter in requiring much less current for a given torque at starting. In a motor equipped with the Zani device we have no changes whatever in the electrical connections of the



Figs. 343, 344 and 345.—Zani Method, with Inductance Coils to Limit Starting Current.

rotor, hence any possibility of sparking is excluded. The only change which takes place in the rotor in starting, after the main switch is closed, is of a magnetic nature resulting from the movement of solid pieces of iron.

The device will be understood by referring to Figs. 343 to 349. In series with each phase of the rotor winding *r* (Fig. 343) we

have a spool *a*, and in parallel with it the regular non-inductive starting resistance *w*. The ends of the spools and of the resistances are brought respectively to the short-circuiting rings K_1, K_2. The spools *a* are situated upon a limb E (Figs. 344 and 345), which is keyed to the shaft of the rotor, and which has projecting poles continued by the pieces *b*, thus forming magnet and yoke. The pieces *b* are guided radially by two brass pins *f* (Figs. 346 and 347, Plate 20), allowing a movement of about 20 millimetres before striking against the flanges of the rotor.

When the motor is at rest the pieces *b* are pressed by the springs *c* (Figs. 347 and 348) against the armature E, the three segments coming at the same time in contact with each other, and completing with the armature E the three magnetic circuits of the spool *a*, which thus have a very high impedance.

The operation of the apparatus is as follows: At starting, the impedances of the spools *a* are large compared with the resistance *w*, and the current flows chiefly through the resistance, thus giving a high starting torque.

As soon as the motor has reached a predetermined speed, the magnetic attraction of the segments towards each other and towards the armature E, and the tension of the springs *c*, will be overpowered by the centrifugal force of the pieces *b*, which will fly apart in a radial direction, striking against the flanges of the rotor. The reluctance of the magnetic circuit of the spools *a* is thus greatly increased, due to the introduction of the air paths between the three pieces *b* and between them and E, and the impedances of the spools become very small compared with the starting resistances *w*, which are thus practically short circuited.

The mechanical construction of the apparatus may be seen from the illustrations, which represent a 15 horse-power rotor equipped with the device.

The spools *a* are kept in place against the centrifugal force by the spool holder D (Figs. 346 and 348, Plate 20), in form of a star, and on the other side by the short-circuiting ring *k*, to which they are riveted. No further support is required for the spool, since it is very rigid (consisting in this particular case of six turns of copper bar 28 millimetres × 4 millimetres). The rotor illustrated is in all other respects normal, and of the same type employed for using internal resistances, as in Fig. 301, page 251.

The results of comparative tests of this 15 horse-power motor, against an otherwise identical motor with slip rings, are shown in

FIG. 346.—Component Parts of Zani Starting Device (see page 288).

FIG. 347.—Zani Starting Device, Partly Assembled (see page 288).

FIG. 349.—Zani Starting Device in Place within the Rotor (see page 287).

FIG 348.—Zani Starting Device. Completely Assembled (see page 288).

the curves of Figs. 350 and 351. The motor had 6 poles, and was for 500 volts at 50 cycles, thus running at a no load speed of 1000 revolutions per minute. It will be seen from these results that the maximum output is about the same in both cases. The efficiency is some 3 per cent. higher in the Zani motor, due to the avoidance of slip ring friction and the decreased resistance of the rotor, which in the Zani motor is permanently and solidly short circuited, whereas, in the other case, it is short circuited through a switch. The Zani motor had a 1 per cent. lower power factor.

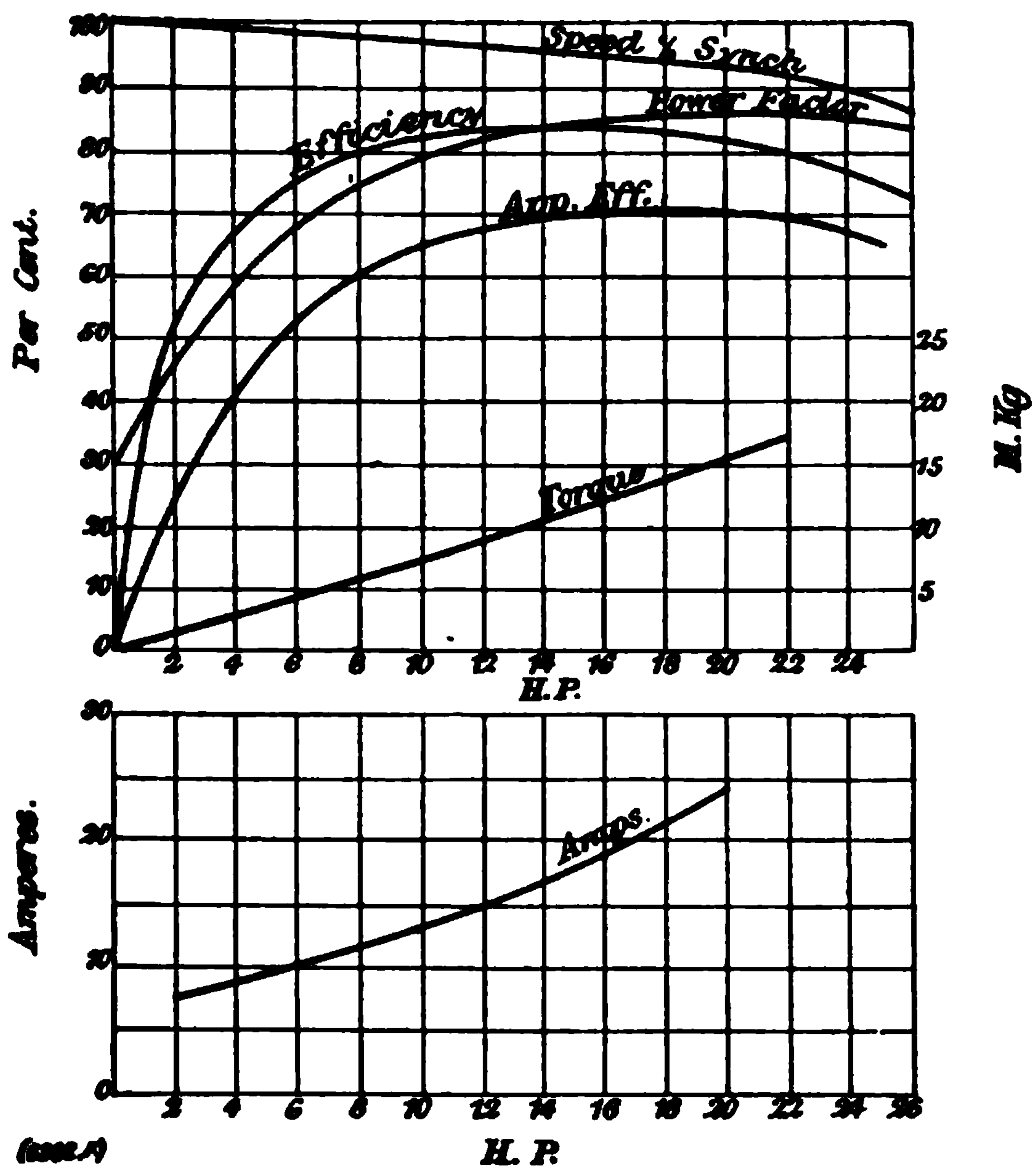


FIG. 350.—Test Curves of Standard Motor with Slip Rings.

It is stated that where the torque required is not more than 1·5 of normal, the resistances w will be found unnecessary, the required energy current in starting being given by the hysteresis, and the eddy currents in the iron of b and E, which would be made solid. This, besides cheapening the construction of the starting device, has the advantage that the energy losses in the rotor in starting do not decrease proportionally to the square of the decrease in the frequency in the rotor circuit, as would be the case with a constant resistance in series, but less rapidly, being the resultant losses from eddy currents, which decrease proportionally

with the square, and of hysteresis, which decreases proportionally with the first power of the decrease in frequency, the density in the iron being assumed constant. This should lead to a more uniform starting.

The starting current is for small motors some 30 per cent. higher than with starting resistance in the rotor for the same torque. In large motors it is about 20 per cent. higher. An important feature of this method, especially for starting at a distance,

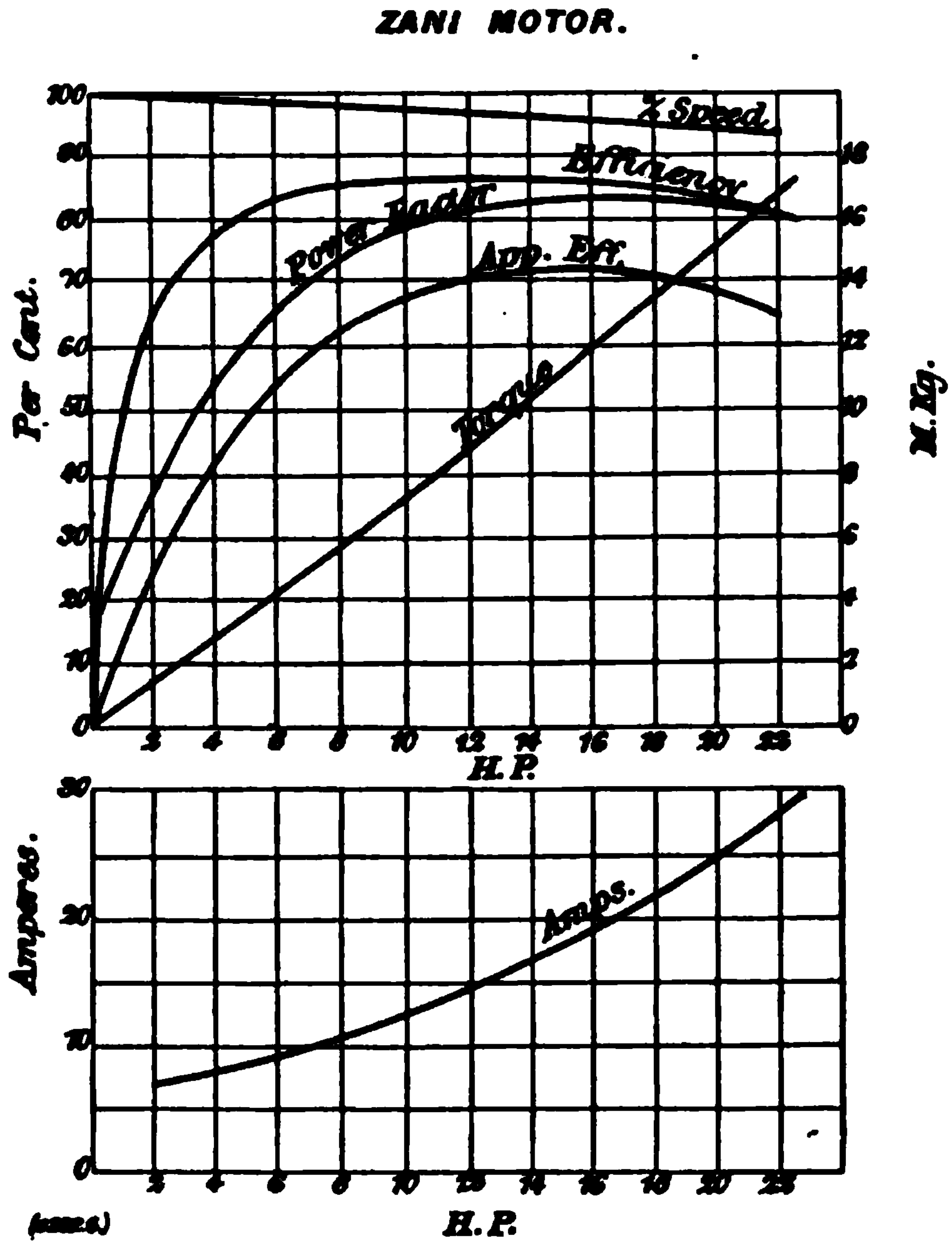


FIG. 351.—Test Curves of 15 Horse-power Zani Motor.

or where high potentials are used, lies in the simplicity of starting, which consists merely in closing the main switch.

Mr Zani has obtained good results in employing this starting device in motors for the fairly high frequencies of from 40 to 60 cycles per second. For 25-cycle motors it becomes difficult to make the device sufficiently compact to be brought within the rotor, since not only are 25-cycle motors of smaller diameter, but a given reactance can only be secured by a larger device than at higher periodicity. The device should be serviceable in cases where the air is charged with highly explosive gases, and where absolutely safe working is required with simplicity of operation

and high "specific" starting torque. A Zani motor has the characteristic of being self-protecting against overloads. If the motor pulls up on account of an overload, the current can evidently not increase above the normal starting current, since the three movable iron pieces are attracted with sufficient force to close the magnetic circuit should the springs opposing the centrifugal force fail to act. Of course, in running up the motor, when the movable pieces fly apart, the starting current suddenly

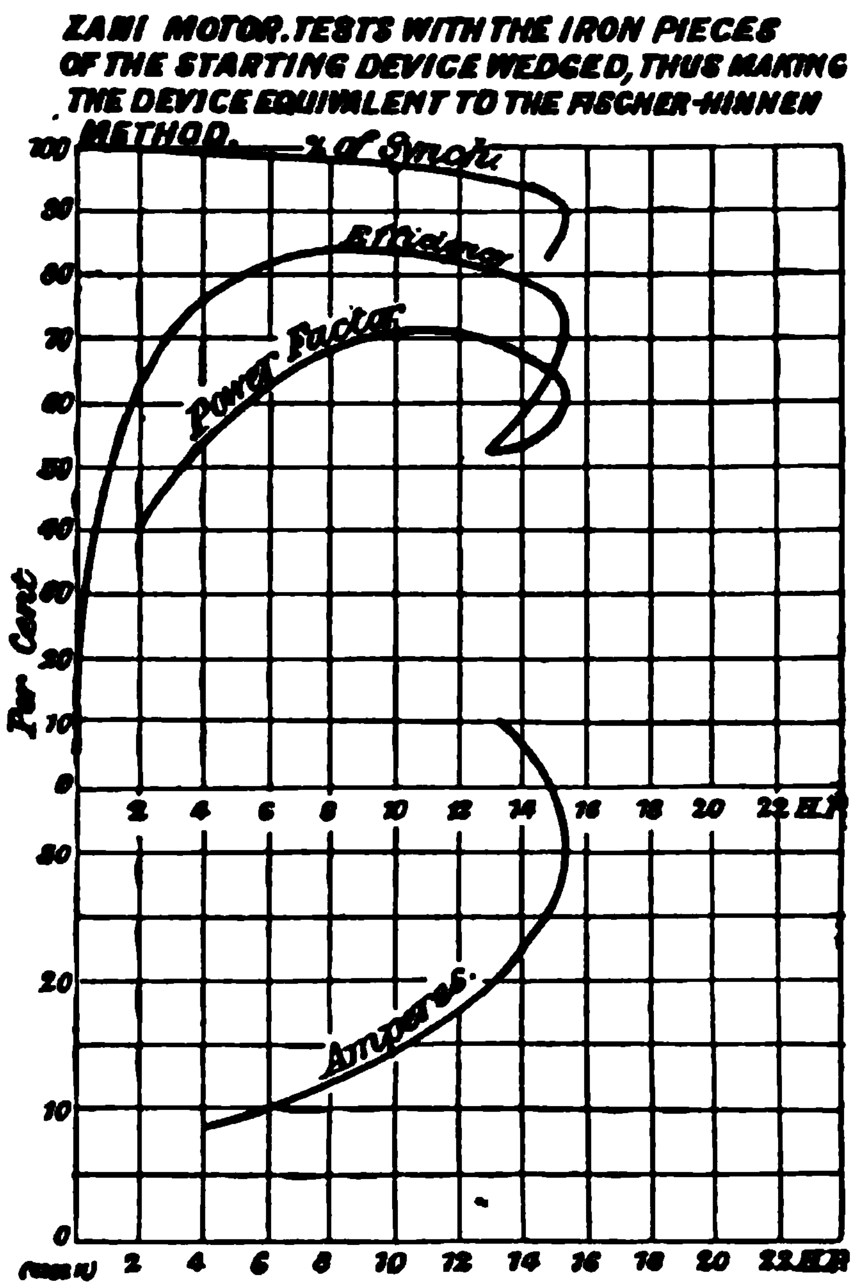


FIG. 352.—Test Curves of 15 Horse-power Zani Motor specially arranged to investigate the Fischer-Hinnen Method.

increases to the value attained in a short circuited armature at the corresponding speed.

§ 11. **Fischer-Hinnen's Method.**—The curves of Fig. 352 represent the results of tests on this same motor, but with the iron pieces of the starting device wedged so as not to be able to open with increased speed, thus keeping the magnetic circuit closed, and realising a starting device with constant magnetic reluctance, such as that described in the Fischer-Hinnen patent above referred to. It will be observed that the maximum output and power factor are very much decreased.

Fischer-Hinnen's method—Hungarian patent No. 6308—is described in vol. xxiv. of *L'Eclairage Electrique*, July 28th, 1900, page 131, from which Fig. 353 is taken. Having a fixed magnetic

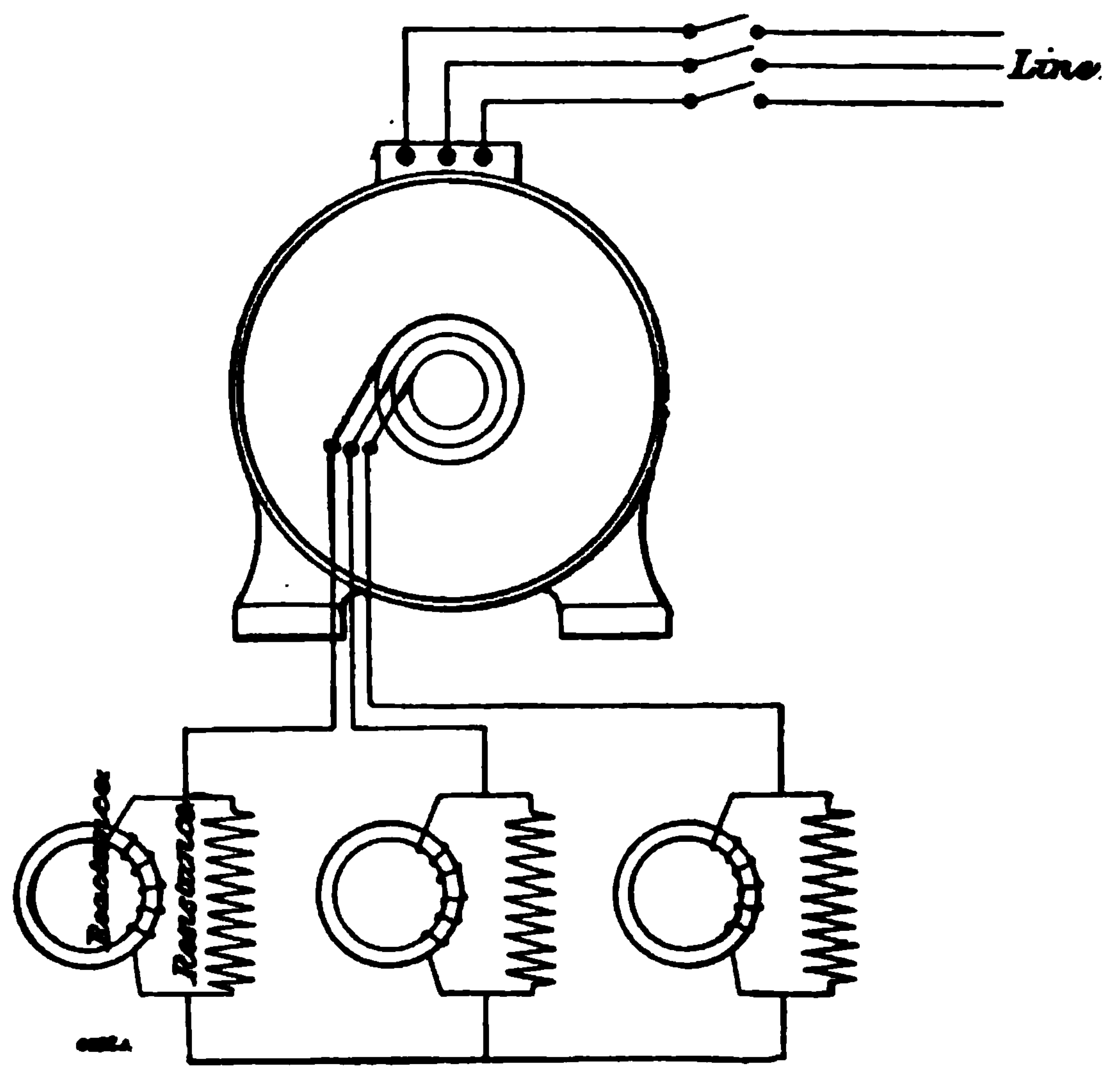


FIG. 353.—Diagram of the Fischer-Hinnen Method.

reluctance, the inductance during permanent running cannot be decreased to so great an extent as by the Zani method, and this accounts for the less satisfactory test results of Fig. 352.

CHAPTER XIV

COMPARISONS BETWEEN INDUCTION AND CONTINUOUS CURRENT MOTORS

§ 1. **Comparative Data of Constant Speed Motors.**—Having given a general idea of the various types of induction motors, and of the methods employed for adapting them to different classes of work, it may be well, before examining closely into their theory, to give some curves, data, and other comparisons between induction motors and continuous current motors. The curves in Fig. 354, page 294, were contributed by Mr Gano S. Dunn to the discussion of Mr C. F. Scott's paper on "Alternating Currents as a Factor in General Distribution for Light and Power" before the American Institution of Electrical Engineers (*Proceedings*, p. 872, vol. xviii., 1901), to show the relative performance of so-called "constant speed" motors of the induction and continuous current types. The curves for the induction motors were made from data from the publications of one of the prominent companies constructing such machines. The curves for the direct current motors were from tests, made especially for this comparison, under Mr Dunn's personal supervision, and were from ordinary commercial machines of standard commercial rating, drawn directly from the storehouse and unmodified in any particular. Mr Dunn explained that "the three sets of curves show the performance of the motors with respect to running current, efficiency, and speed. The abscissæ are torques expressed in percentage of full load running torques, and the heavy line at 100 per cent. represents normal full load for all the motors. The ordinates are current, efficiency, and speed, also expressed in percentages for comparison. The speed and current tests of the direct current motors were made by applying a Prony brake to the motor running free, and gradually tightening it until the motor stalled.

"It will be noted that at full load the running current of the induction motor is about 10 per cent. greater than that of the

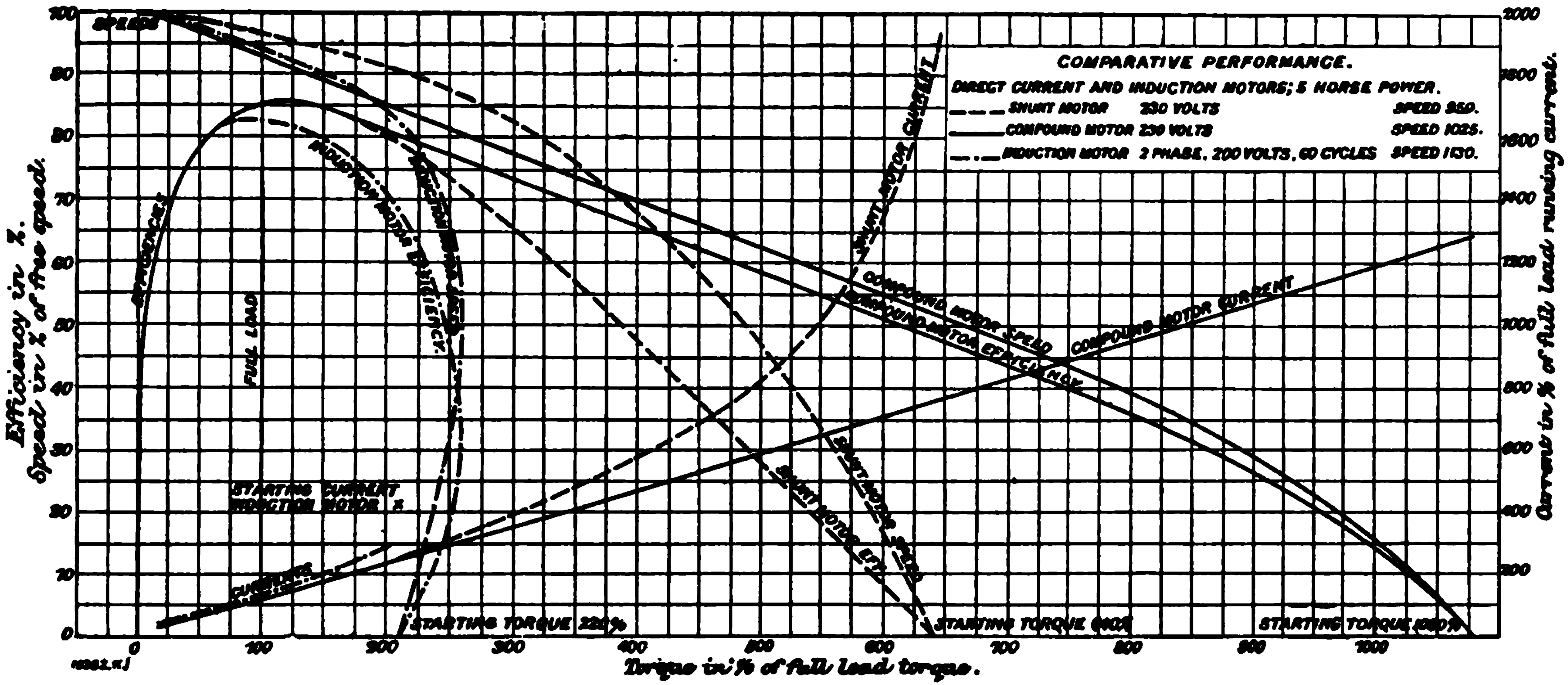


FIG. 354.—Comparative Performance of 5 Horse-power Continuous Current and Induction Motors.

direct current motor. The starting current for the induction motor is at the mark x, and is nearly twice as much as required to give the same torque in the direct current motors. The efficiencies of all the motors are about the same up to two-thirds load, after which the efficiency of the induction motor falls off very rapidly, while those of the direct current motors are tolerably well maintained. The speed curves, which originate in the upper left-hand corner, show that at full load the shunt motor falls off about 4½ per cent., the induction motor about 6 per cent., and the compound motor about 7½ per cent. At 2·6 times normal torque, the induction motor, however, stalls, while both of the direct current machines continue to run, the shunt motor until its torque reaches over six times normal torque, and the compound motor more than ten times normal torque. There was sparking in the middle ranges of the speed curves of the direct current motors (not such as would injure the commutator unless the severe overloads were maintained), but beyond this, at the points of severest overloads, the sparking almost disappeared, because the speeds were so low that the commutation reactance was greatly reduced."

Mr Dunn justified the inclusion of the compound motor in this comparison of "constant speed motors," since its speed regulation is 7½ per cent., and its use for so-called "constant speed" work is rapidly increasing. He also pointed out that these curves beyond 2·6 times normal torque comprise very important features which cause continuous current motors to be appreciated for steel mills and similar establishments where overloads are frequent and severe. For induction motors to give equivalent service much larger sizes would have to be resorted to, and this, together with the fact that the cost per horse-power is higher, would make the cost of installation very much greater. It is the unfavourable characteristics of induction motors in this extended region of the curves which has thrown the elevator business into direct current.

After pointing out that speed control, in which the induction motor is admittedly inferior, is a very important element in the constitution of power plants to-day, and that we must not think that in the future we shall be contented with fixed speeds, when we can have motors that will give us speeds controlled with perfect efficiency and stability throughout wide ranges, Mr Dunn summed up to the effect that the continuous current motor may require a little closer attention, but it is a great deal cheaper, and can do a great deal more.

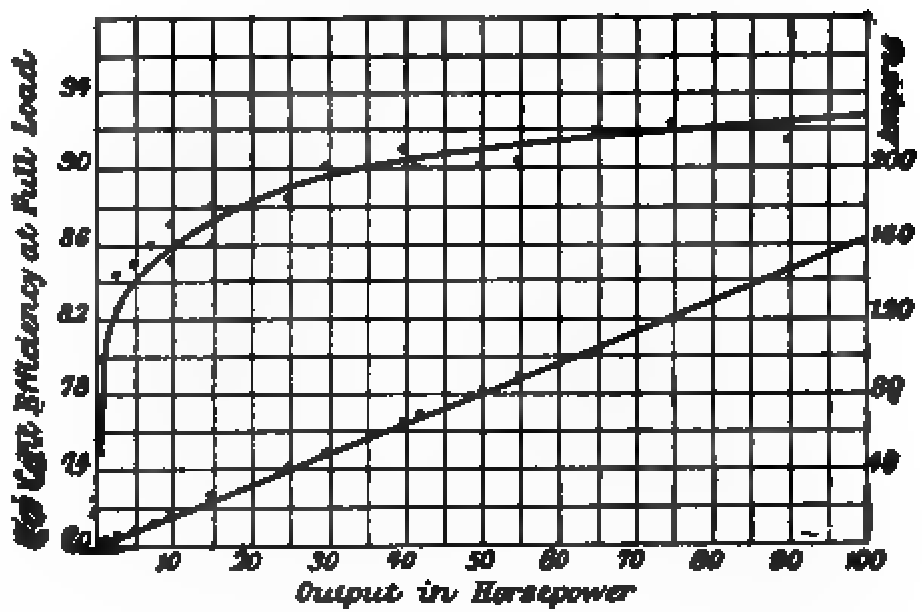
In the course of this discussion Mr Steinmetz emphasised the superiority of continuous current distribution for large low-tension networks in densely-populated districts, and maintained that the true field for alternating current supply networks is in small towns and in the outlying districts of large cities. The weight of opinion on this occasion was to the effect that alternating current was generally superior for generation and transmission, but inferior for the low-tension distributing network, at any rate in densely-populated districts.

§ 2. **Comparison of English and American Induction Motors.**—Professor W. E. Goldsborough also contributed to this discussion some interesting curves comparing induction motors with continuous current motors. The motors compared were of the so-called "moderate speed" rating, it being stated that for this rating the speeds are nearly the same for the two classes. The absence of precise information as to the speeds of the motors compared detracts somewhat from the value of his data.

For a range of ratings from 0 up to 100 horse-power the efficiency and current consumptions are given in Figs. 355 and 356, page 297. The curves of Fig. 355, it is stated, have been plotted from tests made upon machines which are the output of three of the best American factories. In Fig. 356 the results are separated into those for motors of three classes, namely, 25-cycle and 60-cycle American induction motors, and 50-cycle European induction motors. The relation to horse-power of power factor and of apparent efficiency at full load in these three groups of induction motors are given in Figs. 358 and 359. Professor Goldsborough also stated that from these curves it appears, in so far as American motors are concerned, that standard construction gives rise to higher efficiency in low frequency machines than in high frequency machines. The European motors show uniformly a higher full load efficiency than the American motors, which might seem to be a point of superiority, but by reference to Fig. 358 it will be seen that these European motors have also a considerably lower full load power factor than the American motors. We cannot have both high full load efficiency and high power factor, and the American engineer appears to have decided in favour of the latter and the European in favour of the former.

"Fig. 359 shows also that the apparent efficiencies of the 25-cycle American motors and the 50-cycle European motors follow practically the same variations with change of capacity (the American motor has power factor relatively high and apparent

EFFICIENCY AND CURRENT CONSUMPTION OF CONTINUOUS CURRENT MOTORS.



EFFICIENCY AND CURRENT CONSUMPTION OF

Per Cent Efficiency at Full Load

Output in Horsepower.

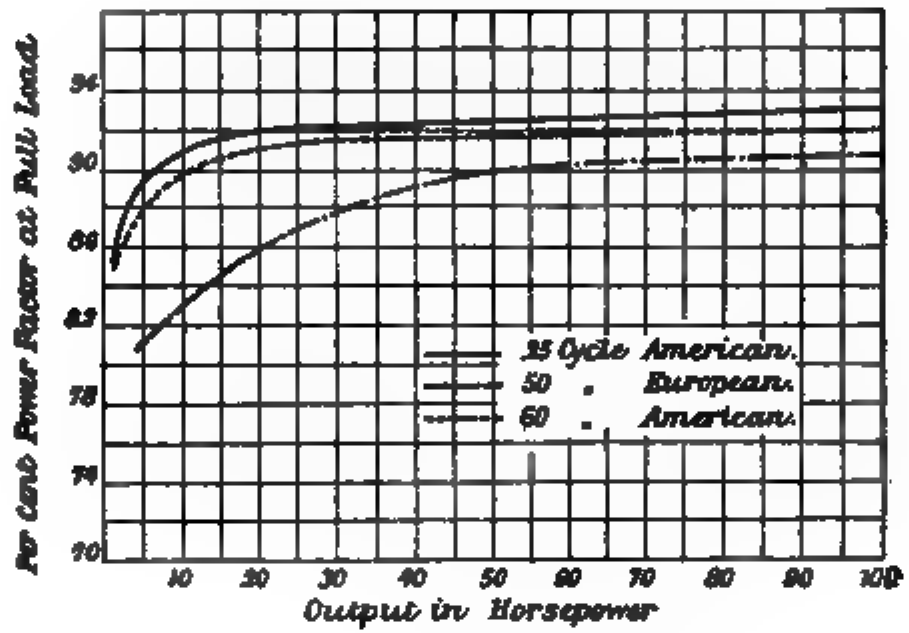
FIGS. 355 and 356.

RELATIVE EFFICIENCY & CURRENT CONSUMPTION OF

Per Cent Efficiency

Output in Horsepower

FULL LOAD POWER FACTORS OF INDUCTION MOTORS.



FIGS. 357 and 358.

efficiency relatively low). The apparent efficiency of an induction motor is important. The capacity of prime movers is usually gauged by the actual power required to operate the motors. The capacity of electric generators supplying power to induction motors cannot, however, be rated in this way. Their capacity is fixed on the basis of the apparent watts, and not on that of the real watts taken by the motors, since they are required to supply the necessary voltage as well as the necessary current. Their armatures must, therefore, be large enough to prevent the lagging currents in the system from causing overheating."

A direct comparison of the continuous current motors and induction motors is then made by means of the curves of Fig. 357,

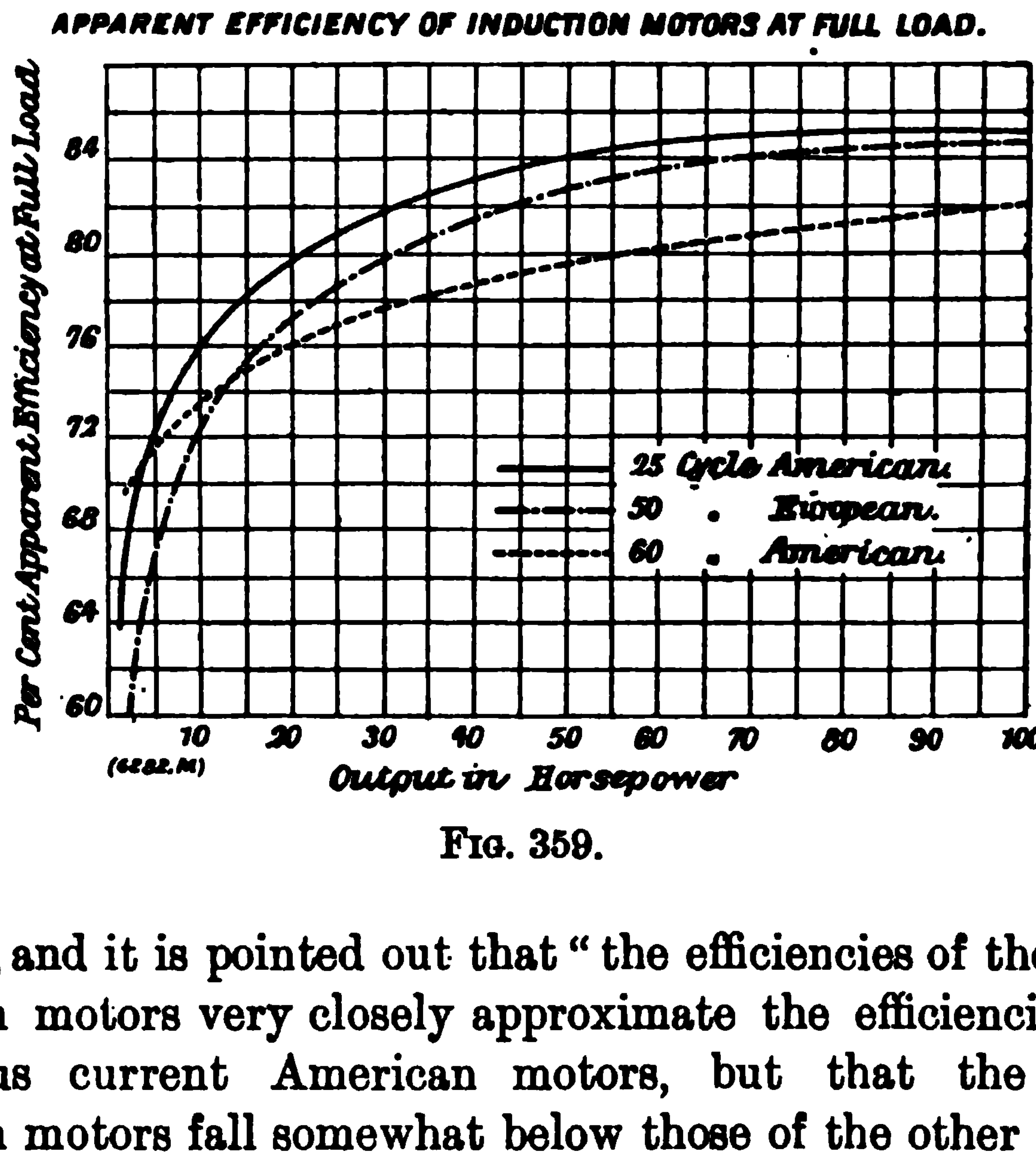
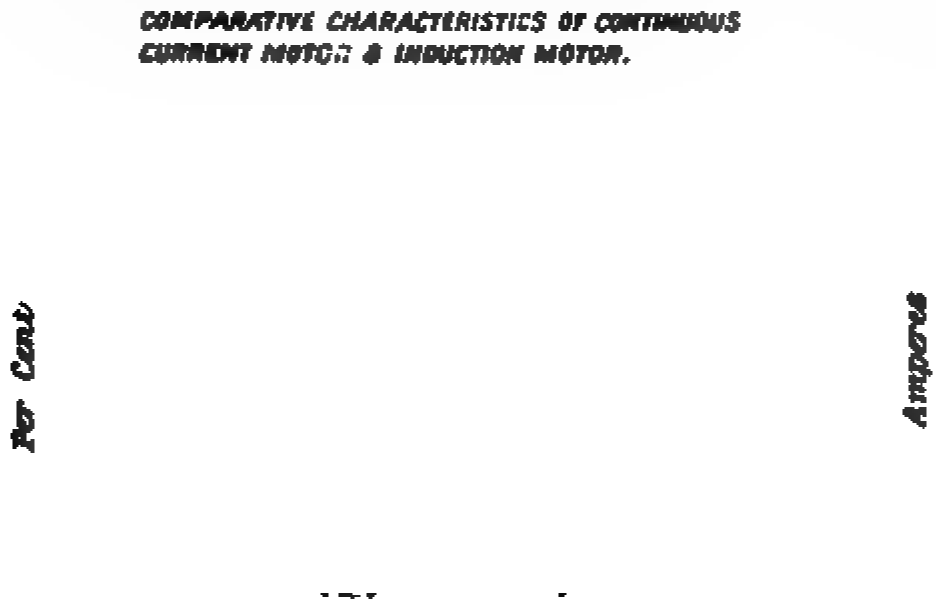


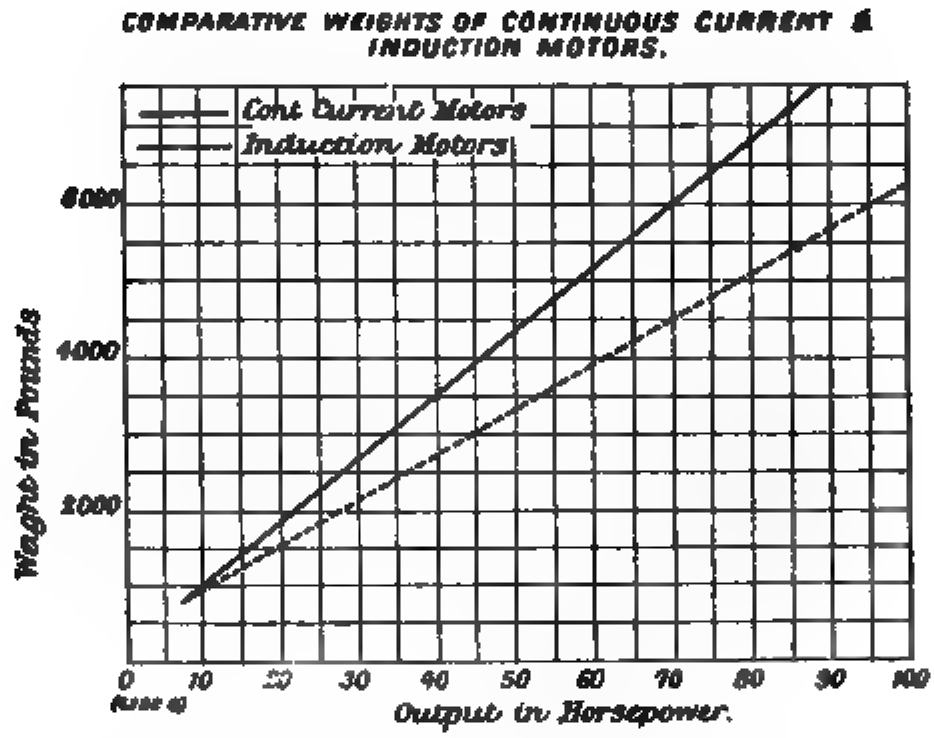
FIG. 359.

page 297, and it is pointed out that "the efficiencies of the 25-cycle American motors very closely approximate the efficiencies of the continuous current American motors, but that the 60-cycle American motors fall somewhat below those of the other machines, *a fact probably to be accounted for by the higher frequency at which they operate.*"

In Fig. 360, page 299, are given curves of a 140 horse-power continuous current motor, and of a 150 horse-power three-phase induction motor. Attention is called to the fact that "while for both very light and heavy loads the efficiency of the continuous current motor is better than that of the alternating current motor, the efficiency of an alternating current motor shows up to the best advantage for intermediate loads, above and below half load. Further than this, the curves illustrate very well the fact that it

is possible to heavily overload an alternating current motor, and, at the same time, have it develop excellent efficiency and power factor characteristics. This is a point directly to the advantage of the alternating current machine, as, at excessively heavy loads, the direct current machine is apt to give trouble at the commutator."





FIGS. 360 and 361.

Finally, in Fig. 361, Professor Goldsborough exhibits a set of curves illustrating the relative weights of continuous current and induction motors, and states that, "In small sizes, of 10 horse-power and under, it will be seen that the weights of continuous and alternating machines are very nearly the same. For larger powers, however, above 30 horse-power, the alternating current motor is, as a rule, considerably lighter than the continuous

current motor. In point of weight, then, the alternating current motor has a distinct advantage. As far as the general construction of the motors is concerned, the same degree of excellence in workmanship and insulation is obtainable in each class. In fact, the methods of manufacture may be said to be identical. The introduction of the commutator into the continuous current machine is the point of greatest difference, and the commutator is, by many, regarded as the weakest point in continuous current machines."

The remarks are concluded by a summary: "In point of weight the alternating current machine has the advantage. In point of full load efficiency and equivalent currents taken at full load there is some advantage to be credited to the continuous current machines. In point of efficiency at medium loads the average performance of standard commercial machines shows that the alternating current motor has the advantage over continuous current motors. At these loads, however, the equivalent line currents taken by the alternating current motors are considerably in excess of the currents taken by continuous current motors. In point of starting torque the continuous current motor is undoubtedly superior to the alternating current motor for exceptionally heavy duty. Against this advantage of the continuous current motor must be placed the elimination of the commutator in the alternating current motor, which, in some cases at least, might make it worth while to instal a 20 horse-power induction motor in order to obtain the required starting torque, where otherwise a 10 horse-power continuous current motor would suffice."

Mr Chas. F. Scott, whose paper served as the basis for the discussion during which these views were expressed, was of opinion that the induction motor can be used very extensively to advantage. It is, however, very clear from his remarks that even he regards the induction motor as not to be readily adapted to efficient operation at varying speeds. Thus he states:

"You cannot do everything with the induction motor that you can with the continuous current motor, but there are lots of things that you can do with it that you cannot do with the continuous current motor. Each has its own characteristics. After careful consideration of the characteristics of the induction motor it has been selected for industrial establishments, and when some speed change is necessary this has been provided in the motor, or external means of changing the speed have been provided while the motors run at constant speed. In the operation of machine tools, for

example, a belt between the motor and the tool with cone pulleys gives a much-to-be-desired flexible connection between the motor and the machine, and devices are arranged for readily shifting the belt. I give this as a single means which is in satisfactory use for accomplishing some of the speed changes."

§ 3. **The Use of Induction Motors for Lifts.**—With regard to lifts, one would think that the alternating current motor could never get an elevator up. As a matter of fact, there are hundreds of elevators which are operated by such motors, and some of them have been running for several years.

The application of the induction motor to the operation of lifts has received, it is true, considerable attention. The results, however, do not compare at all favourably with those obtained by the use of continuous current motors. On this point very interesting data have been brought together in a paper by Professor Sever of Columbia College, on "Power Consumption of Elevators, Operated by Alternating and Direct Current Motors," read before the American Institute of Electrical Engineers, April 25th, 1902, from which we learn that in 1902 there were in New York City approximately three thousand lifts operated by continuous current and three hundred operated by alternating current motors. This disproportion alone does not reflect upon the merits of the induction motor, since continuous current supply has, in New York City, been available for a very much longer time. On the other hand, there are some sections of the city where it is at present impossible to obtain a direct current supply, and the constructors of lifts have had no alternative but to employ induction motors. Under Professor Sever's direction numerous tests were made upon lifts performing similar service but equipped with various types of motors and of motor control systems, and Table XLIV., pages 302 and 303, gives the results obtained by him on eleven representative installations. In the first five cases, direct current motors were employed; in the six other cases, induction motors.

For induction motors, the power factor at starting is singularly high, but during running the average for the various tests gave a power factor of ·36, "showing that the induction motor was much larger than the average conditions would call for, but it seems necessary to provide this capacity in order to accommodate any heavy loads that must occasionally be carried."

Fig. 362, page 304, is taken from Professor Sever's paper, and the left-hand curves "show the average energy for all lifts through a rise of 50 feet, carrying two passengers, the energy being expressed

TABLE XLIV.—RESULTS OBTAINED BY PROFESSOR SEVER WITH MOTORS AND CONTROL SYSTEMS FOR LIFTS.

Test No.	Location.	Date Installed.	Type of Control.	Rise. Ft.	Rise. ins.	Velocity in Feet per Minute. Designed.	Velocity in Feet per Minute. Actual. Down.	Velocity in Feet per Minute. Actual. Up.	Duty in lbs.	Overweight in lbs.
1	Horace Mann School, 120th St., N.Y.	June 29, 1901.	Magnet control, style P.A.	70	1	200	200	200	2500	900
2	Mason & Hamlin, 136 5th Ave.	May 1894.	Magnet control.	58	3	125	125	125	1500	275
3	S. Stein & Co., 5th Ave. and 18th St.	Feb. 1902.	Direct regulated control by hand.	..		..	..	..	1500	..
4	Transit Building, East 42nd St.	Feb. 1901.	Magnet control, style 67.	121	6	300	300	300	2000	500-600
5	Bryant Park Studio Building.	March 1901.	Magnet control.	162	10	300	300	300	1500	250
6	Aliman Apartment, 925 West End Ave.	May 1899.	None.	75	6	150	156	151	1500	250
7	Katahdin, 567 West 18th St.	Dec. 1899.	None	74	0	125	129	124	1500	600
8	Wharfedale, 606 W. 115th St.	..	None.	..		..	..	..	1500	..
9	Foxhall, 116th St. and Amsterdam Ave.	March 1902.	None.	74	11	150	150	150	2000	300
10	Elizabeth Apartment, 105th St. and Broadway.	1901.	Two-voltage control.	75	0	200	181	185	..	725
11	Brambach Piano Co., 133rd St.	June 1901.	Rheostat control.	56	6	100	2% variation with load.	2% variation with load.	2000 freight	500

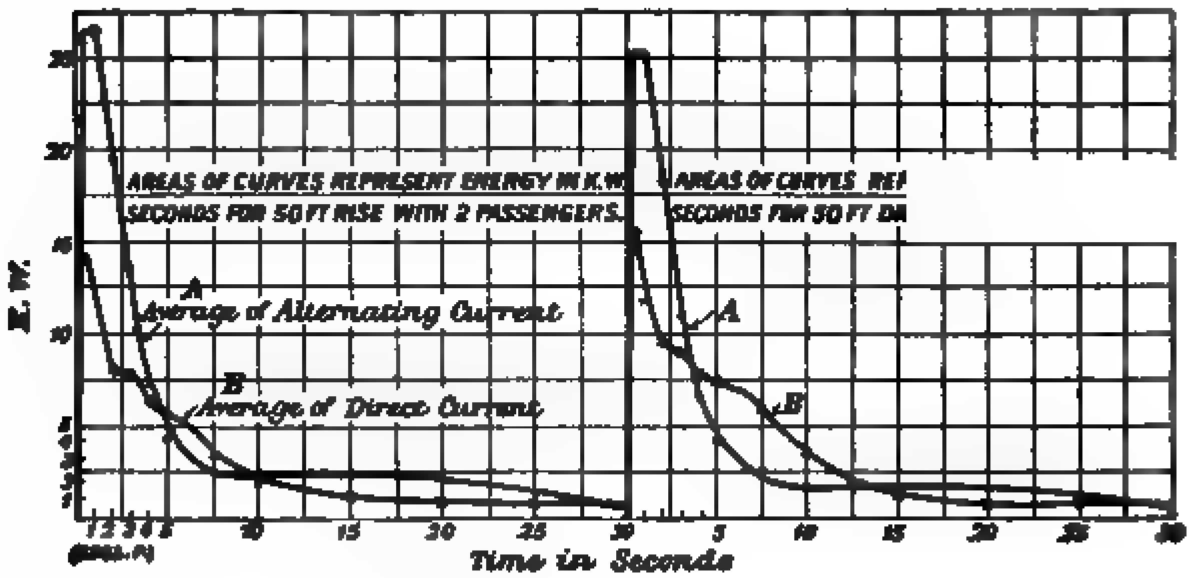
TABLE XLIV.—*continued.*

Test No.	Motor Capacity and Type.	Load.	Volts of Line.	Current. Starting.	Current. Running.	Power in Kilowatts. Starting.	Power in Kilowatts. Running.	Power Factor. Starting.	Power Factor. Running.
1	110 volts, 175 amperes, 800 r.p.m.	2 people up.	110	141	1·54	15·5	17·1		
		2 ,, down.	..	144	77	15·8	8·68		
		3 ,, up.	..	140	25	15·4	2·75		
		3 ,, down.	..	145	67	15·9	7·3		
		10 ,, up.	..	150	80	16·5	8·8		
		10 ,, down.	..	140	20	15·4	[illegible]		
		23 ,, up.	..	190	150	19·8	16·5		
		23 ,, down.	..	145	..	15·4	..		
2	7 h.p., 220 volts, 1000 r.p.m.	2 ,, up.	240	49·6	5·5	11·9	1·32		
		2 ,, down.	..	47·5	8·5	11·4	2·04		
		3 ,, up.	..	50	8·0	12·0	1·092		
		3 ,, down.	..	49	6·0	11·75	1·044		
		4 ,, up.	..	52	10·0	12·48	2·4		
		4 ,, down.	..	48	4·0	11·5	0·96		
3	230 volts, 75 amp., 630 r.p.m.	2 ,, up.	240	74·8	21·6	17·95	5·18		
		2 ,, down	..	77	32·5	18·5	7·8		
		7 ,, up.	..	..	43	..	10·3		
		7 ,, down.	..	72	10	17·3	2·4		
		14 ,, down.	..	66·5	12·5	15·97	3·0		
4	220 volts, 100 amp., 800 r.p.m.	2 ,, up.	240	82	10·7	19·7	2·57		
		2 ,, down.	..	85	34	20·4	8·16		
		4 ,, up.	..	81·3	21·8	19·5	5·23		
		4 ,, down.	..	..	24	..	5·76		
		6 ,, up.	..	[illegible]	20	20·6	4·8		
5	20 h.p., 230 volts, 800 r.p.m.	2 ,, up.	240	82	29	19·7	6·97		
		2 ,, down.	..	83	22·6	19·9	5·43		
		4 ,, up.	..	83·2	36	20	8·65		
		4 ,, down.	..	81	18	19·45	4·32		
6	2-phase motor, 200 volts, 60 cycles.	2 ,, up.	200	170	18	30·5	1·42		
		2 ,, down.	..	193	36	34·7	[illegible]		
7	2-phase motor, 200 volts, 60 cycles.	2 ,, up.	200	150	[illegible]	25	1·3		
		2 ,, down.	..	150	6	24	0·5		
8	2-phase motor, 200 volts, 60 cycles. Variable speed, no load speed, 900 r p.m.	2 ,, up.	200	172	33·6	[illegible]	2·66		
		2 ,, down.	..	170	29	29·5	1·15		
		3 ,, up.	..	193	30·2	29·2	2·96	·79	·415
		3 ,, down.	..	..	30	30·0	[illegible]	..	·236
9	2-phase motor, 200 volts, 60 cycles.	2 ,, up.	210	189	38	38	3·4	[illegible]	·43
		2 ,, down	..	188	40	38	1·8	·97	·41
		3 ,, up.	..	184	46	36	6·6	·93	·63
		3 ,, down.	..	188	35	38	0·8	·96	·14
10	2-phase motor, 200 volts, 60 cycles.	2 ,, up.	60	[illegible]	..	8·5	..	..	..
			175	146	4·4	14·6	1·6	..	.
		2 ,, down.	60	104	..	8·6	..	..	..
			175	182	6·4	14·4	1·0	..	..
		3 ,, up.	60	118	..	..	..	..	..
			175	152	11·2	..	2·0	Starting cur-	
		3 ,, down.	60	104	..	..	..	rents on	
			175	144	2·0	..	0·4	points of con-	
11	15 type F, 2-phase motor, 200 volts, 60 cycles.	1 man and car up.	..	..	86	25·2	4·8	trol ;	
		1 ,, ,, down.	200	..	70	23·6	9·4	1 2 3 4	
		1200 lbs. up.	..	..	29	25·2	4·4	70 90 60 140	
		1200 lbs. down.	..	..	26	23·6	2·0	70 70 80 130	
		2 people in car up.	..	..	..	25·2	4·7	70 90 60 140	
		2 ,, ,, down.	..	..	..	22·6	8·5	70 70 80 130	

in kilowatt-seconds"; the right-hand curves show similar values for the descent of the lift. Curve A shows the average of the alternating current, and curve B that for the continuous current tests. The areas of these curves are to each other as 1·5 to 1.

For the curves relating to the descent of the lift, the energy consumption of the induction motors is about 8 per cent. greater than that of the continuous current ones. It should, however, be noted that the curves for the continuous current test are made from the results of five motors with automatic control, while the results for the alternating current machines are made from six uncontrolled motors.

Professor Sever's conclusion is that it should be more eco-



Lifts rising. Lifts descending.

FIG. 362.—Energy Curves of Lift Motors.

nomical to use continuous current lifts in those places where both continuous and alternating current supply are available: "If the controlling apparatus is more simple with the alternating current system, and the absence of a commutator on the motor can be counted as an advantage, the extra expense of operating an alternating current system might be offset by the decrease in repairs and attendance. Apart from the last consideration, there seems to be every advantage in favour of the use of the direct current motor."

Urban traction work, in several respects quite analogous to lift work, but with the further obstacle to the introduction of polyphase motors of requiring three supply conductors (two and the track rail), is also a field where the use of the polyphase induction motor still offers grave difficulties, and where the

continuous current motor is undoubtedly, in the present circumstances at all events, greatly superior.

§ 4. **The Induction Motor's Sphere of Usefulness.**—Nevertheless, the induction motor has its own sphere of usefulness, and, as has been emphasised by the writer, is far better adapted to high speeds than is the continuous current motor. It is, in fact, with high speeds that its disadvantages become far less striking; its power factor is then high, its no load current small, its overload capacity great, and the necessity for a small clearance between stator and rotor less imperative. All these quantities are also improved the lower the periodicity of the network from which it is supplied.

In the *Engineering Magazine* for April 1903, Mr Fred. M. Kimball, who has been in close touch with the electric motor industry from its earliest days, in speaking of the relative advantages of continuous current and induction motors, expresses the opinion that, "while there still is, and undoubtedly for many years will be, a large demand for continuous current motors, it seems probable that, relatively, the larger business of the future will be in motors operated from alternating circuits." But he points out that "the only real limitation to their use, in nearly all applications, arises from the difficulty of obtaining variation of speed. This can be accomplished, but it is a matter of some considerable expense, and the results are not so satisfactory as in the case of continuous current motors." Mr Kimball mentions the "zone system," used on the Continent and in the United States, with a view to facilitating the employment of whichever type of motor is economically adapted to the distribution conditions. By this system the distribution is by continuous current within a radius of a mile or two from the central station, the outlying districts being supplied by alternating currents. He states that "the result of these changes has been to enlarge greatly the field for induction motors, both in replacing machines which were previously operated on the continuous current circuits, formerly extending beyond the present boundaries of the inner zone, and also for the new power business which is being developed in the outer zone."

§ 5. **The Relations between Frequency and Speed.**—The most extreme statement in advocacy of the induction motor, which the writer has found coming from an especially well-informed source, is Mr Eborall's statement in his fourth Howard Lecture (Society of Arts, May 17th, 1901), where he maintained that "the manner of working and performance of alternating current motors is entirely analogous to that of shunt wound

continuous current motors, and every class of work adapted for the latter machine *is equally well adapted to induction motors*, being in some cases even more suitable for them."

But in the matter of the desirability of low frequencies and high speeds for induction motors, Mr Eborall's views and the writer's (as expressed in the earlier section of this article) quite thoroughly coincide. Thus, in Mr Eborall's Howard Lecture, the following conclusion is most important, as it goes right to the most essential consideration for securing good results in induction motor work: "The pole pitch will be made as great as possible, or, in other words, the number of stator poles for a given diameter will be as small as possible, in order to increase the length of the leakage paths from pole to pole; this implies either a low frequency or a high speed, and thus motors operating at definite frequencies, such as 40, 50, or 60 cycles, must be run at as great a speed as practicable." At the same time it must not be overlooked that the inductance of the end connections is greater the greater the polar pitch. Hence it is not unconditionally better, from the magnetic leakage standpoint, to have short cores of large diameter in all cases.

On another occasion ("Some Notes on Polyphase Machinery," read before the Manchester Section of the Institution of Electrical Engineers, March 25th, 1902) Mr Eborall again alluded to the importance, or, rather, the necessity, of low frequency for induction motors which must be operated at low speed. It is so important, in the interest of a correct use of induction motors, that this point should be thoroughly realised, that it seems well to again quote his words: "The other point connected with polyphase induction motors which may be profitably referred to here is the great advantage of low frequency for slow-speed motors. This is best illustrated by means of an example taken from practice. The curves in Fig. 363 refer to a slow-speed motor of 350 brake horse-power directly coupled to a mining ventilator running at 310 revolutions per minute, while those in Fig. 364 refer to a slow-speed 115 brake horse-power motor directly coupled to a Riedler high-speed pump running at 200 revolutions per minute. The general arrangement of the latter motor comprises a starting resistance which is interlocked with the brush gear in such a manner that, when the resistance is all out (the motor thus being at full speed), a few more turns of the resistance handle causes the rings to be short circuited by an internal clutch, and the brushes to be raised as previously described. In Table XLV. is given a comparison of the two motors:—

TABLE XLV.—COMPARISON OF LOW-SPEED MOTORS.

Normal rating of motor in B.H.P.	350	115
No load speed	315	210
Frequency	21	42
Terminal pressure	500	250

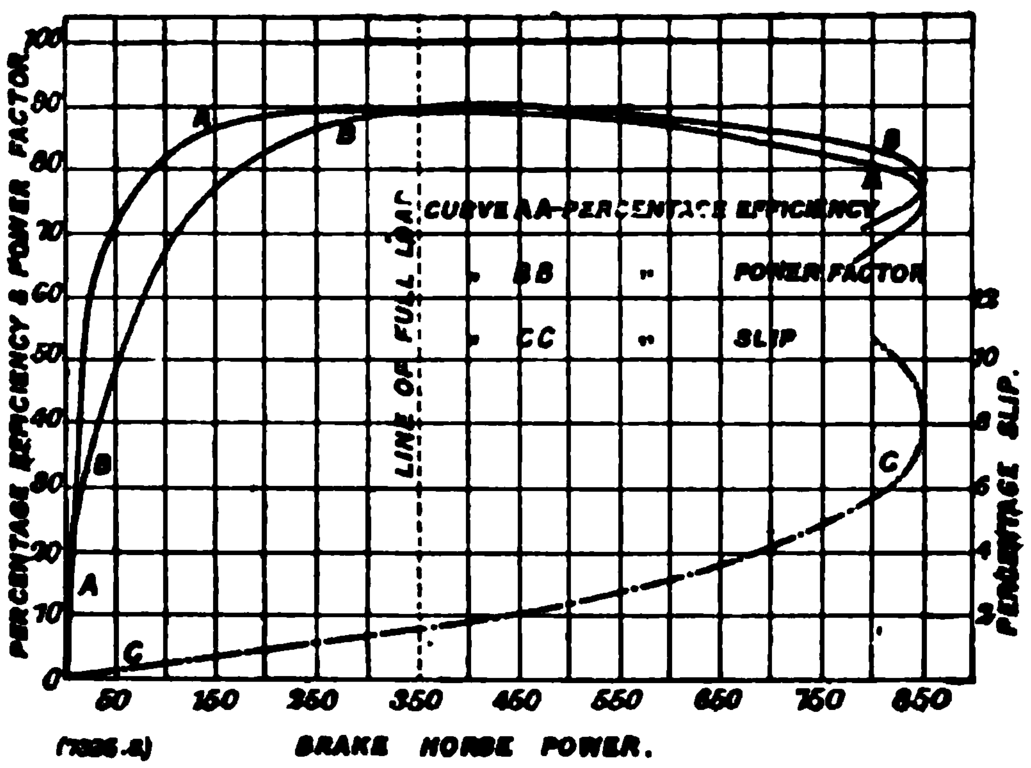


FIG. 363.—Curves of a 350 B.H.P. 310 r.p.m. Induction Motor

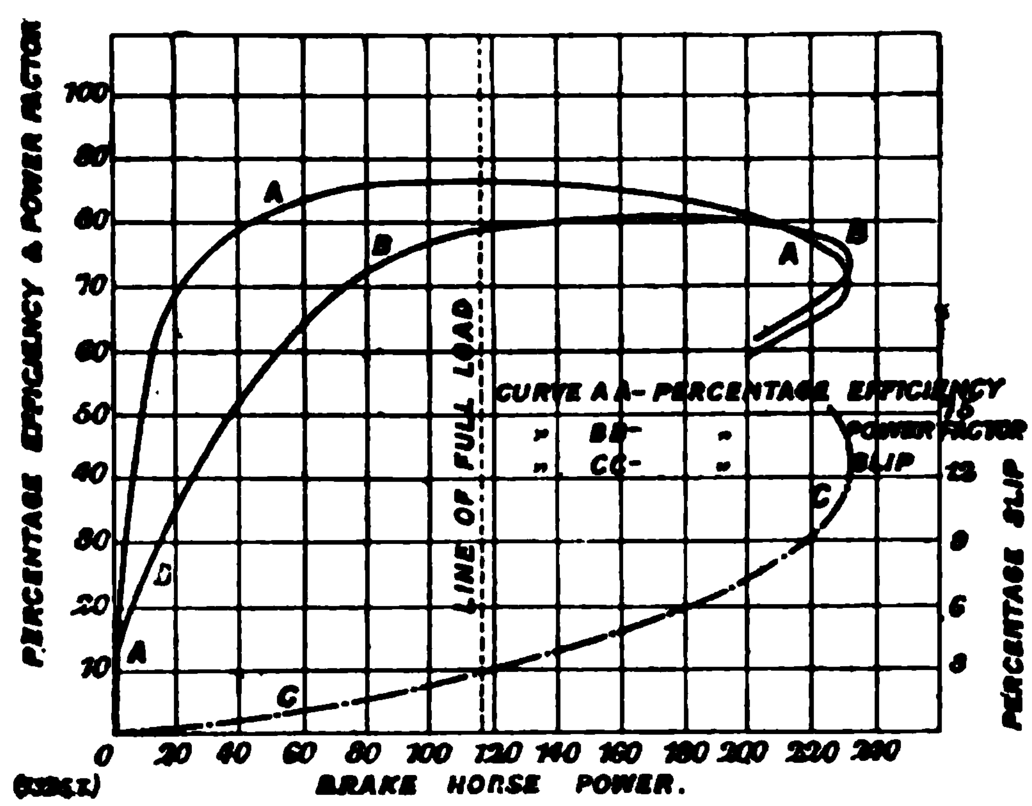


FIG. 364.—Curves of a 115 B.H.P. 200 r.p.m. Induction Motor.

Full load current per phase	373	289
„ efficiency per cent.	90	87
„ power factor per cent.	90	79
„ slip per cent.	1·5	3·0
Maximum load before falling out of step (brake horse-power)	850	230
Net weight of motor (lbs.)	19,400	14,150
Net over-all dimensions (diameter and length in inches)	78 × 29	92 × 17·4
Net weight per rated horse-power (lbs.) ...	55·5	123

"Table XLV. shows at a glance that not only is the 21-cycle motor lighter and cheaper when considered on the same basis of output and speed, but it is far and away a better motor. Its power factor, efficiency, slip, and overload capacity are all superior to the 42-cycle motor. These differences are not due to the fact that the low frequency motor is much the larger; this would make but little difference. The differences are almost entirely due to the abnormal number of poles necessary with the 42-cycle motor in order to bring down its speed to the value required by the pump; this motor has 24 stator poles, while the 21-cycle motor has only 8 poles. The result for the former is a heavy stator core of large diameter, relatively large iron loss, considerable leakage from pole to pole along the air gap, and a considerably lower saturation of the teeth than is desirable in such a motor. For these reasons, three-phase pumping and similar plants, in which slow speeds for the motors are essential, should be laid out, whenever possible, on the basis of a low frequency, such as 25 cycles."

§ 6. **Relation between Frequency and Power Factor.** —Mr Behrend also takes occasion to emphasise the importance of low frequency in order to obtain the best results from induction motors. The following paragraph is quoted from his treatise, *The Induction Motor*:—

"If the circumferential speed of the armature is limited, and this is generally the case, then the pole pitch is also limited for a given number of revolutions per minute. The air gap cannot indefinitely be diminished, hence a high frequency necessitates a large leakage factor. We labour here under the same difficulties that we have met with in the design of alternators for high frequencies. It is possible to build motors for frequencies between 60 and 100, but *the higher the frequency the lower will be the power factor, and the larger will be the lagging currents.* It has also to be borne in mind that motors *for high frequencies* must be made *not inconsiderably larger.*" On certain assumptions, not of general applicability, but still illustrating the importance of low frequencies, Mr Behrend illustrates a certain case by the following comparison:—

Frequency in Cycles per Second.	Maximum Power Factor.
25	0·91
50	0·83
100	0·72

As a matter of fact, 25-cycle motors, for high speeds and medium capacities, may be more cheaply built for power factors of,

say, 0·91 to 0·94, than equivalent 50 cycle-motors for power factors of 0·89 to 0·92, and will, at the same time, require a lower current when running at light loads, and will have a higher overload capacity.

§ 7. The Importance of Low Frequency and High Speed. —The writer has considered it desirable to not only present his own views as to the importance of low frequency and high speed from the standpoint of obtaining the best results from induction motors, but also to support this position with these references to other articles on the subject of the induction motor, since if, as seems probable, this type of motor is about to be extensively used in this country, not only for work for which it is well suited, but for a large amount of other work, it becomes of increased importance to let it at least have the advantage of conditions suited to its best performance. Although these several authorities differ in detail and in emphasising various points, the general opinion is unmistakable, namely, that the induction motor is in many respects at a disadvantage, as compared with the continuous current motor; but that it is decidedly cheaper and more satisfactory the lower the frequency and the higher its normal speed; and, which follows from these conclusions, if it *must* be employed for low speeds, *low frequencies* are the more important; when it *must* be employed at high frequencies, *high speeds* are very important. In the subsequent diagrammatical treatment of the subject this will become much more evident.

Having now compared induction and continuous current motors, also the comparative merits of high and low frequencies and speeds for induction motors, the type of induction motor remains to be referred to.

CHAPTER XV

THE DESIGN OF INDUCTION MOTORS

§ 1. **Squirrel Cage *versus* Slip Ring Motors.** — The writer finds himself at variance with general practice and opinion, in that he would employ the squirrel cage type of motor much more generally, as having such great advantages on the score of simplicity, robustness, better economy in operation, and cheapness in manufacture, as to justify its use, not only in very small sizes, which is the present practice, but also in practically any size—in fact, limiting the use of slip ring motors to special cases. It is true that squirrel cage motors must be started with but little or no load, and even then must be started from a compensator, or with temporary star connection, in order to limit the starting current to a permissible amount; further, by the use of mechanical devices designed to apply the load after starting, they are made not only equal to, but more satisfactory than the slip ring motor, being characterised by higher efficiency, power factor, and overload capacity, and requiring a minimum of attention. The slip ring motor, on the other hand, is not only less satisfactory with respect to these constants, but it is inherently more expensive, more liable to break down, gives at the slip rings and brushes practically as much trouble as a commutator, and must have troublesome and expensive auxiliary attachments for internally short circuiting the slip rings and raising the brushes after starting. All these disadvantages are encountered simply in order to obtain improved starting.

Mr Eborall's recommendations in his Howard Lecture, May 17th, 1901, on "Polyphase Electric Working," probably best represent present practice. They are as follows:—

Thus, with an ordinary squirrel cage rotor, the motor will take a starting current equal to three or four times the full load current when starting with full load torque; this current is, of course, doubly objectionable, firstly on account of its magnitude, and secondly, on account of its being so much out

of phase, which will produce an excessive pressure drop in the generating and transforming plant, and also in the line. For these reasons squirrel cage rotors and their modifications must be given up for all but small motors (unless for special purposes) in favour of a construction which will allow the starting current to be effectively controlled.

Mr Eborall then gives data of the comparative weight and performance of low pressure, three-phase, 50-cycle, 4-pole, 1500 revolutions per minute, squirrel cage and slip ring motors of medium sizes, as manufactured by Messrs Kolben & Co., of Prague, from which the writer has prepared Table XLVI.

TABLE XLVI.—CHARACTERISTICS OF KOLBEN MOTORS.

	Designation of Motor.					
	D M 5.		D M 6.		D M 8.	
	Slip Ring Type.	Squirrel Cage Type.	Slip Ring Type.	Squirrel Cage Type.	Slip Ring Type.	Squirrel Cage Type.
Brake horse-power	4·5	5	5·5	6	7	8
Full load efficiency	84	84	85	85	87	87
Half load efficiency	76	76	77	77	79	79
Full load power factor ...	·88	·88	·88	·88	·89	·89
Full load apparent watts ...	4550	5000	5550	6000	6780	7800
Nett weight of motor (lbs.) ...	352	330	440	420	692	660
Weight per B.H.P. (lbs.) ...	78	66	80	70	99	83
Ratio of weights per B.H.P., squirrel cage : slip ring	·85		·88		84	

But not only is the squirrel cage motor lighter per brake horse-power, but the construction is cheaper in a still greater proportion, and the motor requires less attendance and can stand much more rough handling. Although the above designs show the same efficiency and power factor for both types, it is generally the practice to take up the advantage partly in improved efficiency and power factor in the squirrel cage type.

Mr Eborall returned to this subject in his Institution paper in 1902, and stated (*Engineering*, June 27th, 1902, page 859):—

Regarding the question of the starting capabilities of induction motors, there still seems to be a certain amount of misunderstanding prevalent among those engineers who have been used more particularly to continuous current work. The polyphase induction motor constructed with a permanently short

circuited rotor winding (that is, with a squirrel cage rotor or a modification of it) is naturally an *ideal* motor from the user's point of view. On account of its *simplicity*, it is *low in first cost* and *easy to operate*, and the *cost of upkeep* is *negligible*, inasmuch as there is nothing to get out of order or to require attention or renewal. But principally on account of the starting of motors of this type, it becomes necessary to limit the size of such motors when a large scheme for the distribution of power is concerned; the power station superintendent has, in fact, to insist that all induction motors connected to the mains shall be provided with rotor resistances for starting, when these motors exceed a certain size—say about 5 brake horse-power.

Induction motors with permanently short circuited rotors require an altogether abnormal starting current, especially if they have to start against any considerable load. For, on account of the relatively low resistance of the rotor bars and connecting rings (or strips), there is nothing to limit the current taken at the moment of starting, except the magnetic leakage from pole to pole of the stator, and the lowering of the pressure on the motor terminals; the former will increase so enormously that the useful rotor flux (that flux actually getting into the rotor and cutting the bars) will be relatively small. Thus the starting torque (which is proportional to the useful flux and to the rotor current) will be quite small, although the starting current of the motor is so great; moreover, this large starting current is (on account of the great leakage) of very low power factor, and thus causes very harmful effects on the pressure regulation of the system. Naturally the pressure on the terminals on the motor also falls, which in turn affects the starting torque, as the latter is proportional to the square of the terminal pressure. For these reasons the very best induction motors of this type on the market, when starting against the load offered by a belt and loose pulley only, take a current equal to at least twice the full load current; when starting with full load torque this will increase to three or four times the full load current. In the former case the rush of current is only momentary, but in the latter case the large current is required for some time, as the starting is certainly not good (unless the motor is provided with a special high resistance rotor, which means low efficiency at load), and hence it follows that under the latter conditions of operation the motor is liable to burn out; it will do so if there are any weak points in it, or if it takes too long to speed up.

It will therefore be readily understood, from what has been said above, that the successful operation of a large power distribution would be impossible if large motors of this type were used indiscriminately; even if there were no lighting work the effect of the starting of such motors would be detrimental to the proper performance of other motors connected to the system. For this reason it is absolutely necessary that all induction motors of any size should be fitted with slip rings, and be operated (at starting) with rotor resistances. In the author's opinion all motors above 5 brake horse-power having to start against load should be so fitted; if they are not required to start against load, permanently short circuited rotors can be used for motors up to 8 brake horse-power inclusive.

§ 2. Osnos' Divided Short-Circuiting Rings for Rotors. —A sound objection to squirrel cage motors, with a means for overcoming it, is discussed by Osnos in the *Zeitschrift für Elektro-*

technik for August 10th, 1902. The objection lies in the fact that, owing to the small air gaps required in induction motors, and the low resistance of the squirrel cage windings, a very small eccentricity of the rotor, within the stator, will give rise to wasteful "equalising" currents. Osnos proposes to overcome this by dividing the end rings preferably into as many parts as there are pairs of poles. Thus for a 4-pole machine the end rings would consist of two parts, as illustrated diagrammatically in Fig. 365.

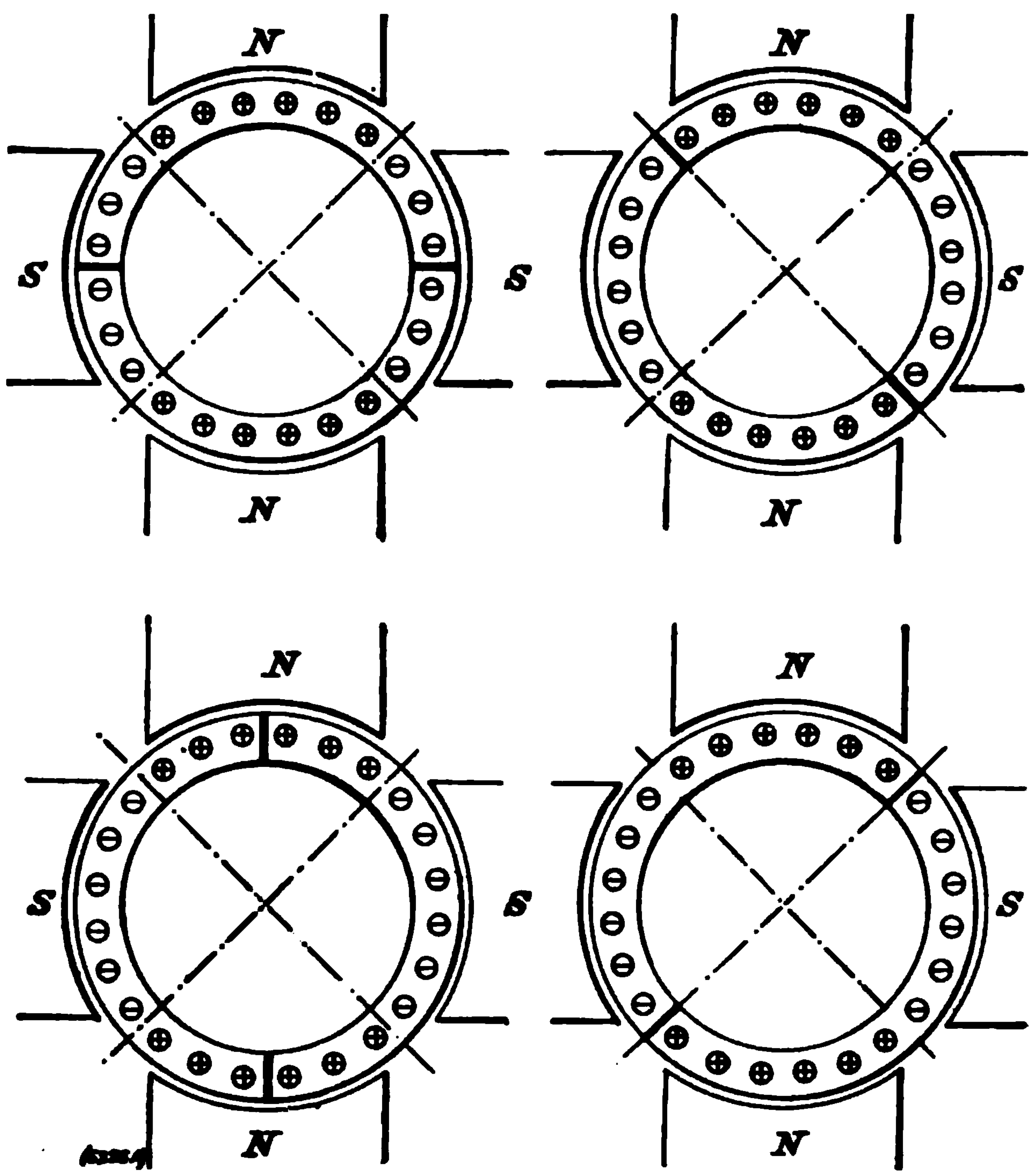


FIG. 365.—Diagrams of Divided End Rings for a 4-pole Squirrel Cage Rotor.

An 8-pole machine could have four sections per end ring, as shown in Fig. 366, page 314. (Merely for descriptive purposes, the external stator poles are shown in these figures as distinct salient poles.) From Fig. 365 it may easily be seen that, in any position of the end ring with reference to the stator field, the number of face conductors influenced by a north pole, and the number influenced by a south pole, are equal to one another, and this is the case for each half end ring; and, with a low resistance in the ring segments, all the face conductors are as fully and uni

formly employed as would be the case for an ordinary squirrel cage construction, with undivided end rings. In the positions of the rotor corresponding to the two right-hand diagrams in Fig. 365, eccentricity in the rotor will not give rise to equalising currents; in the positions corresponding to the upper left-hand diagram, equalising currents can only flow between equivalent conductors opposite south poles; and, corresponding to the lower left-hand diagram, only between equivalent conductors opposite north poles. The net result is that the wasteful equalising currents are reduced by this construction to about one-fourth of their amount in a rotor with undivided end rings.

In machines with more than 4 poles the improvement to be secured by such subdivision of the end rings into a number of

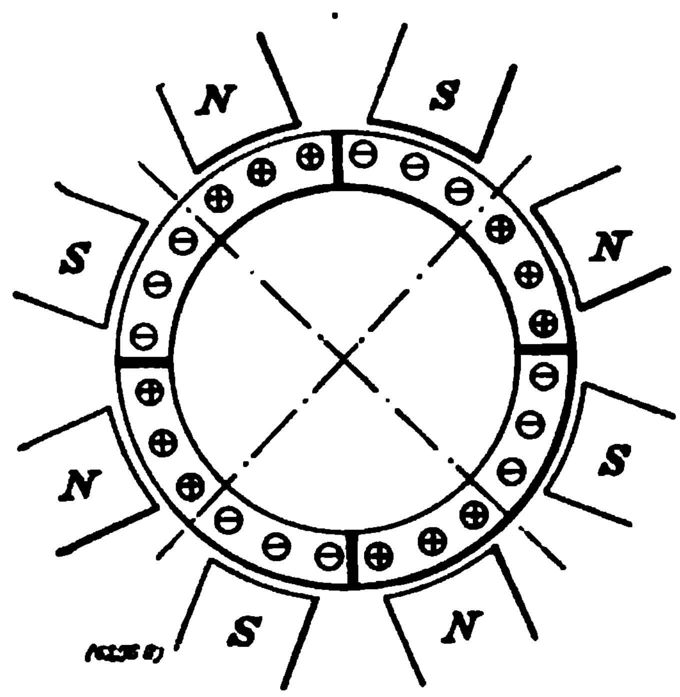


FIG. 366.—Diagram of Divided End Rings for an 8-pole Squirrel Cage Rotor.

segments equal to the number of pairs of poles is still greater, for the length of the air gap between the nearest neighbouring poles of like polarity is the more nearly equal the greater the total number of poles, as they are distant from one another by a smaller angle. Thus in the 8-pole motor of Fig. 366 the equalising currents are practically eliminated by a sub-division of the end rings into four segments. Hence, in motors with more than 4 poles, it might be a sufficient precaution to subdivide the end rings into fewer segments than correspond to the number of pairs of poles. It is sufficient to subdivide one end ring, leaving the other continuous.

§ 3. Ziehlke's Divided Short-Circuiting Rings for Rotors.—Ziehlke has built motors with both end rings subdivided, but in quadrature with one another, as shown diagram-

matically in Fig. 367. In this case, in a 4-pole motor, the current has to pass through four bars in series, and situated opposite four different poles, in order to complete its circuit. Good results are reported to have been obtained by this method of subdividing the end rings, the starting torque being improved.

§ 4. **Varying the Speed of Induction Motors by Resistance in Secondary Circuit.**—The most satisfactory method, and that generally employed, consists in inserting a resistance in the secondary circuit, as has been shown in the diagram, Fig. 297, page 249, for starting. But for running constantly with resistance in circuit, the carrying capacity of the rheostat must, of course, be far greater, and rheostats for this purpose are large and expensive. The greater the resistance the lower the speed, and

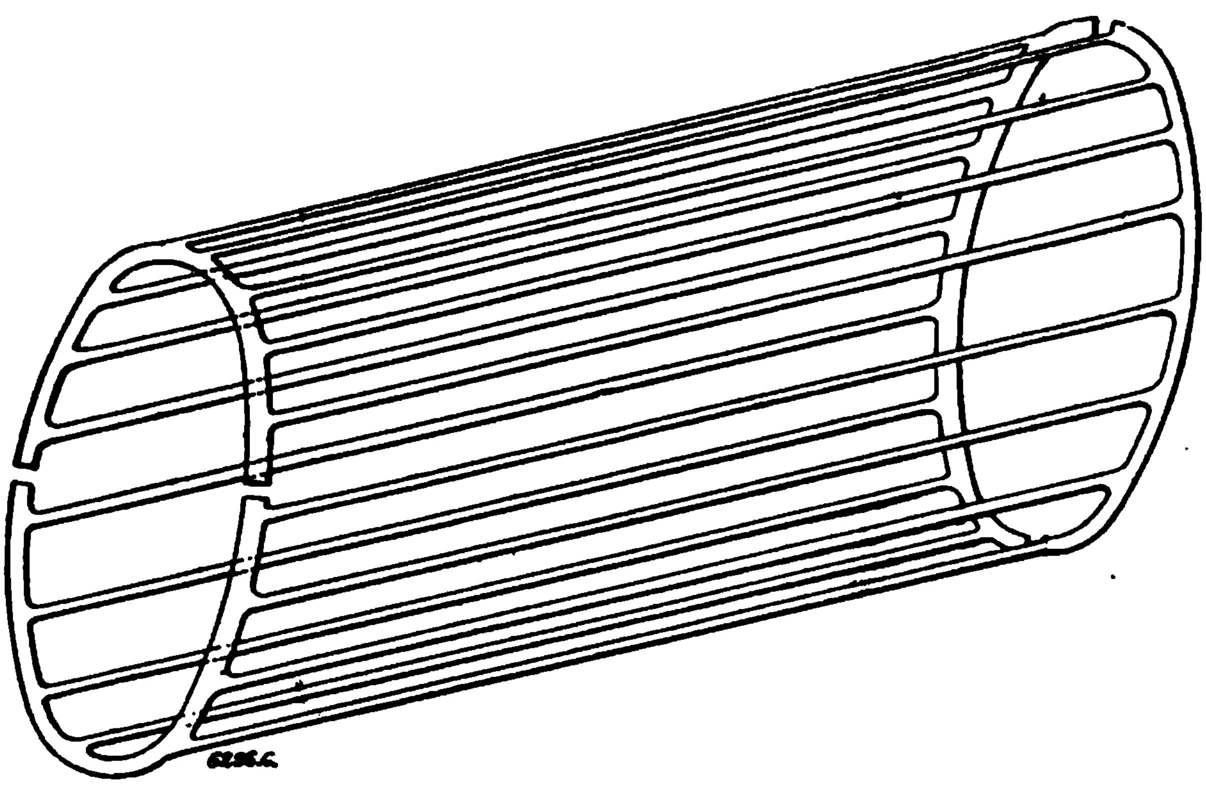


FIG. 367.—Diagram showing Subdivision of End Rings for Squirrel Cage Rotor.

the lower also the efficiency—in fact, when running on such a resistance as to give half the normal speed the efficiency is about half the normal figure, and is in the same ratio for other speeds. The speed is, moreover, not definite for any one resistance step, but changes with the torque put upon the motor. When running light, the speed will be but little below synchronous speed, even though running on a resistance point corresponding to, for example, quarter speed at full load torque. This is analogous to a continuous current series wound motor arranged for speed control by variation of the amount of resistance in series with it.

§ 5. **Variable Speed by Potential Control.**—In order to reduce the expense of the controller for a variable speed induction motor, the resistance is sometimes arranged to be changed successively in each of the three sets of resistances, thus obtain-

ing, for a given number of contacts, three times as many speed steps as when the steps in all three sets of resistances are taken simultaneously. This is indicated diagrammatically in Fig 368. The slight dissymmetry thus introduced occasions no difficulty.

Fig. 368.—Diagram showing Method of Controlling Speed of Induction Motors by successive changes in different Phases.

The speed of induction motors may also be controlled by resistance or inductance in series with the primary windings, and this offers the advantage of avoiding the use of collector rings—provided the motor may start unloaded—and enables a squirrel

cage rotor to be used. Generally, however, the cases requiring variable speed motors also require considerable starting torque. This last method, which has been styled that of "variable speed by potential control," since the speed variation is obtained in virtue of the increased slip corresponding to the decreased voltage at the primary terminals of the motor, is shown diagrammatically in Fig. 369. Oudin has given, in *Standard Polyphase Apparatus and Systems*, page 83, the following data setting forth the relative efficiencies of these two methods :—

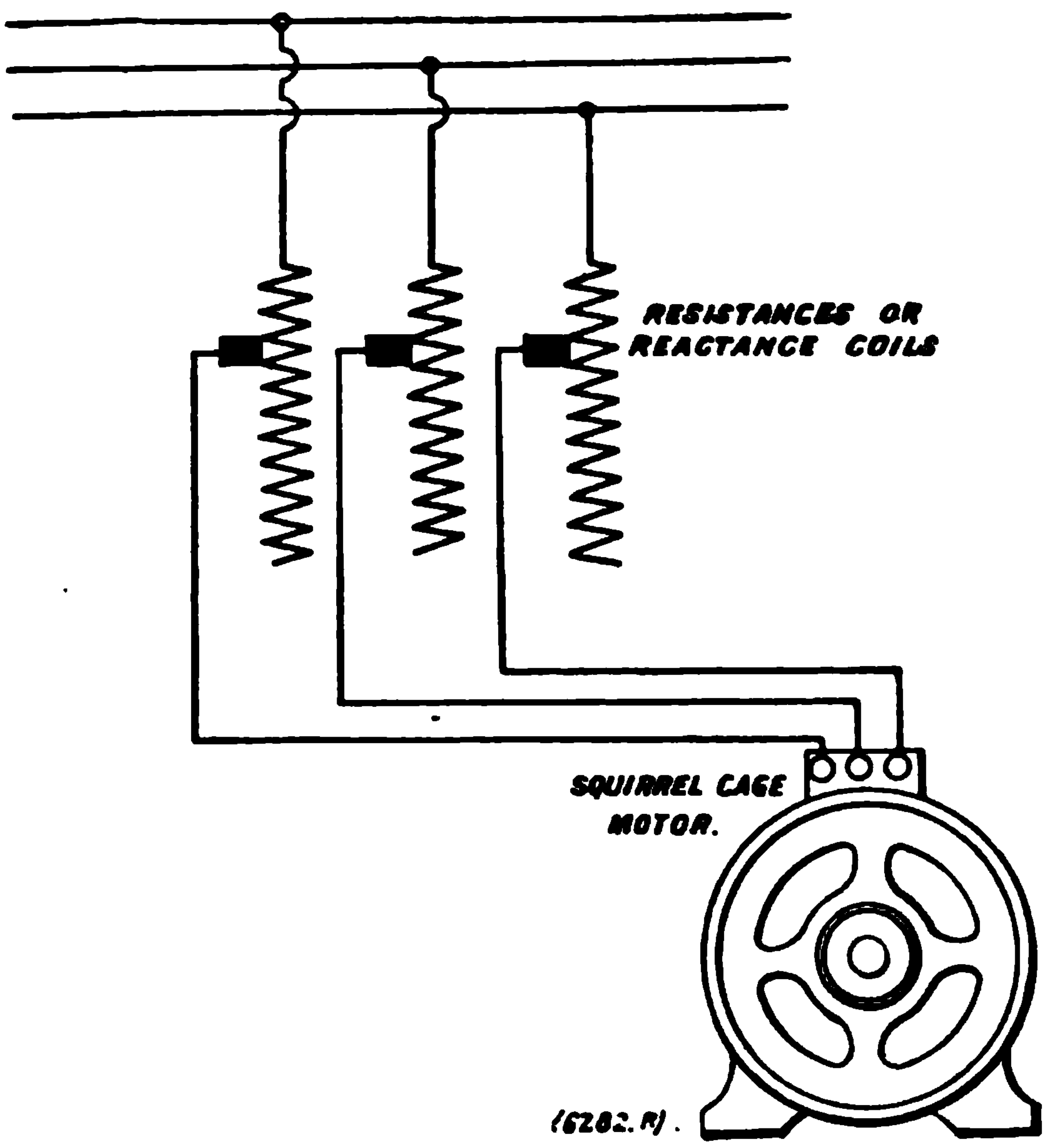


Fig. 369.—Variable Speed of Induction Motor by Potential Control.

TABLE XLVII.—COMPARISON OF EFFICIENCIES OF RHEOSTATIC AND POTENTIAL METHODS OF SPEED CONTROL OF INDUCTION MOTORS.

Speed.	Method of Control.	True Efficiency.	Power Factor.	Apparent Efficiency.
Full ...	Rheostatic	83	86	72
	Potential	83	86	72
Half... ...	Rheostatic	41·5	86	36
	Potential	36	57	20·5
Quarter ...	Rheostatic	21	86	18
	Potential	16	48	7·7

The torque is assumed to be constant at all speeds.

Since the overload capacity of an induction motor is proportional to the square of the impressed voltage, it is obvious that speed variation by potential control is only practical for motors designed for high overload capacities. But there are certain ranges of sizes, speeds, and periodicities, notably in small motors for high speed and low periodicity, where very high overload capacities are perfectly consistent with economical and good designs in other respects.

The remaining method of obtaining variable speed consists in such arrangements of windings as to permit of varying the number of poles. This has the disadvantage of providing but a limited number of definite speeds. These speeds are, however, practically definite for all loads, *i.e.* practically independent of the load upon the motor, hence cases arise where the problem is most suitably met by this arrangement. It is troublesome enough when it involves merely commutation of the primary windings—if the secondary (rotor) windings must also be arranged for more than one number of poles, the motor must necessarily be very expensive and complicated—hence squirrel cage rotors are extremely desirable for such cases. The motor may, for example, for two speeds, have either two separate windings, which would be wasteful of space, or some arrangement of winding whereby a commutation of the connections will change the number of poles. Many different ways of accomplishing this latter purpose have been proposed. These will be taken up in detail in a later section relating to the general subject of windings for induction motors.

§ 6. **General Properties of the Three-phase Induction Motor.**—We will now consider the case of the ordinary three-phase induction motor, with respect to its underlying properties. The no load current, power factor, and overload capacity are all ultimately dependent chiefly upon the values of two quantities—one the magneto-motive force corresponding to the amount of the magnetic flux linked with the primary turns, and the other the inductance of the windings. These quantities are both of a nature precluding any exact predetermination except by means of practically empirical methods based upon experimental data. There is plenty of such data available, and on account of the advantage of working from the average of several different manufacturers' constructions and proportions, the writer proposes to illustrate the subject at this stage by means of data derived from the fairly complete descriptions and tests of motors which have been published by Thompson, Kapp, Arnold, Eborall, and others. This

discussion will be followed by descriptions of a number of recent motors by various designers in different countries, much in the way followed in the preceding articles on continuous current motors.

§ 7. **Determination of the Magnetic Flux and the Magnetising Current.**—Unlike most dynamo-electric machinery, almost the entire magneto-motive force is, in induction motors, that required to overcome the reluctance of the air gap. The iron part of the magnetic circuit rarely requires more than from 10 per cent. to 15 per cent. of the total magneto-motive force. Hence very considerable variations in the permeability of the material are accompanied with but trifling variations in the magnetising current, and the considerations controlling the choice of material relate almost exclusively to obtaining a sufficiently low core loss. But this elimination of the question of varying permeability by no means renders definite the estimation of the magnetising current, and this is chiefly because such small clearances are used in induction motors that any exact control of the true average radial depth of the air gap is quite out of the question. The radial depths employed range from 0·5 millimetre in small motors for high periodicity and low speed, up to 1·5 or 2 millimetres in large motors for low periodicity and high speed. In practice a 10 per cent. variation in the depth is very difficult to avoid, and greater uniformity is probably commercially impracticable. Another obstacle to exactness consists in the irregular distribution of the magnetic flux. Were the density uniform at all points of the periphery, the conditions could be represented as indicated in Fig. 370, where the portion of a periphery corresponding to two poles is developed on a straight line, so that abscissæ represent distances along the air gap, and ordinates represent density of magnetic flux across the air gap. This distribution would be approached were the magneto-motive force supplied exclusively by the windings of one phase, when the conductors of which its

Fig. 370.—Diagram of Uniform Density of Magnetic Flux.

windings consist are concentrated in one small slot per pole. Thus a 6-pole stator, with its windings concentrated in six small slots, as shown diagrammatically in Fig. 371, would, when traversed by single-phase currents, set up a magnetic flux across the gap, which would have fairly equal density at all points of the periphery. For any two adjacent poles the distribution would approach that indicated in Fig. 370. But in practical three-phase motors the windings of any one phase are not concentrated in one slot per pole, but are subdivided in several slots per pole, distributed over one-third of the periphery, as shown in Fig. 372, where there are five slots per pole per phase. The slots of one phase only are drawn, as, for the moment, attention should be concentrated upon

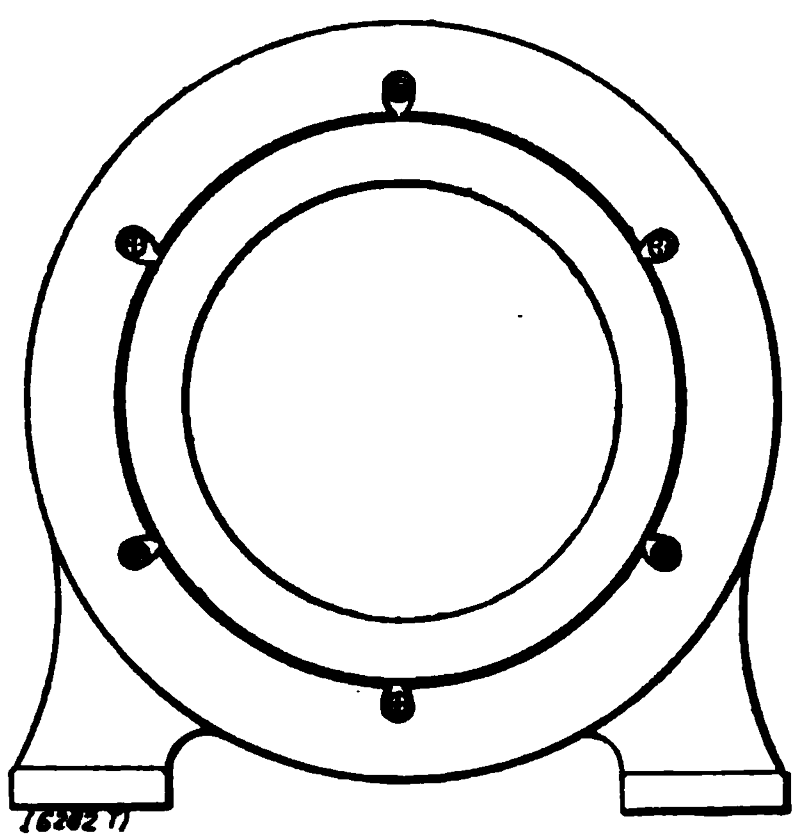


FIG. 371.—A 6-pole Stator with Concentrated Winding.

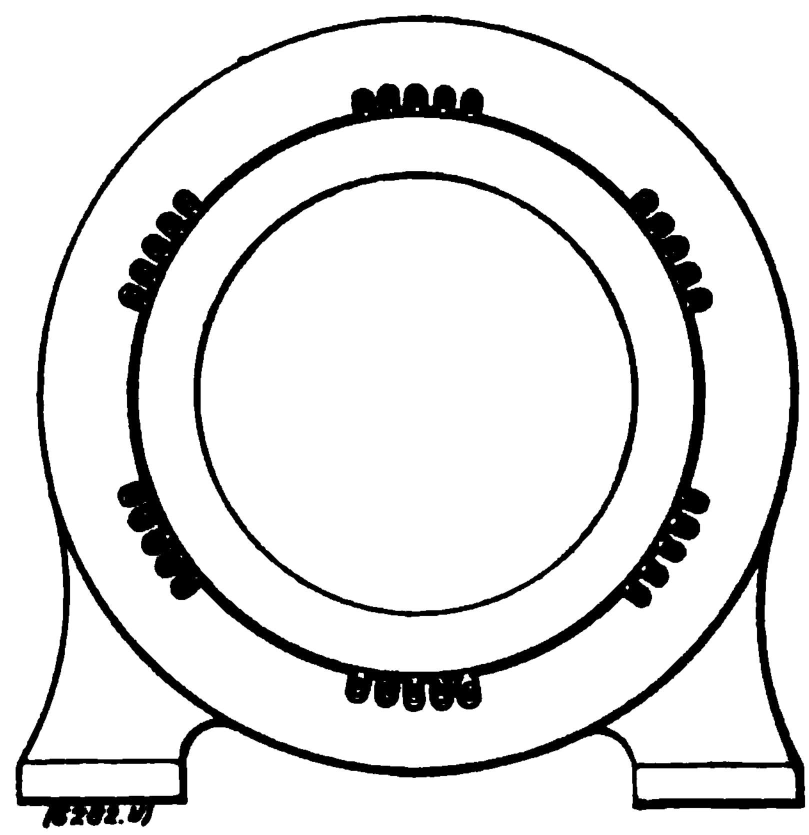


FIG. 372.—A 6-pole Stator with Distributed Winding.

the more simple occurrences relating to the current in the windings of but one phase. In Fig. 373, *A*, abscissæ represent distances along the internal periphery of the stator in Fig. 372, and ordinates represent the magneto-motive force set up at each point by the current in the windings of one phase.

The curve of flux distribution will also tend to conform to the shape shown in Fig. 373, *A*, the flux density at each point being proportional to the magneto-motive force at that point, since the reluctance of the iron path is so nearly negligible. Still confining our attention to one phase, we may take the curve *A* of Fig. 373 to represent the distribution of magneto-motive force and flux at the instant when the current in the winding is at its maximum value. Assuming that the current follows a sine

rate of variation, we shall find that one-twelfth of a cycle later $\left(\frac{360}{30}=\right.$ 30 "degs." of time later; one complete cycle being called 360 electrical degrees for convenience in using trigonometrical functions), the curve of flux distribution along the periphery will have changed from that shown in curve *A*, Fig. 373, to that shown in curve *B*, where the maximum value is but 0·867 as great as in Fig. 373, *A*, since sine 30° = 0·867. Fig. 373,

A

B

C

D

E

Fig. 373.—Diagram of Magneto-motive Force and Flux Distribution.

curves *C*, *D*, and *E*, and Fig. 374, curves *F* to *M*, page 322, represent further successive conditions, one-twelfth of a cycle 30 "degs." apart, and in Fig. 374, *M*, the cycle is seen to have been completed, the original direction and intensity having again been attained.

These, then, would be the changes through which the flux distribution in the gap would go, were there current in the windings of but one phase. The currents in the three lines leading to a three-phase motor are related to one another in phase, as

F

G

H

I

J

K

L

M

FIG. 374.—Diagram of Magneto-motive Force and Flux Distribution.

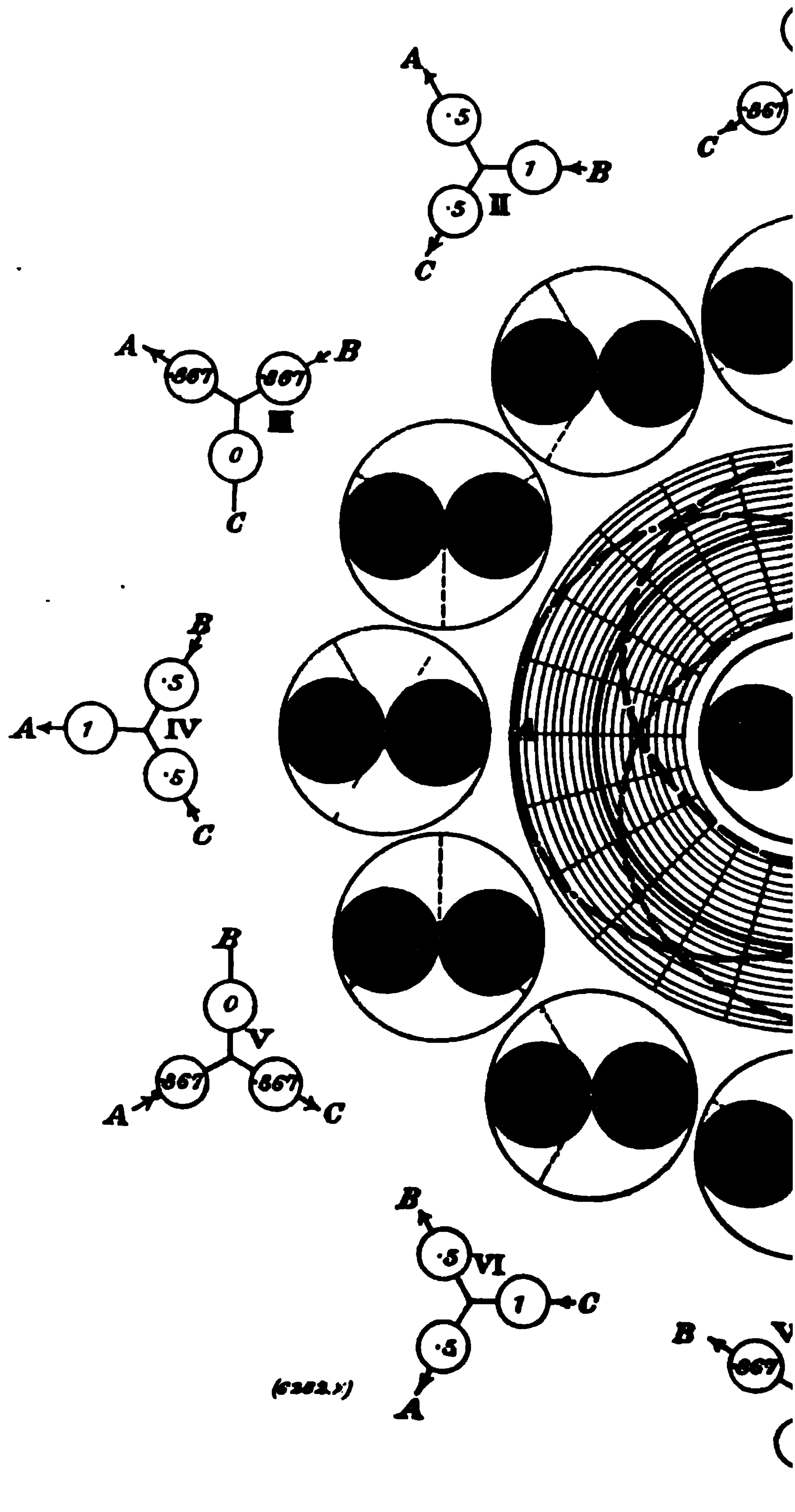
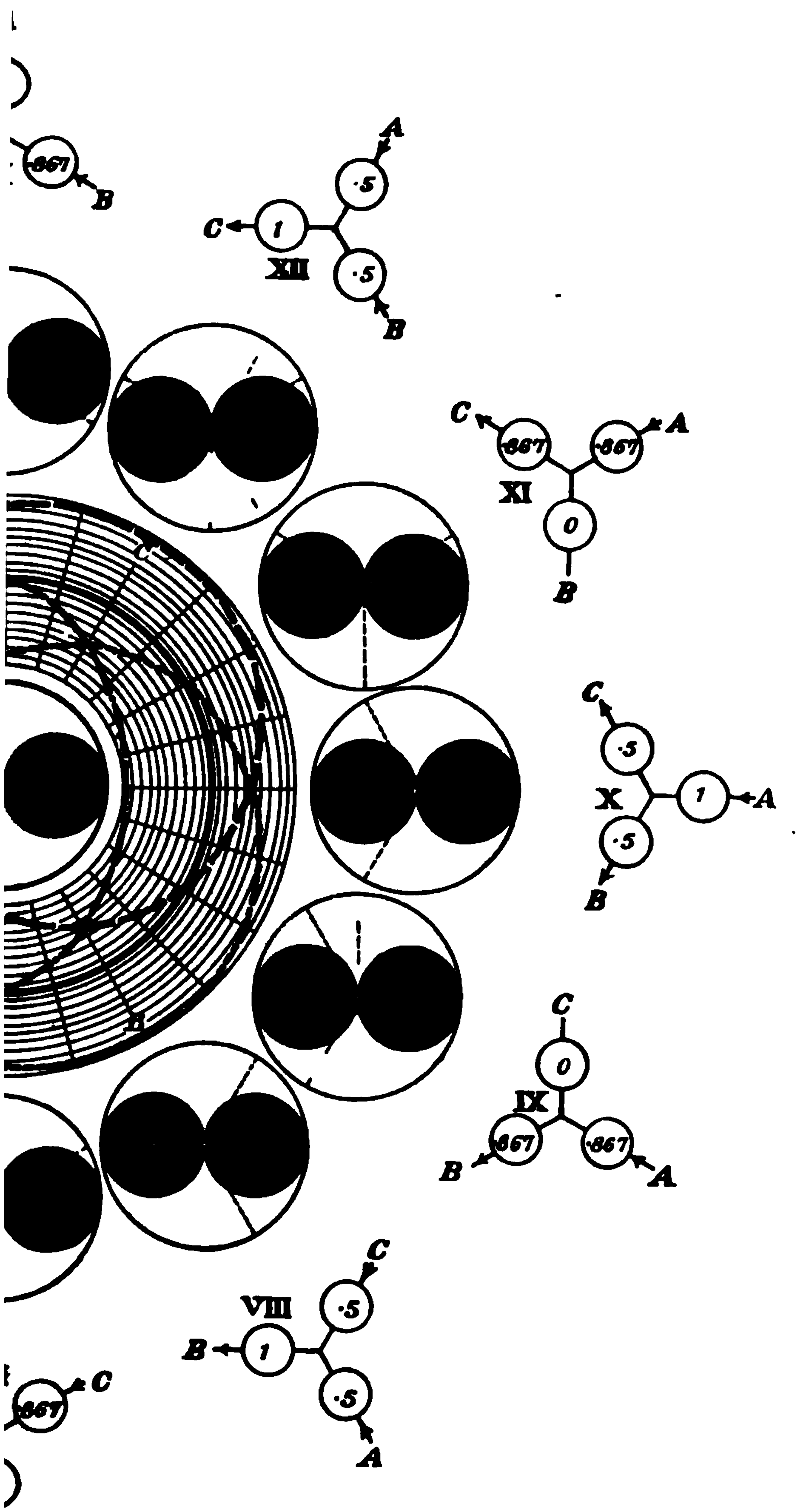


FIG. 376.—Diagram of Cycle of Current (



ıanges in Induction Motor (see page 323).

represented in Fig. 375, where abscissæ represent successive instants of time, and ordinates represent corresponding values of the current. Let Curve *A* of Fig. 375 represent the current in the windings of the phase we have been considering. At the instants indicated by I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, and I^1, the magneto-motive force and flux distribution occasioned by the current in the windings of phase *A* are distributed along the air gap, as indicated respectively by curves *D*, *E*, *F*, *G*, *H*, *I*, *J*, *K*, *L*, *M*, *B*, *C*, and *D*, in Figs. 373 and 374. We now want to ascertain the combined effect of the currents in all three phases, A, B, and C, Fig. 375, and we see that the sum of the ordinates is at any instant nought, from which it follows that the current in any one phase is always equal in amount and opposite in direction to the sum of the currents in the other two phases. In Fig. 376, Plate 21, the twelve outer diagrams represent a complete cycle of

Fig. 375.—Diagram of Time and Current Values.

changes of the currents in the three stator windings, A, B, and C, of an induction motor. The twelve diagrams on the next inner circle are self-explanatory, and have been added as of interest in considering these occurrences. Further, towards the centre, the curves *A*, *B*, *C* of Fig. 375 are plotted to polar co-ordinates. We are now in a position to investigate the magneto-motive force and flux distribution in space resulting from these current-time relations in the three phases. At the instant I (Figs. 375 and 376), when the current in phase A is zero and that in phases B and C is ·867, the magneto-motive force and flux due to B or C alone would, with space as abscissæ, be distributed peripherally about the gap, as shown in Fig. 373, curve *B*. These two magneto-motive forces are, however, 60° apart in space, as shown in Fig. 377, curve *N*, page 324, and the resultant magneto-motive force and flux distribution is shown in Fig. 377, curve *P*, page 324. Now for the instants corresponding to I, III, V, VII, IX, XI, and I^1,

the current in two of the phases is of the value ·867, and in the remaining phase is 0. Hence Fig. 377, curve *P*, represents in magnitude the resultant magneto-motive force and flux for all these cases, the flux being merely shifted 60° in space from time I

FIG. 377.—Curves of Magneto-motive Force and Flux Distribution.

FIG. 378.—Component and Resultant Distribution Curves.

to time III, time V, etc. The condition at the intermediate times II, IV, VI, VIII, X, and XII correspond to a current of ·5 in the windings of each phase and a current of 1 in that of the remaining phase. This gives the three component magneto-motive force distribution curves of Fig. 378, *Q*, and the resultant curve of

distribution of magneto-motive force and flux shown in Fig. 378, *R*, page 324.

Figs. 377, *P*, to 378, *R*, represent the limits between which the magneto-motive force and flux vary while travelling around the gap. The precise areas enclosed by the curves of these two figures are to one another as 100 to 110. Had we analysed the conditions more minutely—taking, for example, for the distribution of the magneto-motive force the more precise curves of Fig. 379—we should have obtained precisely the same maximum values as in Figs. 377, *P*, and 378, *R*, but the areas of the curves of Fig.

S

T

Fig. 379.—Curves of Magneto-motive Force and Flux Distribution.

379 will be found to be practically equal to one another; hence we see that the rotating flux does not always change in total value (expressed, for instance, in number of megalines), but only in the shape of its distribution, which is such as to give a maximum value of the flux density, varying between the limits of 100 (Figs. 377, *P*, and 379, *S*), and 116 (Figs. 378, *R*, and 379, *T*). Smooth curves, A and B, Fig. 380, are shown, having areas and maximum values equal respectively to those of Fig. 379, and placed at an angular distance of 30 "magnetic degrees" from one another, as it is while moving over this angle along the gap that the change in shape from A to B occurs. After travelling another 30° the curve will have returned

to the shape A, Fig. 380; hence we see that in each complete cycle six complete fluctuations in wave form, with 16 per cent. variation in maximum intensity, occur.

The maximum value of curve *R*, Fig. 378, also of curve *T*, Fig. 379, and of curve B of Fig. 380, which is the maximum value occurring in the air gap, is just twice the maximum value of curve *A*, Fig. 373, so that for magnetising current determinations we may take *the resultant magneto-motive force of three phases as equal to twice the magneto-motive force of one phase.*

A

B

C

Fig. 380.—Curves of Magneto-motive Force and Flux Distribution.

The magnetising current is estimated from the maximum flux density, which depends, for a given *total* flux, upon the shape of the curve of distribution of that total flux. This, we have seen, is constantly varying between the limiting forms shown by curves A and B of Fig. 380. But it remains to determine the value of the total flux M, which will be set up when we have a given voltage per phase, E, a given number of turns per phase in series T, and a given periodicity of N cycles per second.

As the curves A and B of Fig. 380 are identical in area,

differing only in shape, it is most convenient and amply exact, for the purpose of electro-motive force determinations, to take for the curve of flux distribution the equivalent sine wave shown in curve C. But, in determining the required magnetising force, we must allow for the 8 per cent. by which the maximum value of the flux density of curve B exceeds that of the equivalent sine curve C of Fig. 380. Thus the maximum flux density to be dealt with is *not* $\frac{1}{\cdot 637} = 1{\cdot}57$ times the average value for this sine curve C, but $1{\cdot}08 \times 1{\cdot}57 = 1{\cdot}70$ times the average value.

It is evident that the entire flux per pole, M, is only linked with all the turns per phase, T, when the conductors are concentrated in one slot per pole per phase, as shown in the diagram of Fig. 381, representing a "uni-slot" winding. When the conductors are distributed in many slots per pole per phase (a so-called "distributed" winding), as shown in the diagram of Fig. 382, a "breadth coefficient" must be introduced in order to make allowance for the incomplete linkage of flux and turns. This "breadth coefficient" for three-phase motors approaches the value 0·95 the greater the number of slots per pole per phase. It is desirable to take it as ·95 for all cases. For a sine curve of flux distribution, and a uni-slot winding such as that of Fig 381, for which the "breadth coefficient" is 1·00, the effective (R.M.S.) electro-motive force in the windings of one phase is $E = 4{\cdot}44\ T\ N\ M \times 10^{-8}$. Introducing the "breadth coefficient," we obtain as a general

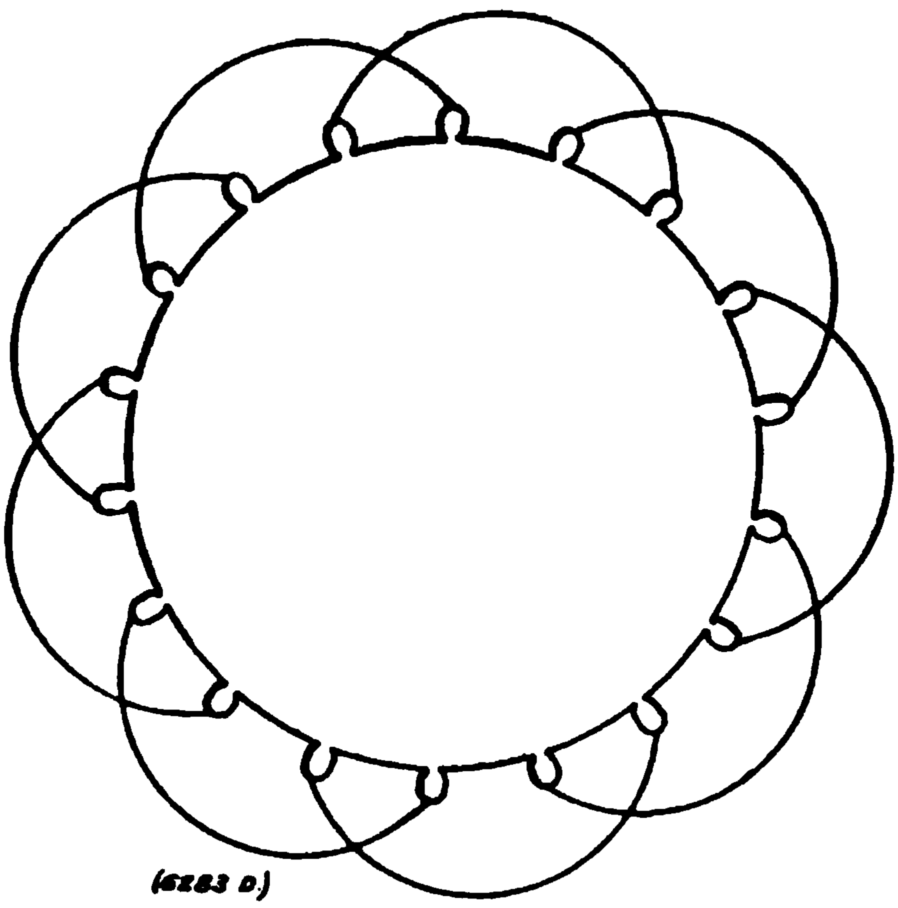


FIG. 381.—Diagram of Uni-slot Winding.

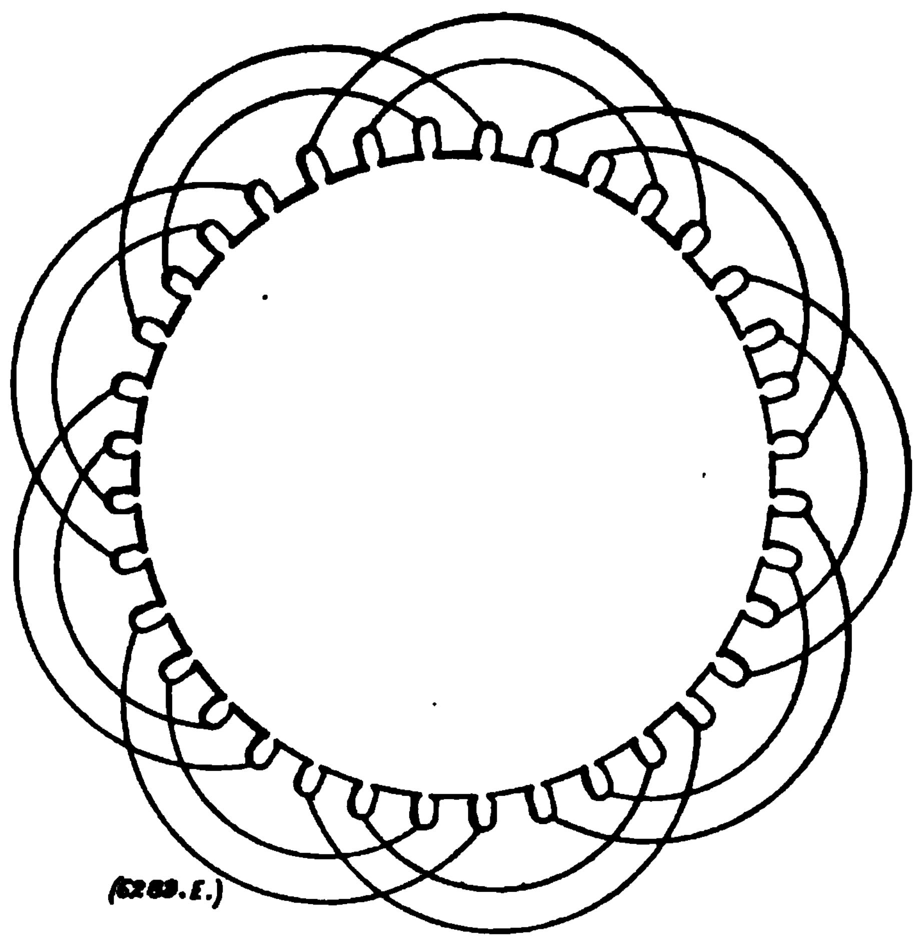


FIG. 382.—Diagram of Distributed Winding.

formula for three-phase motors, with distributed windings, such, for example, as Fig. 382,

$$E = 4{\cdot}44 \times {\cdot}95 \times T\,N\,M \times 10^{-8}$$
$$\text{or } E = 4{\cdot}2\,T\,N\,M \times 10^{-8},$$

where T, N, and M have the same significance as in Part I. E is the effective value of the electro-motive force—*i.e.* the square root of the mean of the squares of the instantaneous values of the electro-motive force, the so-called "root-mean-square" (R.M.S.) value.

§ 8. **Calculations for an 8-Pole 50-Cycle Three-phase Motor.**—Let us now take the case of an 8-pole 50-cycle three-

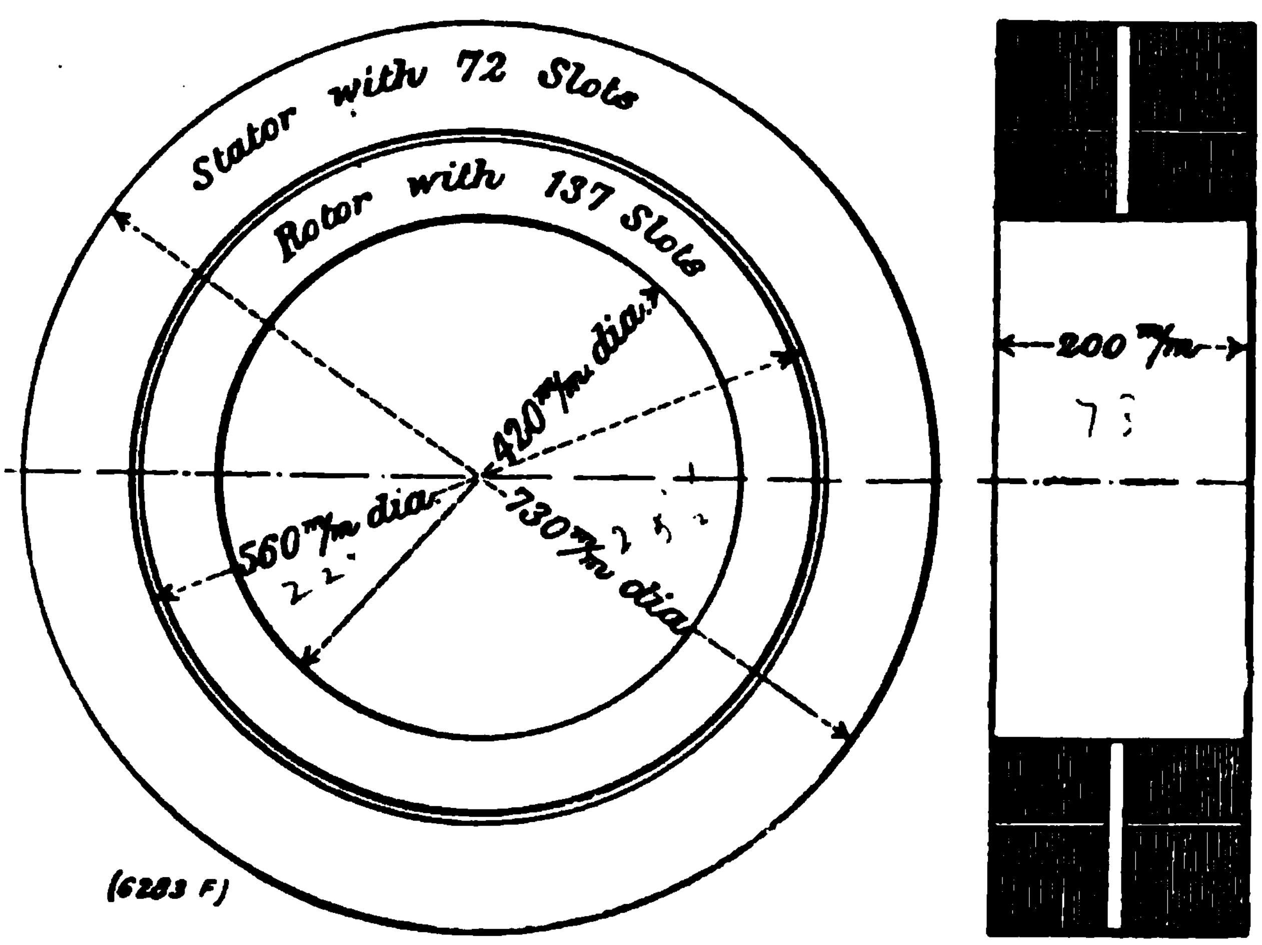


FIG. 383.—Core of 8-pole, 50-cycle, Three-phase Induction Motor.

phase induction motor with the primary wound in seventy-two slots, three slots per pole per phase. The core of this motor is shown in Fig. 383. The primary (stator) windings are Y connected, and the motor is designed for 500 terminal volts, the winding consisting of ten conductors per slot. The voltage per phase $= \frac{500}{\sqrt{3}} = 288$. The counter electro-motive force per phase may be taken at 280 volts.

Total number of face conductors	$72 \times 10 = 720$
Total number of turns	360
Total number of turns per phase	120

$$E = 4{\cdot}2\,T\,N\,M \times 10^{-8}$$
$$280 = 4{\cdot}2 \times 120 \times 50 \times M \times 10^{-8}$$
$$\therefore M = 1{\cdot}11 \text{ megalines.}$$

Gross length of core between flanges	200 millimetres
One ventilating duct	15 ,,
Net effective length of core $185 \times \cdot 90 =$	166 ,,
Diameter of bore of stator	560 ,,
Polar pitch $= 560\pi \div 8$	220 ,,
Stator slot pitch $= 560\pi \div 72 =$	24·5 ,,
Stator slot opening	3·5 ,,
Exposed iron at stator surface, in per cent. of slot pitch $(24{\cdot}5 - 3{\cdot}5) \div 24{\cdot}5 =$	87 per cent.
Number of rotor slots	137
Rotor slot opening	2 millimetres
Rotor slot pitch $= (560 \times \pi) \div 137$	12·9
Exposed iron at rotor surface $= \dfrac{12{\cdot}9 - 2{\cdot}0}{12{\cdot}9} =$	84 per cent.
Mean exposed iron for stator and rotor	85 ,,
Mean cross section exposed iron at air gap $= 16{\cdot}6 \times 22{\cdot}0 \times \cdot 85$	310 sq. cms.

The cross section of the air gap is taken 15 per cent. greater than the "mean cross section of the exposed iron at the air gap," and 1·15 may be regarded as a "spreading coefficient." It should be taken somewhat greater the greater the radial depth of the gap and the greater the width of slot openings. From 1·15 to 1·25 will, for all customary cases, be found to be a fairly satisfactory range of values.

Air gap cross section $= 1{\cdot}15 \times 310 =$ ...	356 square centimetres.
Average density $= 1{,}110{,}000 \div 356 =$ ...	3120 lines per sq. cm.

We have seen that the maximum value of the magnetic flux is *not* equal to the average value divided by ·637 (which would be the case for a sine wave of distribution, such as C, Fig. 380), but is the maximnm value found in curve B of Fig. 380, which will be 8 per cent. greater. Hence the maximum value must be found from the average value, *not* by multiplying the latter by $\frac{1}{\cdot 637} = 1{\cdot}57$, but by $1{\cdot}08 \times 1{\cdot}57 = 1{\cdot}7$. Therefore the maximum density $= 1{\cdot}7 \times 3120 = 5300$ lines per square centimetre. The minimum width of stator tooth is about 10 millimetres, and of rotor tooth about 5 millimetres. Maximum density in any point of any stator tooth $= 12{,}800$; of any rotor tooth $= 13{,}300$.

These densities only occur at one point of stator and rotor tooth respectively, and will not be associated with any considerable magneto-motive force.

Density in the laminations above the stator slots = 6500.
Density in the laminations below the rotor slots = 9000.

Instead of attempting to calculate the magneto-motive force for the teeth and cores more accurately than to ascertain by inspection that it will not increase the total required magneto-motive force by more than some 10 or 15 per cent., we obtain an "equivalent" depth of air gap by multiplying the actual radial depth by, say, 10 per cent. (This convenient method of substituting an "equivalent" depth of air gap is due to Kapp. See *Elektromechanische Konstruktionen*, pages 35 and 179, Second Edition. Julius Springer, Berlin, 1902).

Actual radial depth of air gap $= 1{\cdot}10$ millimetres.
"Equivalent" depth $= 1{\cdot}10 \times 1{\cdot}10 = 1{\cdot}21$ millimetres.

Magneto-motive force required by a flux density of 5300 through an air length of ·121 centimetres equals $0{\cdot}8 \times 5300 \times {\cdot}121 = 515$ ampere turns. We have seen that the resultant magneto-motive force of three phases is twice the magneto-motive force per phase, hence $\frac{515}{2} = 260$ ampere turns per phase are required. There are 120 turns in series per phase, hence $\frac{120}{8}$, or 15 turns per pole per phase. Hence there are required $\frac{260}{15} = 17{\cdot}3$ maximum amperes per phase, and $\frac{17{\cdot}3}{\sqrt{2}} = 12{\cdot}2$ R.M.S. amperes per phase, for magnetising.

The value varies with the wave shape of the circuit from which the motor is operated. Furthermore, since it is difficult to determine with any accuracy the radial depth of so short an air gap, it may in general be considered impracticable to predetermine values of the magnetising current in induction motors within a greater accuracy than 15 per cent. or 20 per cent. Suppose this motor is rated at 35 horse-power with an "apparent efficiency" of 80 per cent. Then its full load current is $\frac{35 \times 746}{{\cdot}80 \times 3 \times 288} = 38$ amperes, and the magnetising current is $\frac{12{\cdot}2}{38}$, or 32 per cent. of full load current. The total current consumed by the motor when running unloaded is the resultant of the magnetising current as determined above, and the current required to supply the losses occasioned by hysteresis, Foucault currents, and bearing friction. This latter current is in phase with the impressed electro-motive force, whereas the magnetising current lags by 90 per cent. Now

the core loss is made up of hysteresis and Foucault current losses, and these can vary through very wide limits, even in motors built from the same specification.

§ 9. **Curve for Estimating Core Losses in Induction Motors.**—Iron for induction motors is much more carefully chosen, and receives, during the construction of the motor, much more careful handling than the iron used for the cores of continuous current machinery. The teeth are rarely touched with a file after assembling. Hence, instead of the curve given in Fig. 21 on page 30, the curve in Fig. 384 may be used in induction motor work for determining the core loss. Even lower values should be readily obtainable by the exercise of reasonable care in specifying the quality of iron required and in testing samples from the

Fig. 384.—Curve for Determining Core Losses.

material received. From the dimensions in Fig. 383 we obtain for the weights of stator and rotor laminations 200 kilogrammes and 130 kilogrammes respectively. The density in the stator core, back of the slots, works out at 6500 lines per square centimetre, and from the curve of Fig. 384 we obtain for $\frac{\text{C D}}{100} = \frac{50 \times 6{\cdot}5}{100} = 3{\cdot}25$, the value of 5·1 watts per kilogramme. Thus the total stator core loss is $200 \times 5{\cdot}1 = 1020$ watts. In the rotor the reversal of magnetisation is at the slow rate corresponding to the "slip," and could almost be neglected even at full load. It is, however, good practice to increase the stator core loss by 10 per cent., and take for the total core loss $1020 \times 1{\cdot}10 = 1120$ watts.

Taking the bearing and air friction at 400 watts, we have for the total watts input, for the motor running light, $1120 + 400 =$

1520 watts, requiring an energy current of $\frac{1520}{3 \times 288} = 1{\cdot}8$ amperes per phase. Hence the total current input of the motor, running light, is equal to $\sqrt{12{\cdot}2^2 + 1{\cdot}8^2} = 12{\cdot}3$ amperes; volt-amperes input $= 3 \times 12{\cdot}3 \times 288 = 10{,}600$; power factor running light $= \frac{1520}{10{,}600} =$ $0{\cdot}143$. The current at no load lags 82° behind the terminal voltage, since cos. 82° = ·14.

§ 10. **Inductance of the Windings.**—Coming next to the question of the estimation of the inductance of the windings, we find it to be a much more inaccurate matter even than the predetermination of the magnetising current, and we must keep in close touch with observed results in constructing our theory.

It *is* practically the predetermination of the magnetising current, but for one phase of the stator winding, or, more precisely, it is the magnetising current which would be required were the reluctance of the magnetic circuit that offered by all the paths exclusive of that path leading through the secondary winding.

In estimating the inductance of the turns short circuited under the brush in continuous current armatures, we have used as a basis the rough values of 4 c.g.s. lines per ampere turn per centimetre of "embedded" length, and 0·8 c.g.s. line per ampere turn per centimetre of "free" length.

In induction motors we are concerned with the inductance, not of small compact groups of conductors occupying 2 or 3 centimetres of the periphery, but of the more or less spread-out groups corresponding to the conductors per pole per phase. In most induction motors the polar pitch—*i.e.* that portion of the periphery devoted to one pole (the gap periphery divided by the number of poles)—amounts, in different designs, according to the periodicity, the normal speed, and the designer's choice, to from 18 to 45 centimetres; hence the belt of conductors belonging to one phase occupies a peripheral width of from 6 to 15 centimetres. The radial depth of winding is not generally different from that customary in continuous current machines. For the greater width considerably smaller values should be employed for the lines per ampere turn per centimetre of "embedded" and "free" length in deriving the inductance. The values given in Table XLVIII. are suggested as affording a fairly satisfactory basis for calculation, though they must be considered only approximate.

TABLE XLVIII.—LINES PER AMPERE TURN PER CENTIMETRE OF "EMBEDDED" LENGTH (APPROXIMATE).

Polar Pitch in Centimetres.	Wide Open Slots.	Half Open Slots.	Completely Closed Slots.
20	0·86	1·08	1·34
25	0·69	0·87	1·08
30	0·57	0·72	0·90
35	0·49	0·62	0·77
40	0·43	0·54	0·67
45	0·38	0·48	0·60

Another variable is introduced by the so-called "zig-zag" flux, which is inversely proportional to Δ, the radial depth of the air-gap; and to H, the average number of slots per pole for stator and rotor. The values in Table XLVIII. are a very fair approximation in the case of designs where $\Delta \times H$ lies between 1·4 and 2·0. For designs lying outside of these values, a correction must be applied, which is greater, the greater the deviation of $\Delta \times H$ from this range of values.

Lines per ampere turn per centimetre of "free" length may be taken at 0·4 for all values of the pitch. Of course the mechanical arrangement (*i.e.* grouping, etc.) of the end connections, occasions considerable variations in this constant.

For motors with squirrel cage rotors, take 0·3 line per ampere turn per centimetre of "free" length, taking as "free" length that of the stator winding.

§ 11. Analysis of the Inductance of a 12-Pole Motor.—Let us employ this data in the estimation of the inductance of a motor to which the following data applies:

Diameter of air gap	109 centimetres
Periphery of air gap	342 „
Number of poles	12 „
Polar pitch	28·5 „

The slots are nearly closed. They are shown to scale in Fig. 385 (page 334), and we shall, for the inductance calculations, employ the values: 0·93 line per ampere turn per centimetre of "embedded" length, and 0·40 line per ampere turn per centimetre of "free" length.

Effective length of core parallel to shaft	24 centimetres
Mean length of one stator turn	159 „
"Embedded" length per turn	48 „
"Free" length per turn	111 „
Lines per ampere turn for "embedded" length ...	$0·93 \times 48 = 44·5$
Lines per ampere turn for "free" length	$0·40 \times 111 = 44·4$

Total lines per ampere turn...	89
Number of stator slots	108
Number per pole per phase	3
Conductors per slot	33
Turns per pole per phase	49·5

But the winding, so far as relates to the estimation of its inductance, is equivalent to 6 coils of 99 turns each, that is, to one 99-turn coil *per pair* of poles (*not* to 12 coils of 49·5 turns each, *i.e.* one 49·5-turn coil per pole).

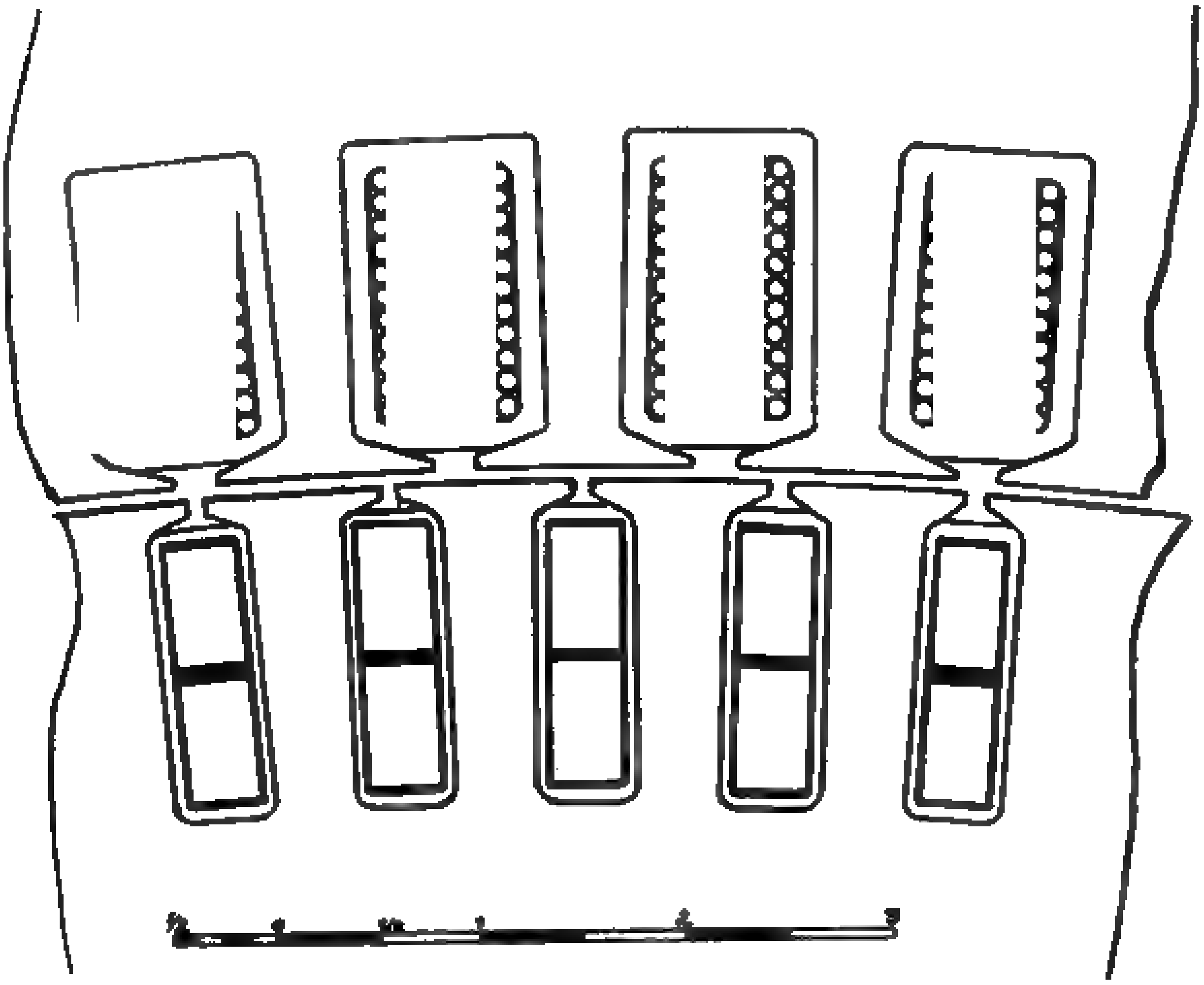


Fig. 385.—Slots of Rotor and Stator.

Lines per ampere = 89 × 99 =	8800·
Inductance per coil = 99 × 8800 × 10 − ⁸ =	0·0067 henry.
Inductance per phase = 0·0067 × 6 =	0·0523 „
Periodicity in cycles per second	50·
Reactance per phase = 6·28 × 50 × 0·0523 = 16·4 ohms.	

The motor is for 5000 terminal volts, and the stator (primary) windings are Y connected, there being therefore $\frac{5000}{\sqrt{3}} = 2880$ volts per phase.

The rotor windings are also connected Y, and there are two conductors per slot. There are 144 rotor slots; hence 8 conductors

in series per pole per phase, as against 99 in the stator. Therefore the true rotor inductance per phase is $\left(\frac{8}{99}\right)^2 \times 0{\cdot}0523 = 0{\cdot}0034$ henry.

But at this stage in induction motor calculations it is more convenient to assume an equivalent rotor, wound for a 1 : 1 ratio of transformation. This is justifiable, since, with sufficient subdivision of the winding in many slots, the operation will be the same whatever the precise number of rotor conductors, provided the product of number of conductors by cross section of each conductor—*i.e.* the total cross section of all the secondary conductors —remains unchanged. Hence we will assume an "equivalent" rotor with 3564 conductors (this being the total number of stator conductors), each conductor having a cross section of 0·116 square centimetres instead of the actual winding with 288 conductors with each a cross section of $1{\cdot}6 \times 0{\cdot}90 = 1{\cdot}44$ square centimetres $\left(\frac{288}{3564} \times 1{\cdot}44 = 0{\cdot}116\right)$. Instead of the actual resistance of 0·010 ohms at 60° cent., each phase of the "equivalent" winding will have a resistance of $\frac{1{\cdot}44}{0{\cdot}116} \times \frac{3564}{288} \times 0{\cdot}010 = 1{\cdot}53$ ohms. The actual primary resistance per phase = 2·20 ohms at 60° cent.

This "equivalent" rotor winding will have about the same inductance per phase as the primary winding, namely, 0·0523 henry.

In practice the inductance estimations are made by a much simpler method, which will subsequently be set forth. For a fundamental understanding of the underlying phenomena, however, such an analysis as the above is preferable.

The flux corresponding to the primary reactance voltage is only one portion of the total flux linked with the primary turns. This is a matter in which error may readily arise.

§ 12. **The Actual Magnetic Fluxes Present at any Load.**—These are:

(1) Primary Flux, M_p.—A flux, corresponding almost exactly with the primary terminal voltage per phase—in fact, differing therefrom only by the small amount caused by vectorially subtracting the primary I R drop. Of this total flux, the component, corresponding in amount to the primary reactance voltage, is linked with the primary winding alone, and does not succeed in entering the secondary winding.

(2) Air Gap Flux, M_g.—The remaining component of this

primary flux—*i.e.* the total primary flux minus that portion corresponding in amount to the primary reactance voltage—would become completely linked with the secondary winding were the latter devoid of inductance. This never being the case, there is also a

(3) Secondary Flux, M_s.—Third flux to be considered. This is the residual which, at any given load, is finally linked with the secondary winding, and which is less than the second flux mentioned, to the extent obtained by subtracting the flux corresponding in amount to the secondary reactance voltage. This third flux suffices, at the slip occurring, to generate in the secondary windings an electromotive force per phase equal to the product of resistance per phase and current per phase.

It is fairly in accordance with the true occurrences to consider that these three fluxes really exist; all others are imaginary components. We denote these three fluxes by M_p, M_g, and M_s. Of the flux M_p originally set up, only M_g crosses to the rotor. M_g would become linked with the secondary windings had this latter no reactance—*i.e.* for this case M_g would equal M_s, but owing to the reactance of the secondary only the portion M_s of the flux M_g finally becomes linked with the secondary windings.

To illustrate this diagrammatically, let us neglect the primary resistance drop, which would be small, and in Fig. 386 represent by O A the primary terminal voltage, and by O A^1 the primary counter electro-motive force which, for zero primary resistance—*i.e.* for $R_p = 0$—is equal and opposite to O A. The total flux M_p linked with the primary windings may be represented by O M, and, since O A^1 is proportional to the rate of change in O M, the latter must be 90° in advance of O A^1. The load corresponding to the conditions represented in the diagram may be taken as being such as to require a primary current O B, which lags by the angle ϕ behind the terminal voltage O A. The product of primary current per phase O B, and primary reactance per phase, is the primary reactance voltage per phase, H_p, and is plotted as O C, 90° behind O B. Under the conditions of load, represented by the diagram, O C is merely an imaginary voltage component which we subtract vectorially from O A, and obtain O D, which is also an imaginary voltage component, the magnitude of which enables us to calculate M_g, that portion O P of the total primary flux which would enter the secondary windings were the latter non-inductive. So O D is proportional to, and 90° behind, O P, the second of the three above-mentioned magnetic

fluxes. Not quite 180° behind the primary current O B comes the secondary current O E, which must be so related in magnitude and direction to the primary current O B as to give a resultant magnetising current O L sufficient to set up the required flux in the magnetic circuit. The voltage required for driving the secondary current O E through the secondary resistance R_s is laid off on the vector O E at O F, and this latter voltage (*i.e.* the secondary C R drop) is a measure of the flux O N (90° in advance of O F) actually linked with the secondary winding. At 90° behind the secondary current, O E, comes H_s, the secondary re-

Fig. 366.—Vector Diagram of Induction Motor.

actance voltage plotted as O G, this being the product of the reactance per phase of the secondary windings at the periodicity corresponding to the slip, with O E the secondary current per phase.

The vector O H, which, compounded with the secondary reactance voltage O G, gives as resultant the internal secondary voltage O F, is also a measure of O P (M_g), the second of the three fluxes—*i.e.* it is a measure of the flux which would be linked with the secondary winding were it absolutely non-inductive. Sometimes O H is looked upon as the internal secondary voltage, and it is explained that it is a voltage sufficient to overcome the resultant of the secondary resistance voltage O F and the re-

actance voltage O G, but since the flux, O P (M_g), corresponding in magnitude to the imaginary voltage O H, never becomes linked with the secondary winding, but only the smaller flux, O N (M_s), corresponding to the smaller voltage O F (the secondary I R voltage), this is an undesirable designation, and this latter voltage O F is alone correctly to be regarded as the secondary internal voltage, and the larger voltage, O H, is correctly only an imaginary quantity, so long as the secondary has inductance. With decreasing secondary inductance, O F approaches O H in magnitude and direction, as O G decreases, and for a non-inductive secondary they become identical, since O G then becomes zero. The flux O N, corresponding in magnitude to the voltage O F, and 90° in advance of it, is M_s, the third of the three fluxes; it is the fraction of the original flux O M (M_p), which actually becomes linked with the secondary winding.

We shall shortly show by precise constructions that even M_s, the third (and least) of these three fluxes, is, at full load, but little less than at no load; hence also the intermediate value O P (M_g, the second of the three fluxes), which may fairly be taken as representative of the air gap density. This justifies the retention of the no load value for O L, the resultant magneto-motive force for all loads, and the utilising as the locus of its phase position at any load, the direction of M_g, the second of the three fluxes—*i.e.* the direction of the normal to O D.

In Fig. 387 are plotted without the other lines, the primary terminal voltage per phase, O A (V_p), and the flux, O M (M_p), associated with its counter electro-motive force O A^1 (E_p), which latter we have seen is, for zero primary resistance, identical with it in magnitude, and about 180° behind it in phase, so that O M (M_p) is 90° behind O A in phase. There is, furthermore, shown in the diagram, E_s, the actual internal secondary voltage plotted as O F, and M_s, the flux O N, which actually becomes linked with the secondary windings. O M (M_p) is the largest of the three fluxes; O N (M_s) is the least, O P (M_g) having an intermediate value. .O P (M) is laid off at right angles to O D, the resultant of the primary counter electro-motive force E_p, and the primary reactance voltage H_p this being also the direction of the resultant of the secondary internal and reactance voltages, the line O H of Fig. 386. All the six lines drawn in Fig. 387 represent voltages and fluxes actually present. Even the intermediate flux O P (M_g) is present, though less tangibly, as there is no winding from whose voltage we may derive it. For this reason O D is indicated

in Fig. 387 by a broken line. O P (M), however, being in reality the flux corresponding in direction with the resultant magnetising current O L (of Fig. 386), is indicated by a heavy line.

In the following, we will continue to denote these three fluxes by M_p, M_g, and M_s, and the corresponding electro-motive forces by E_p, E_g, and E_s. The primary terminal voltage we will denote by V_p, this seldom differing appreciably in magnitude from E_p, but leading the latter by about 180° in phase. Hence V_p is also often a measure of M_p, so far as relates to magnitude. E_g has, as we have seen, no real existence, but will be used and plotted, and

FIG. 387.—Simplified Vector Diagram of Induction Motor.

denotes equally well the vector sum of secondary I R voltage and reactance voltage, or of primary counter electro-motive force and reactance voltage. It would correspond with the secondary internal voltage were the secondary windings non-inductive. It would correspond with the counter electro-motive force in the primary windings were the latter non-inductive.

E_s is the secondary I R voltage. Hence E_p, E_g, and E_s are measures of M_p, M_g, and M_s, and are, therefore, 90° in phase behind them.

We will denote primary reactance voltage by H_p, and secondary reactance voltage by H_s.

I_p = primary current.
I_s = secondary current.
N_p = primary periodicity.
N_s = secondary periodicity = periodicity of slip.
$\phi = \phi_p$ = angle of lag of I_p behind V_p.
ϕ_g = angle between M_p and M_g.
ϕ_s = angle of lag of E_s behind E_g, and of M_s behind M_g.
R_p = primary resistance per phase.
R_s = secondary resistance per phase.

§ 13. **Calculations for a 150 H.P. Motor.**—Assume now the case of a motor having the following constants, chosen as giving convenient values for graphical discussion, not as representing a good design:—

Rated output	150 horse-power.
Periodicity cycles per sec.	40
Terminal voltage	550
Connection of windings	Y
Volts per phase	318
Primary reactance per phase (estimated as shown)	0·296 ohm.
Secondary reactance per phase	0·296 „
R.M.S. Magnetising current (estimated by the method already set forth on p. 319)	50 amperes.

We will first assume that it is without friction or iron losses, and that the resistance of the primary winding is zero. Thus the only loss at first considered is that due to the resistance of the secondary windings.

The terminal voltage V_p, being constant at all loads, and the primary being of no resistance, M_p must be constant for all loads. The resultant magnetising component of the currents will also be practically constant up to loads greatly in excess of the rated load (since M_g, to which the magnetising component is proportional, decreases but slowly up to heavy overloads), and will be taken throughout as constant. At no load the reactance voltage set up by the 50 primary amperes equals $0·296 \times 50 = 14·8$ volts $= H_p$.

In Fig. 388 these no load conditions are plotted to the scale of 1 centimetre per 100 volts, and per 100 amperes; effective (*i.e.* root mean square) values being plotted in all cases.

V_p = O A = 318 volts.
H_p = O C = 14·7 volts = primary reactance voltage.
$E_g = 318 - 14·7 = 303$ volts.
I_p = O L = 50 amperes.

$I_s = 0$, since the diagram is for no load, and the motor is assumed to be without friction or iron losses, hence no torque is required to be developed in the rotor.

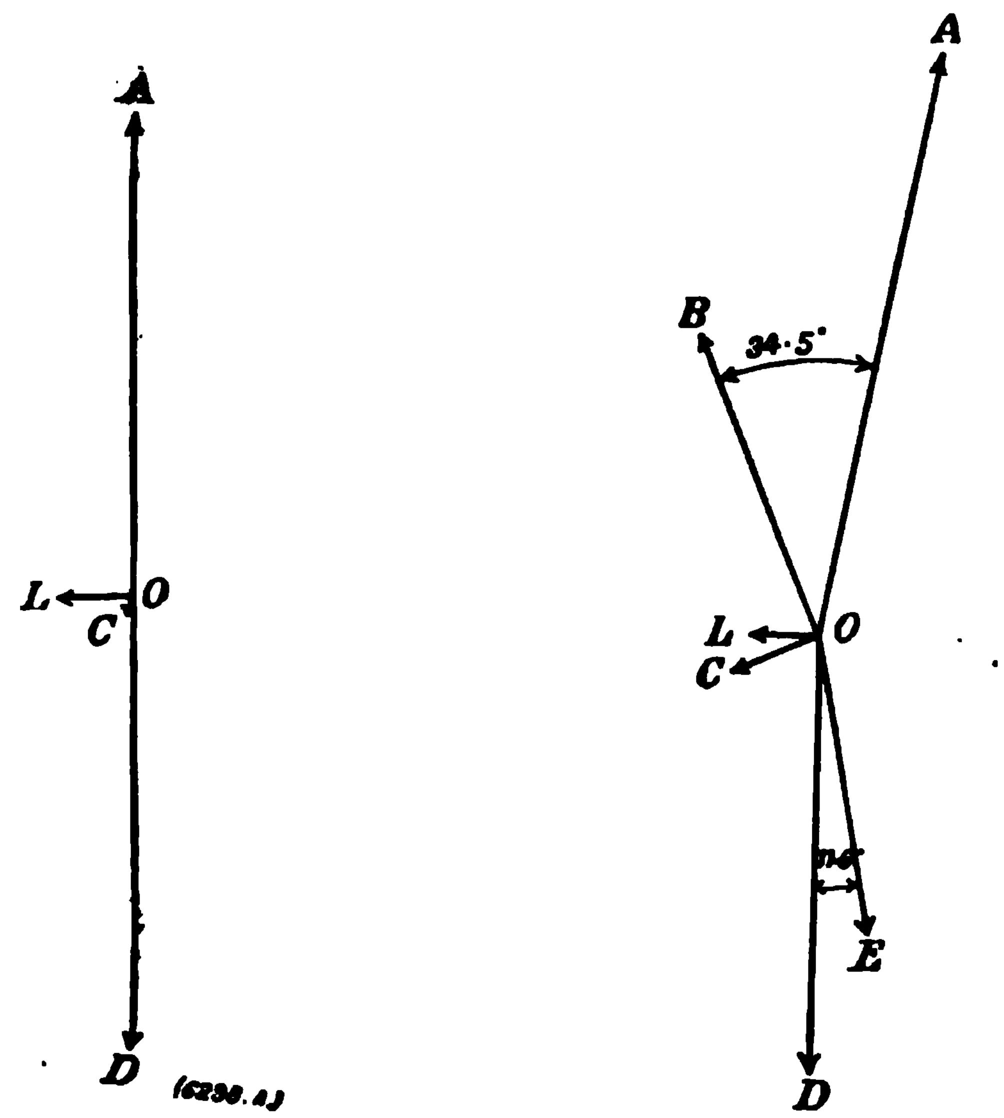
$E = 4{\cdot}2 \times T \times N \times M \times 10^{-8}$.

$E = 318 = V_p$. $T = 80$. $N = 40$.

$$\therefore\ M = M_p = \frac{318 \times 100{,}000{,}000}{4{\cdot}2 \times 80 \times 40} = 2{,}370{,}000 \text{ lines} = 2{\cdot}37 \text{ megalines.}$$

$$M_g = M_s = \frac{303}{318} \times M_p = \frac{303}{318} \times 2{\cdot}37 = 2{\cdot}26 \text{ megalines.}$$

We must next suppose the motor to be given a load. Let this at first be sufficient to cause a flow of current equal to 200 amperes in the "equivalent" rotor (secondary) winding, thus $I_s = 200$. It will be accompanied, in the rotor winding, by an I R drop of $200 \times 0{\cdot}047 = 9{\cdot}4$ volts per phase; $E_s = 9{\cdot}4$. We require



FIGS. 388 and 389.

sufficient "slip" in the rotor—*i.e.* such a value of N_s as to generate 9·4 volts by the cutting of the rotor conductors through the flux M_s. We know only M_p, as M_g can only be exactly derived in the light of a knowledge of the primary current, and M_s by a knowledge of N_s, the rotor "slip." But, except at considerable overloads, M_s will be found to differ from M_p by but a few per cent., and at the present load of 200 amperes per secondary winding we may, as a preliminary trial estimate, take M_s at 10 per cent. less than M_p.

$M_p = 2{\cdot}37$ megalines.

$M_s = {\cdot}90 \times 2{\cdot}37 = 2{\cdot}13$ megalines.

$E_s = 9{\cdot}4 = 4{\cdot}2 \times 80 \times N_s \times 2{,}130{,}000 \times 10^{-8}$.

$$\therefore\ N_s = 1{\cdot}32 \text{ cycles per second, or a "slip" of } \frac{1{\cdot}32 \times 100}{40} = 3{\cdot}3 \text{ p.c.}$$

We may now obtain the secondary reactance voltage H_s, since l, the secondary inductance per phase, has already been estimated at ·00117 henry.

$$H_s = 2\pi \times 1{\cdot}32 \times {\cdot}00117 \times 200 = 1{\cdot}93 \text{ volts per phase.}$$

$$\phi_s = \tan^{-1}\frac{1{\cdot}93}{9{\cdot}4} = \tan^{-1} 0{\cdot}205 = 11{\cdot}6^\circ.$$

In Fig. 389, page 341, draw O E equal to the secondary current I_s (200 amperes), and at an angle ϕ_s (11·6°) behind O D. O B, equal to L E, must be the corresponding primary current I_p, since the resulting magneto-motive force must remain equivalent to that corresponding to the one furnished by 50 amperes in the primary windings at no load, this being the value O L of Figs. 388 and 389.

I_p = O B = 216 amperes.
$H_p = 219 \times {\cdot}294 = 63$ volts.
H_p is laid off at O C, 90° behind C_p (O B).

Knowing the direction of E_g (O D), the *magnitude* of V_p (318 volts), and the direction and magnitude of H_p (O C), we obtain, by construction, the *direction* of V_p (O A), and determine the *magnitude* of E_g (O D), which we find to be 291 volts, expressed in terms of the primary voltage. This possesses for us the first interest, as the degree of agreement between our assumption for M (2·13 megalines), and the value of E_g, is a test of the degree of correctness of that assumption.

$E_g = \sqrt{E_s^2 + H_s^2} = \sqrt{9{\cdot}4^2 + 1{\cdot}9^2} = 9{\cdot}6$ volts, expressed in terms of the secondary voltage.

$$M_g = \frac{9{\cdot}6}{9{\cdot}4} \times 2{\cdot}13 = 2{\cdot}17 \text{ megalines.}$$

Now from the derived value of 291 volts, for E_g we obtain—

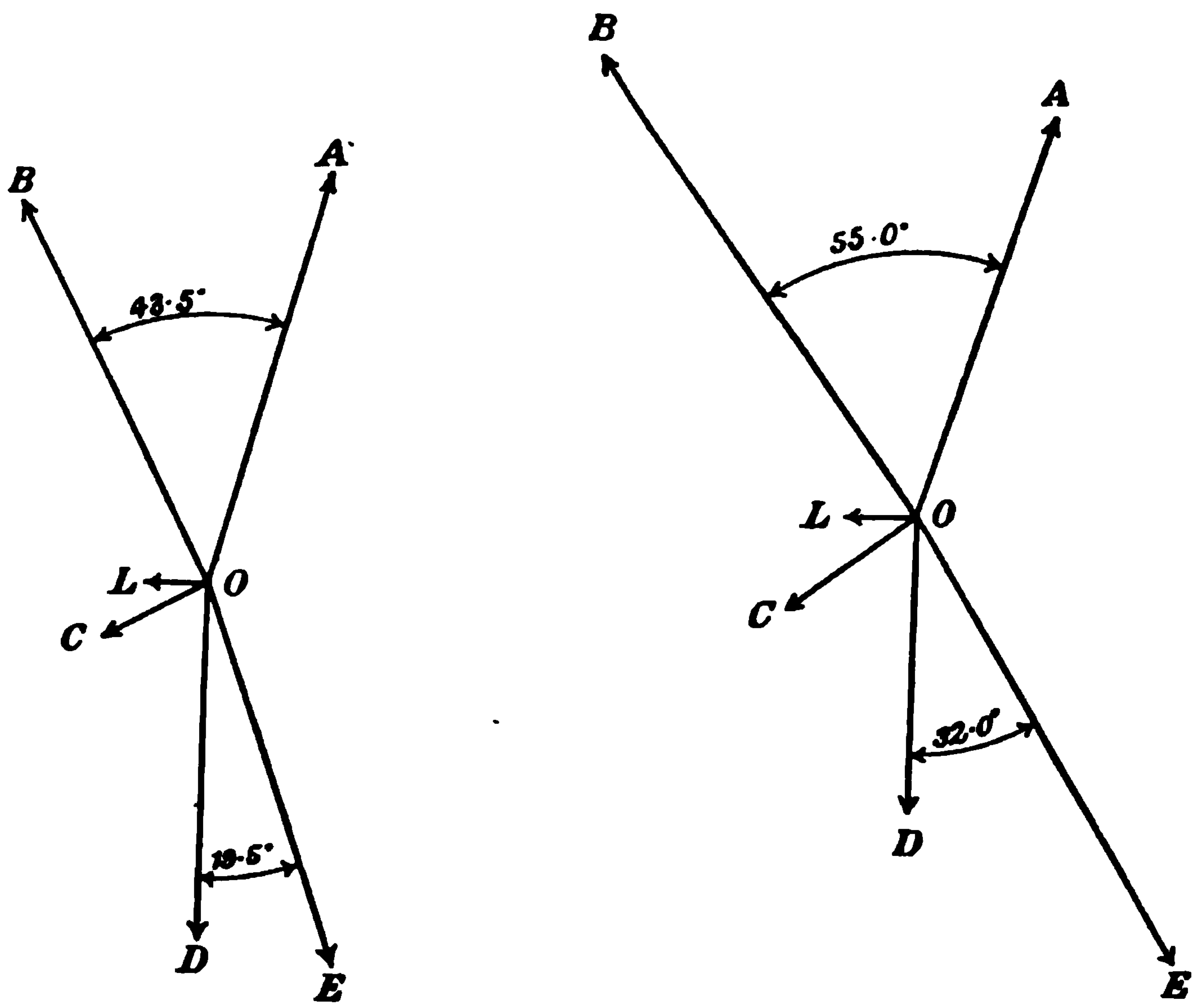
$$M_g = \frac{291}{318} \times 2{\cdot}37 = 2{\cdot}17 \text{ megalines.}$$

The agreement thus justifying our preliminary assumption of a value for M_s 10 per cent. less than M_p. Had this failed to agree closely enough with our final determination, a revision of the calculation, based upon a revised assumption for M_s, would have been required. Of course, in reality, the calculation *as it stands here* is arranged in accordance with results obtained by a more direct construction, to an understanding of which these explanations lead, and which, in practice, is, of course, to be used in preference to such tedious calculations as these, which lead, however, to a more correct understanding of the principles of the inductive motor than is generally obtained by the use, from

the first, of the short and more practical methods at which we shall eventually arrive.

For 300 amperes per secondary winding the diagram of Fig. 390 applies:—

$E_s = 300 \times \cdot 047 = 14 \cdot 1$ volts.
$M_s = \cdot 78 \times 2 \cdot 37 = 1 \cdot 85$ megalines.
$N_s = \dfrac{14 \cdot 1 \times 100{,}000{,}000}{4 \cdot 2 \times 80 \times 1{,}850{,}000} = 2 \cdot 27$ cycles per second $= 5 \cdot 7$ per cent.
$H_s = 2\pi \times 2 \cdot 27 \times \cdot 00117 \times 300 = 5 \cdot 00$ volts per phase.
$\phi_s = \tan^{-1} \dfrac{5 \cdot 00}{14 \cdot 1} = \tan^{-1} \cdot 355 = 19 \cdot 5^\circ$.



FIGS. 390 and 391.

$C_p = 320$ amperes.
$H_p = 320 \times \cdot 294 = 94 \cdot 0$ volts.
$E_g = \sqrt{14 \cdot 1^2 + 5 \cdot 0^2} = 15 \cdot 0$ volts in terms of secondary voltage.
$M_g = \dfrac{15 \cdot 0}{14 \cdot 1} \times 1 \cdot 85 = 1 \cdot 97$ megalines, from assumption.
E_g (from diagram) $= 264$ in terms of primary voltage.
M_g (from diagram) $= \dfrac{264}{318} \times 2 \cdot 37 = 1 \cdot 97$.

For 400 amperes per secondary winding we have in the diagram, Fig. 391:—

$E_s = 400 \times \cdot 047 = 18 \cdot 8$ volts.
$M_s = \cdot 59 \times 2 \cdot 37 = 1 \cdot 40$ megalines.

$N_s = \dfrac{18{\cdot}8 \times 100{,}000{,}000}{4{\cdot}2 \times 80 \times 1{,}400{,}000} = 4{\cdot}00$ cycles per second $= 10{\cdot}0$ per cent. slip

$H_s = 2\pi \times 4{\cdot}00 \times {\cdot}00117 \times 400 = 11{\cdot}8$ volts per phase.

$\phi_s = \tan^{-1} \dfrac{11{\cdot}8}{18{\cdot}8} = \tan^{-1} {\cdot}625 = 32°$.

$I_p = 428$ amperes.

$H_p = 428 \times 294 = 126$ volts.

$E_g = \sqrt{18{\cdot}8^2 \times 11{\cdot}8^2} = 22{\cdot}2$ in terms of secondary volts.

$M_g = \dfrac{22{\cdot}2}{18{\cdot}8} \times 1{\cdot}40 = 1{\cdot}65$ megalines, from assumption.

E_g (from diagram) $= 221$ in terms of primary volts.

M_g (from diagram) $= \dfrac{221}{318} \times 2{\cdot}37 = 1{\cdot}65$ megalines.

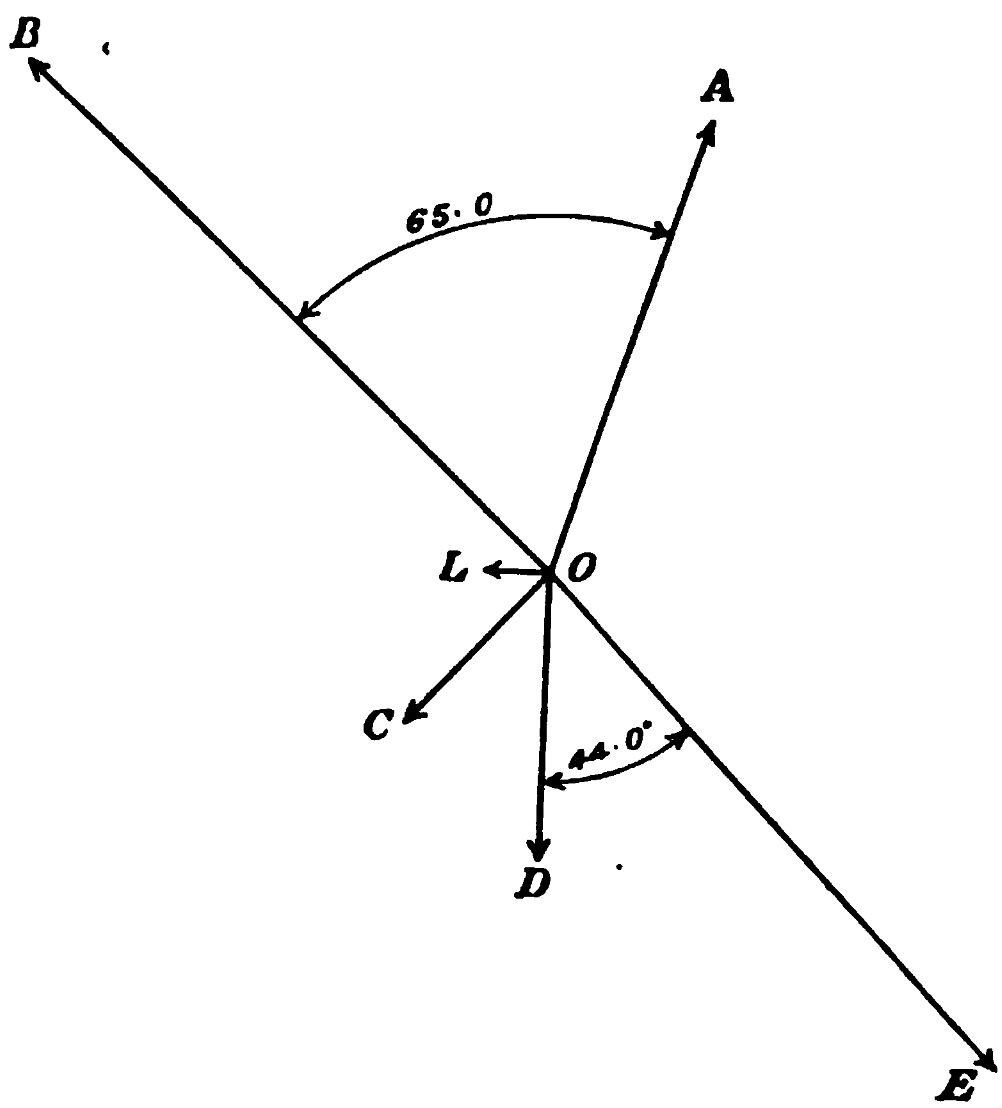


FIG. 392.

For 450 amperes per secondary winding we have in diagram of Fig. 392:—

$E_s = 450 \times {\cdot}047 = 21{\cdot}2$ volts.

$M_s = {\cdot}43 \times 2{\cdot}37 = 1{\cdot}02$ megalines.

$N_s = 21{\cdot}2 \times \dfrac{100{,}000{,}000}{1{,}020{,}000} = 6{\cdot}20$ cycles per second $= 15{\cdot}5$ per cent. slip.

$H_s = 2\pi \times 6{\cdot}20 \times {\cdot}00117 \times 450 = 20{\cdot}5$ volts per phase.

$\phi_s = \tan^{-1} \dfrac{20{\cdot}5}{21{\cdot}2} = \tan^{-1} {\cdot}97 = 44°$.

$I_p = 486$.

$H_p = 486 \times {\cdot}294 = 143$ volts.

$E_g = \sqrt{21{\cdot}2^2 + 20.5^2} = 29{\cdot}5$ in terms of secondary volts.

$M_g = \frac{29 \cdot 5}{21 \cdot 2} \times 1 \cdot 02 = 1 \cdot 42$ megalines, from assumption.

E_g (from diagram) $= 191$ in terms of primary volts.

M_g (from diagram) $= \frac{191}{318} \times 2 \cdot 37 = 1 \cdot 42$ megalines.

For 480 amperes per secondary winding we have the diagram Fig. 393 :—

$E_s = 480 \times \cdot 047 = 22 \cdot 6$ volts.

$M_s = \cdot 26 \times 2 \cdot 37 = \cdot 615$ megaline.

$N_s = \frac{22 \cdot 6 \times 100{,}000{,}000}{4 \cdot 2 \times 80 \times 615{,}000} = 11 \cdot 0$ cycles per second $= 27 \cdot 5$ per cent. slip.

$H_s = 2\pi \times 11 \cdot 0 \times \cdot 00117 \times 480 = 38 \cdot 8$ volts per phase.

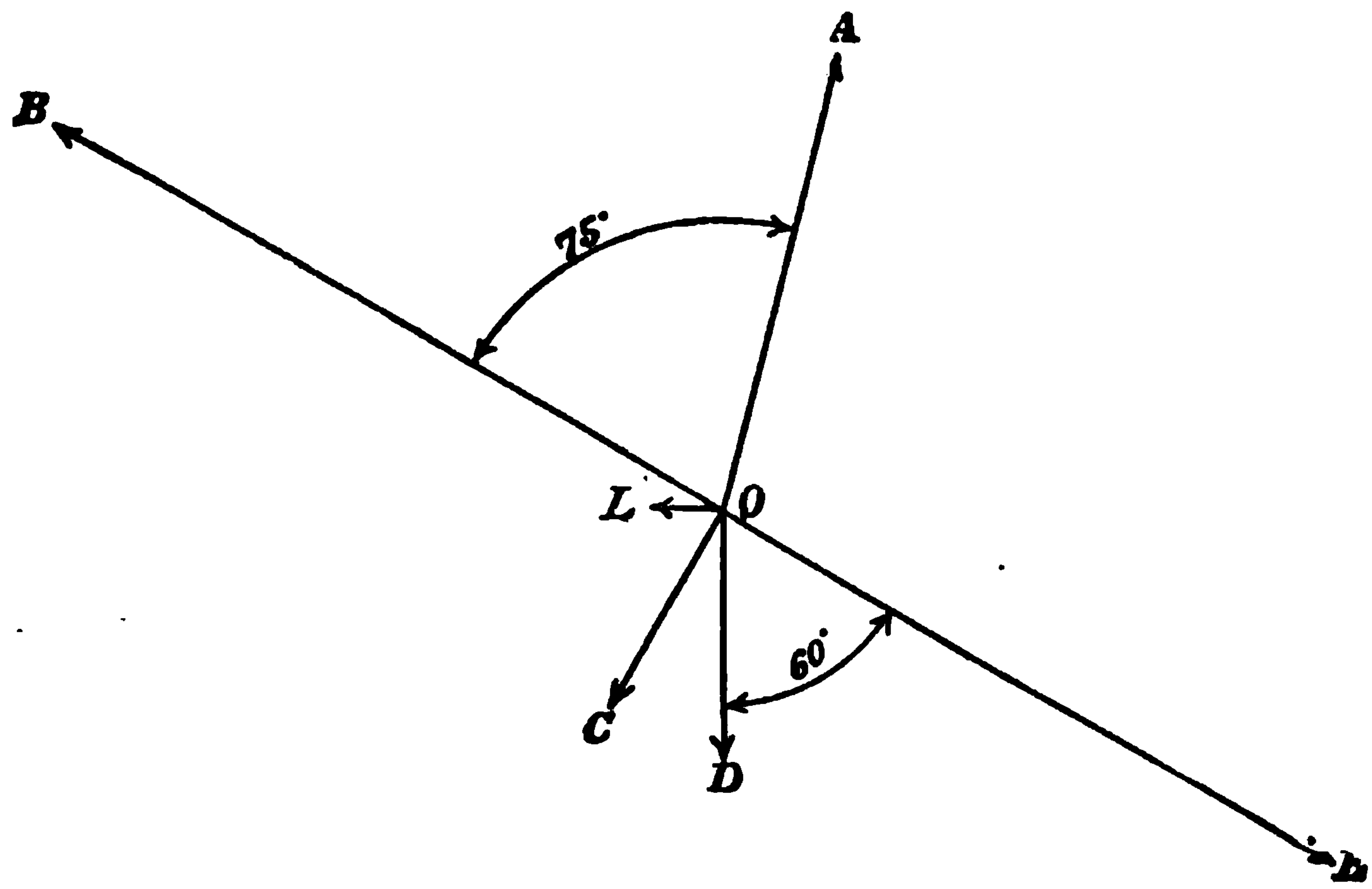


FIG. 393.

$\phi_s = \tan^{-1} \frac{38 \cdot 8}{22 \cdot 5} = \tan^{-1} 1 \cdot 73 = 60°$.

$I_p = 525$.

$H_p = 525 \times \cdot 294 = 154$ volts.

$E_g = \sqrt{22 \cdot 6^2 + 38 \cdot 8^2} = 45 \cdot 0$ in terms of secondary volts.

$M_g = \frac{45 \cdot 0}{22 \cdot 6} \times \cdot 615 = 1 \cdot 23$ megalines, from assumption.

E_g (from diagram) $= 165$ in terms of primary volts.

M_g (from diagram) $= \frac{165}{318} \times 237 = 1 \cdot 23$ megalines.

It will have been observed from this group of diagrams how the slip has increased with ever greater rapidity for each successive increment in current. Thus we have the values in Table XLIX. :—

TABLE XLIX.—SHOWING INCREASE IN SLIP WITH INCREASE IN CURRENT.

	Secondary Current.	"Slip" per Cent.	Per Cent. Increase in Slip per Ampere Increase in Secondary Current.
Fig. 389	200	3·3	...
" 390	300	5·7	·024
" 391	400	10·0	·043
" 392	450	15·5	·110
" 393	480	27·5	·400

The last three diagrams have, in fact, represented unstable conditions such as occur when one has gradually increased the load on the motor beyond the point where the residual flux (M_s) suffices, no matter how great the rotor slip, to generate in the rotor

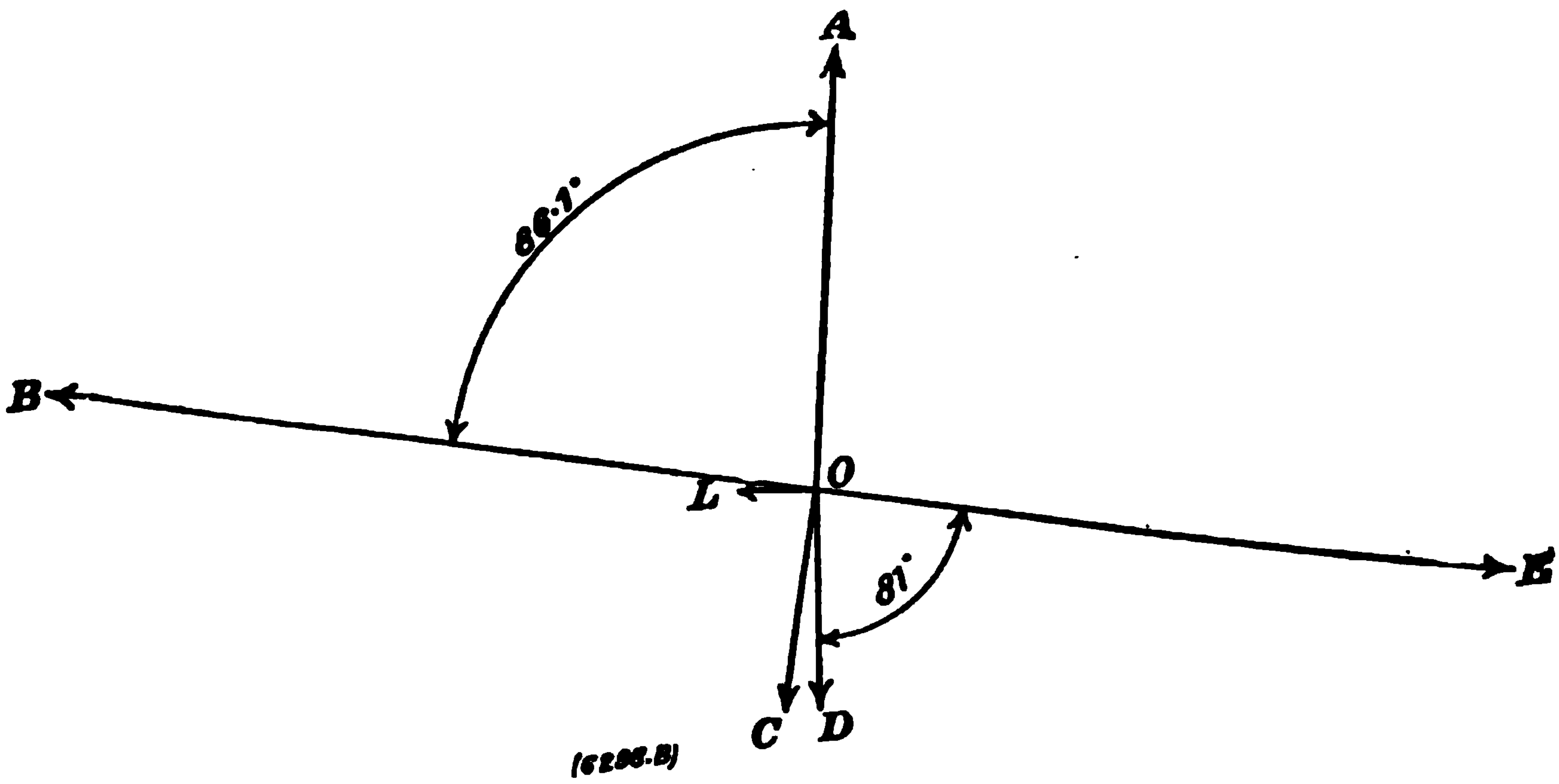


FIG. 394.

conductors sufficient current to give the torque corresponding to the load imposed. The motor comes to rest, and, if left on the circuit, carries about 503 amperes per secondary winding. The conditions are as represented in Fig. 394. The only loss in the motor is, by assumption, the secondary C^2R loss of $3 \times 503^2 \times \cdot 047 = 35{,}500$ watts. Hence this is the watts input:—

$E_s = 503 \times \cdot 047 = 23\cdot 6$ volts.

$N_s = 40$ cycles per second, being now equal to N_p.

$$M_s = \frac{23\cdot 6 \times 100{,}000{,}000}{4\cdot 2 \times 80 \times 40} = \cdot 176 \text{ megaline.}$$

$H_s = 2\pi \times 40 \times \cdot 00117 \times 503 = 148$ volts per phase.

$$\phi_s = \tan^{-1} \frac{148}{23\cdot 6} = \tan^{-1} 6\cdot 25 = 81°.$$

$I_p = 547$ amperes.

$H_p = 547 \times \cdot 294 = 161$ volts.

$E_g = \sqrt{23\cdot 6^2 + 148^2} = 150$ volts.

$M_g = \frac{150}{23 \cdot 6} \times \cdot 176 = 1 \cdot 12$ megalines.

Primary volt-amperes input per phase $= 547 \times 318 = 174{,}000$.

" " for three phases $= 522{,}000$.

Watts input $= 35{,}500 =$ secondary I^2R loss.

$\because$ Power factor $= \frac{35{,}500}{522{,}000} = \cdot 068 = \cos.\ \phi$.

$\therefore\ \phi = 86° =$ angle of lag of primary current behind terminal voltage.

TABLE L.—RESULTS GIVEN BY, AND VALUES DEDUCED FROM, VECTOR DIAGRAMS, FIGS. 388 TO 394.

Figure Number.	Primary Terminal Voltage per Phase—V_p.	Primary Current per Phase—I_p.	Angle of Lag of Primary Current behind Primary Terminal Voltage—ϕ.	Power Factor—Cos. ϕ.	Primary Volt Amperes Input—$3\ V_p\ I_p$.	Primary Watts Input—$3\ V_p\ I_p$ Cos. ϕ.	Secondary Current per Phase—I_s.	Secondary I^2R Loss in Watts.	Watts Output from Motor.			
388	318	50	90°	0	47,600	0	0	0	0	0	0	2·37
389	318	216	34·5°	·825	206,000	170,000	200	5,620	164,380	96·7	220	2·37
390	318	325	43·5°	·725	310,000	225,000	300	12,700	212,300	94·4	284	2·37
391	318	426	55·0°	·574	408,000	234,000	[redacted]	22,600	211,400	90·2	262	2·37
392	318	496	65·0°	·423	464,000	197,000	450	28,600	168,400	85·5	226	2·37
393	318	525	75·0°	·259	500,000	130,000	480	32,500	97,500	75·0	131	2·37
394	318	547	86·0°	·068	522,000	35,500	508	35,500	0	0	0	2·37

"Slip"—(N_s in per cent. of N_p).	Speed of Rotor in Revolutions per Minute.	Primary Reactance Voltage—E_p.	Secondary Reactance Voltage—E_s.	Angle between M_g and M_g—ϕ_g.	Angle between M_g and M_s—ϕ_s.
0	600	14·7	0	0°	0°
3·3%	580	68	1·98	10·0°	11·6°
5·7%	566	94	5·00	16·0°	19·5°
10·0%	540	126	11·8	20·0°	32·0°
15·5%	507	[redacted]	20·5	17·5°	44·0°
27·5%	435	154	38·8	14·0°	60·0°
100%	0	161	14·3	4·0°	81·0°

In Table L. these results are brought together, and there are included a number of values readily deduced therefrom, such as power factor, watts input, secondary I^2R loss, watts output, efficiency, and horse-power output. The calculations being on the assumption of there being no losses other than the secondary I^2R loss, the efficiency values obtained are of but little interest. The chief interest for our present purposes relates to tracing from no load, and the synchronous speed, up to standstill (100 per cent. "slip"), the changes in output, Fig. 395; megalines flux, Fig. 396; primary current, Fig. 397; the angles of lag, Fig. 398; and the "slip," Fig. 399.

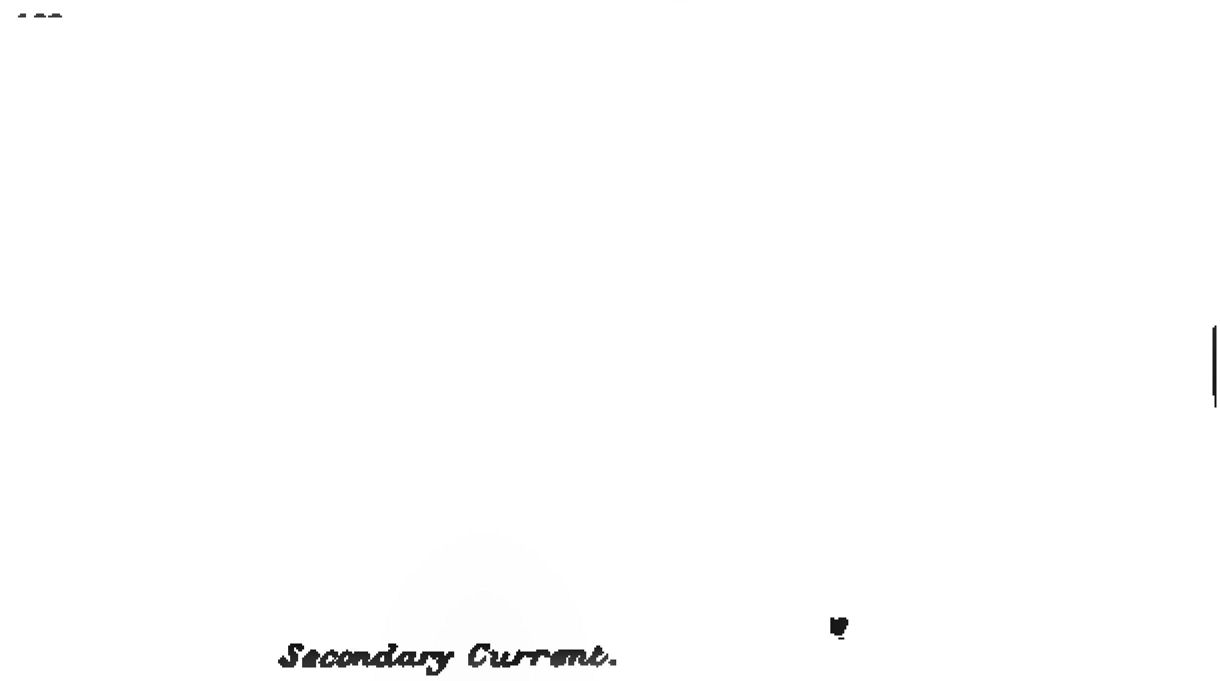


FIG. 395.—Changes in Output.

These are all plotted in terms of the current in the "equivalent" secondary winding—*i.e.* a winding with the same number of turns as the primary winding—substituted for that actually employed. This substitution makes, as we have seen, no difference in the performance of the motor, but greatly simplifies the calculations. First, from Fig. 395, we note that the rated output of our motor (150 horse-power) corresponds to 125 secondary amperes, whereas the maximum load which it will carry (285 horse-power) requires 370 secondary amperes. At that point the motor breaks down, and while coming to rest the current gradually increases up to the final value of 503 secondary amperes, and 547 primary amperes at standstill.

An important point to note in Fig. 396 is that the flux M_s, which is actually linked with the secondary circuit, falls off but

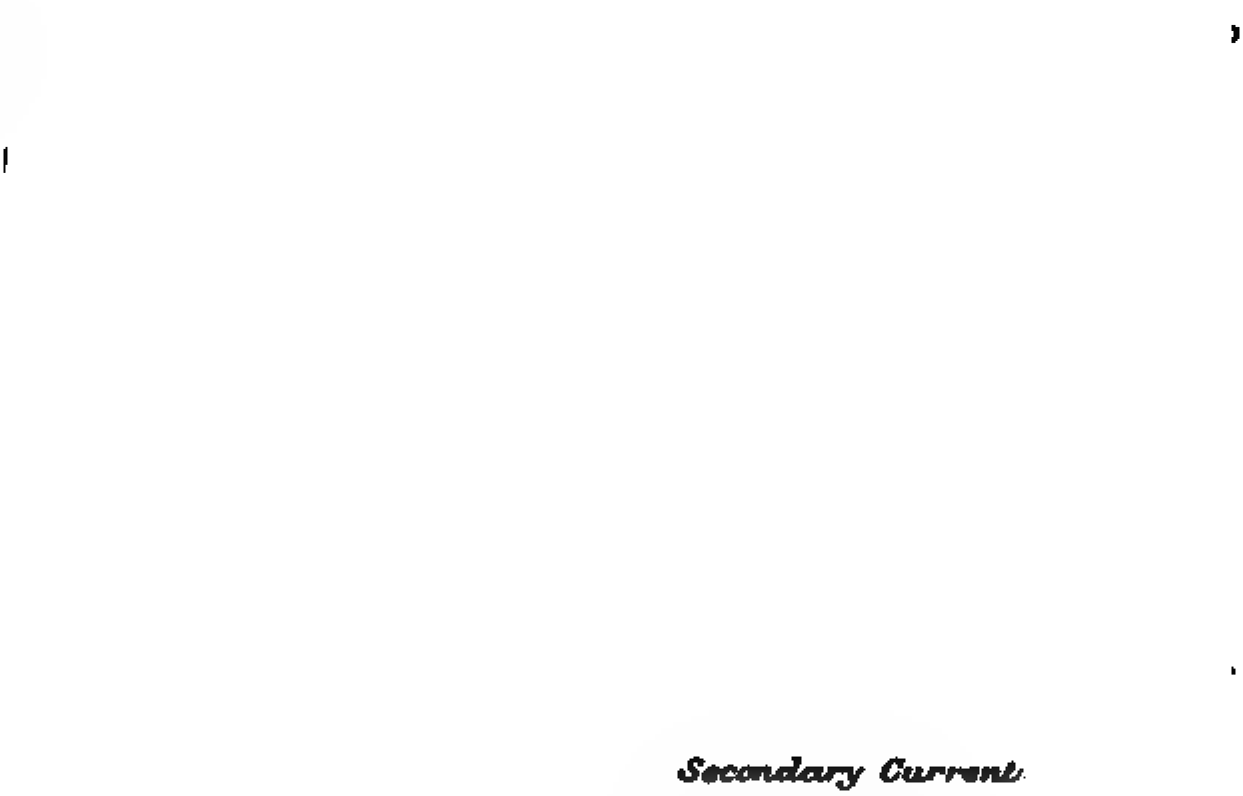


Fig. 396.—Flux.

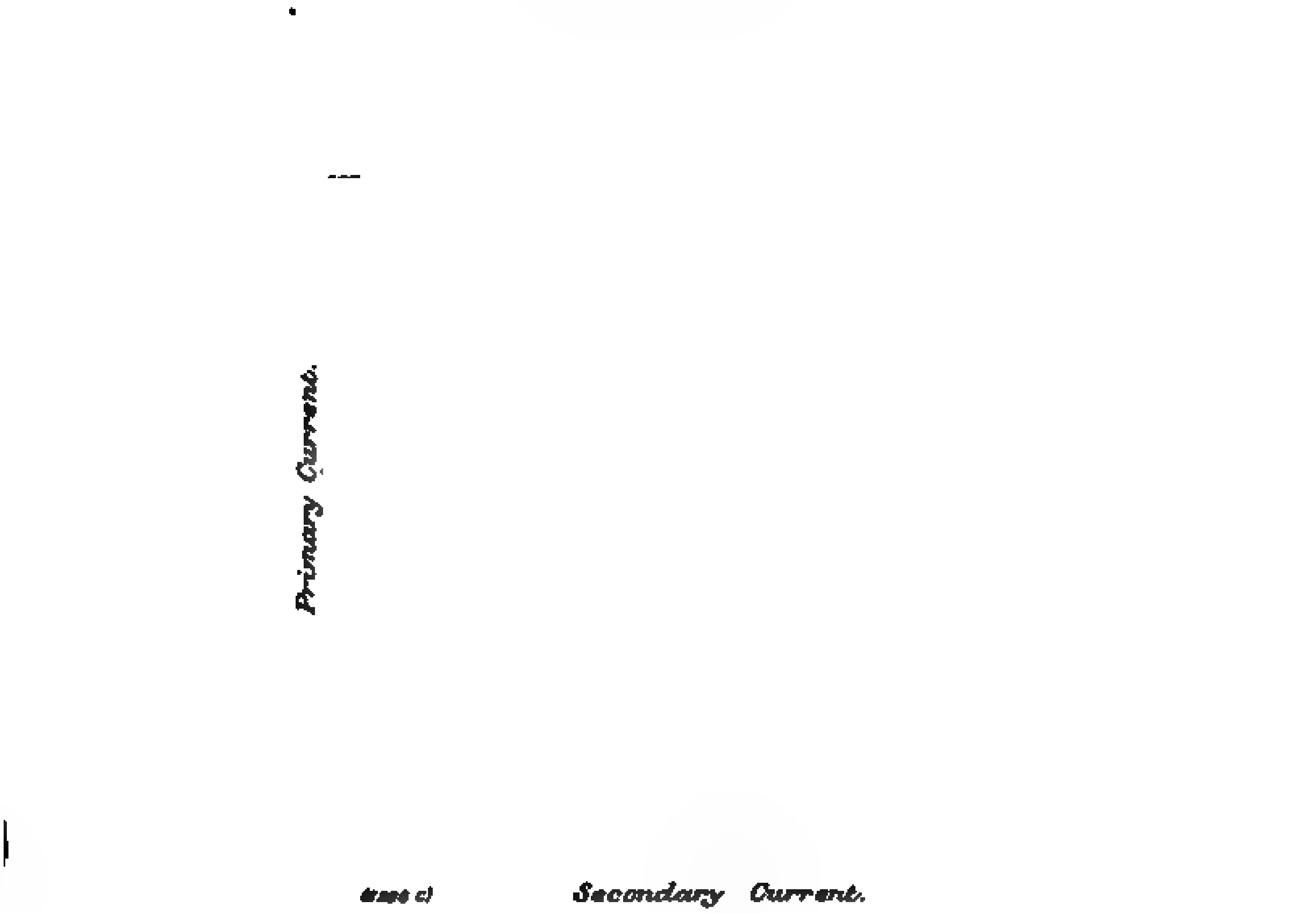


Fig. 397.—Primary Current.

very slightly from no load to full rated load. M_g has fallen off still a trifle less, and M_p has, of course, remained constant, because of the assumption of resistanceless primary windings. At the motor's *maximum* load, however, M_s has fallen off 33·3 per cent., and M_g 25·0 per cent. At standstill, while M_p still remains constant, M_g has decreased 53 per cent., and M_s 93 per cent., the remaining 7 per cent. being the small flux needful to set up

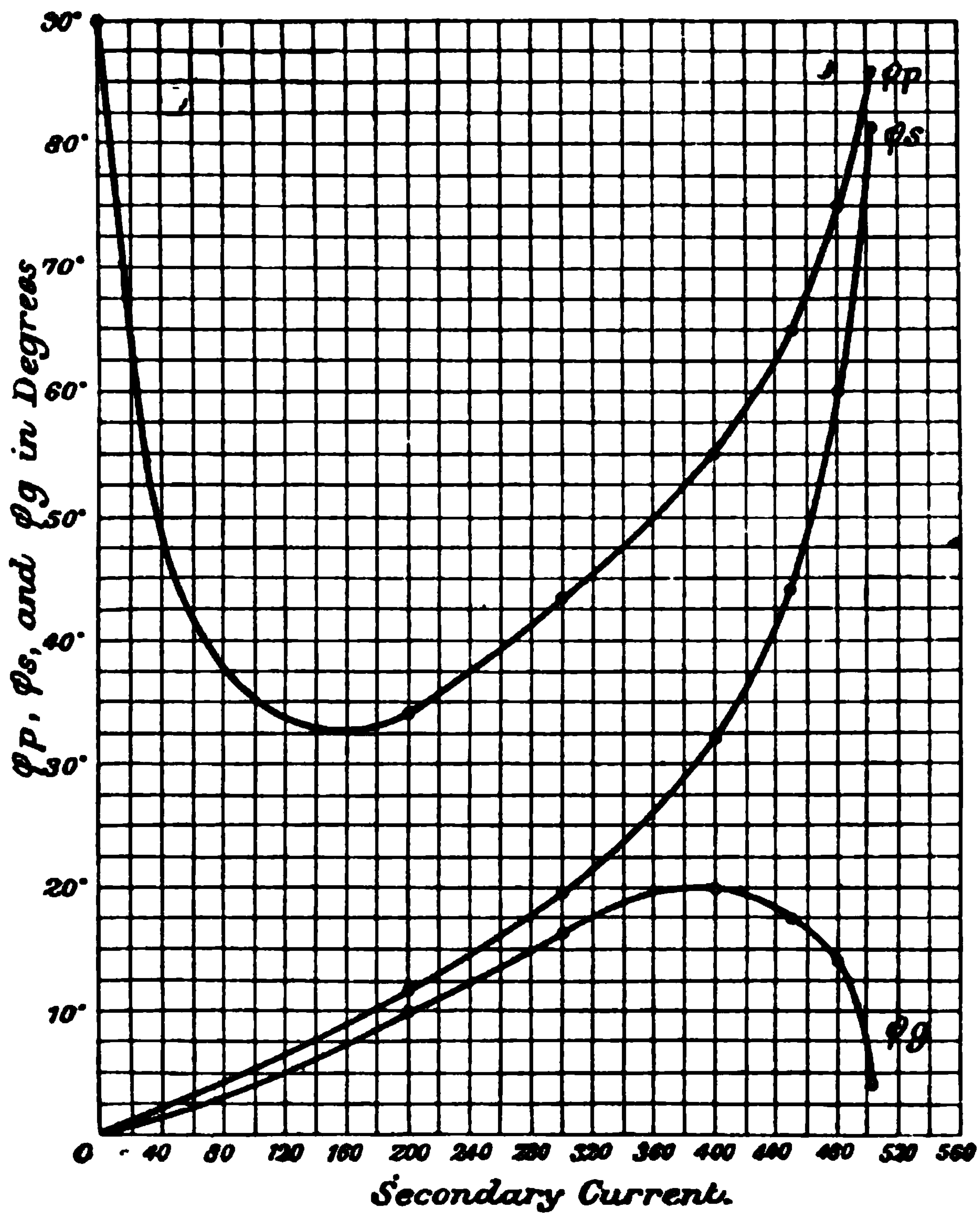


FIG. 398.—Angles of Lag.

the 23·5 volts required to drive 503 amperes through ·047 ohm, this being the I R drop per phase (503 × ·047 = 33·5). In Fig. 397 the primary current at no load is the required 50 amperes magnetising current. At higher loads the primary current is not much in excess of the secondary current, so that when vectorially combined the resultant shall be equivalent to the 50 amperes magnetising current. Thus, at standstill, where primary and secondary currents are nearly in opposition, the primary current

is 547 amperes, thus having closely approached the limit of $503 + 50 = 553$ amperes.

From Fig. 399 we see that the slip is 2 per cent. at rated full load, 8·2 per cent. at maximum capacity, and, of course, 100 per cent. at standstill, the motor then being nothing but a stationary

FIG. 399.—Slip.

transformer with (for that purpose) badly arranged primary and secondary windings, and a poor magnetic circuit.

For the purposes of this preliminary study of the motor from no load to standstill, it has been assumed that the primary windings have no resistance, that the primary and secondary cores have no iron losses, that the motor has no friction; in fact, that there existed only the secondary I^2R loss.

It will be well to draw one more diagram for the condition

of standstill, with the further assumption that not only the primary, but also the secondary, windings have no resistance. This diagram is represented in Fig. 400.

Since both primary and secondary resistances are zero, the primary voltage has merely to overcome the reactances of the primary and secondary windings, which are ·296 ohm each. Therefore $\frac{I_p + I_s}{2} = \frac{V_p}{2 \times \cdot 294} = \frac{318}{\cdot 588} = 540$ amperes. But,

$$I_s = I_p - 50.$$
$$\frac{2\ I_p - 50}{2} = 540.$$
$$I_p - 25 = 540.$$
$$I_p = 565 \text{ amperes.}$$
$$I_s = 515 \text{ amperes.}$$
$$H_p = 565 \times \cdot 294 = 166 \text{ volts.}$$
$$H_s = 515 \times \cdot 294 = 152 \text{ volts.}$$

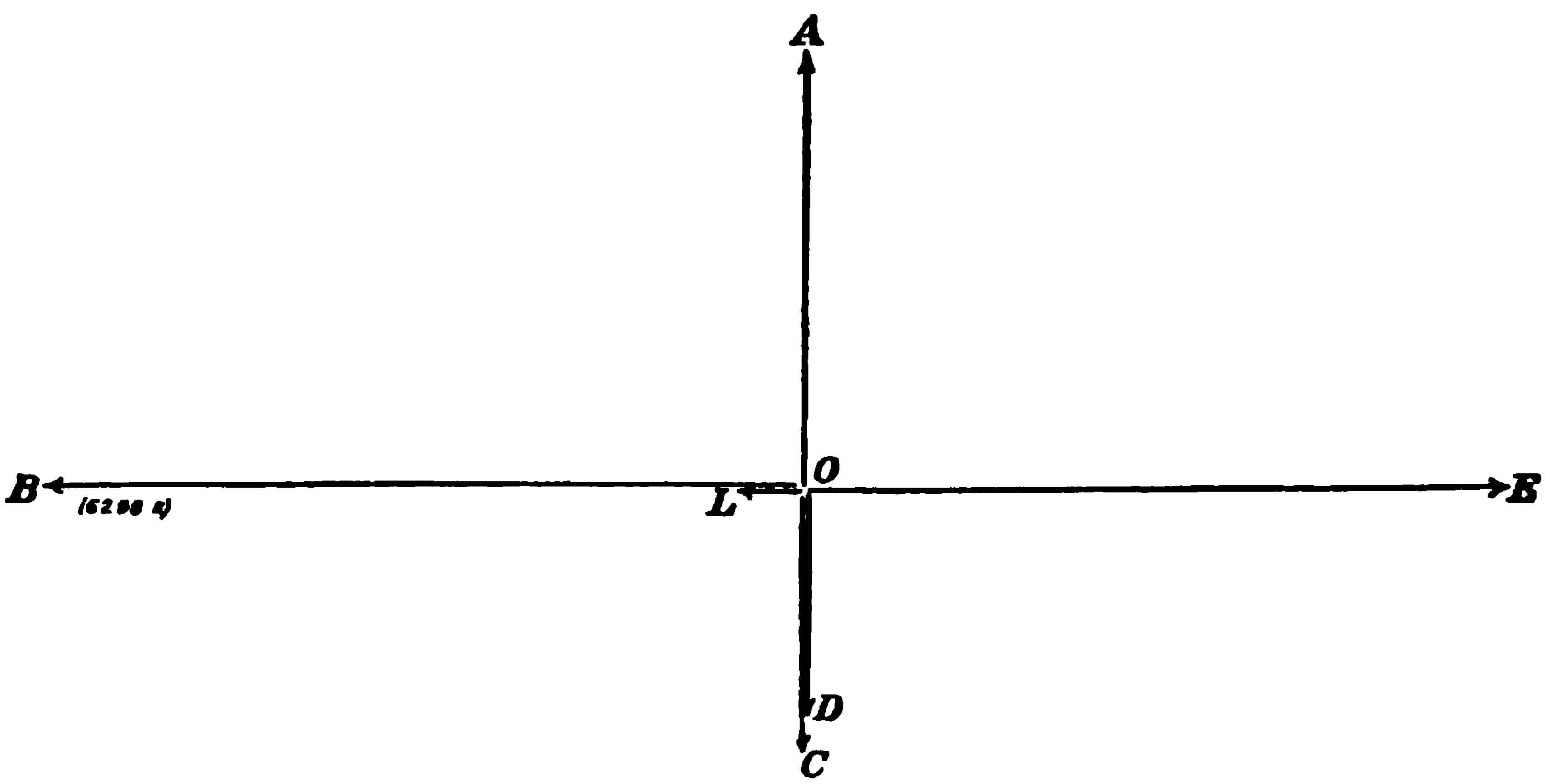


Fig. 400.

Since the secondary resistance is zero, no flux will penetrate through the secondary windings. $M_s = 0$, and $E_g = H_s = 152$ volts.

$$M_g = \frac{152 \times 100{,}000{,}000}{4 \cdot 2 \times 80 \times 40} = 1 \cdot 13 \text{ megalines.}$$
$$M_p = 2 \cdot 37 \text{ megalines.}$$
$$\phi = \phi_p = 90 \text{ degs.}$$
$$Cos.\ \phi = 0.$$

In Figs. 388 to 394 the primary terminal voltage per phase V_p has, as O A, been plotted at a varying angle, ϕ_g with the vertical, the diagrams all having been constructed with E_g(O D)

vertical. In Fig. 401, O A is drawn vertically, and the primary current is laid off in magnitude, and in direction with respect to O A, for the seven values of the seven figures 388 to 394. We find the locus of the primary currents to be the circumference of a circle with a diameter of 503, the value of the secondary current at short circuit.

§ 14. **The Circle Diagram.**—This leads us at once to the "circle diagram," due to the investigations of Bedell, Behrend, Blondel, Heyland, Rothert, Steinmetz, and others. A good many questions having been raised as to the priority in this matter, the list has been arranged alphabetically. It is generally agreed that Heyland has done a very great deal in the devising and perfecting of this important diagram, and it is often referred to as the "Heyland diagram." The writer has led up to it in this gradual way with the hope of better bringing out its real significance and

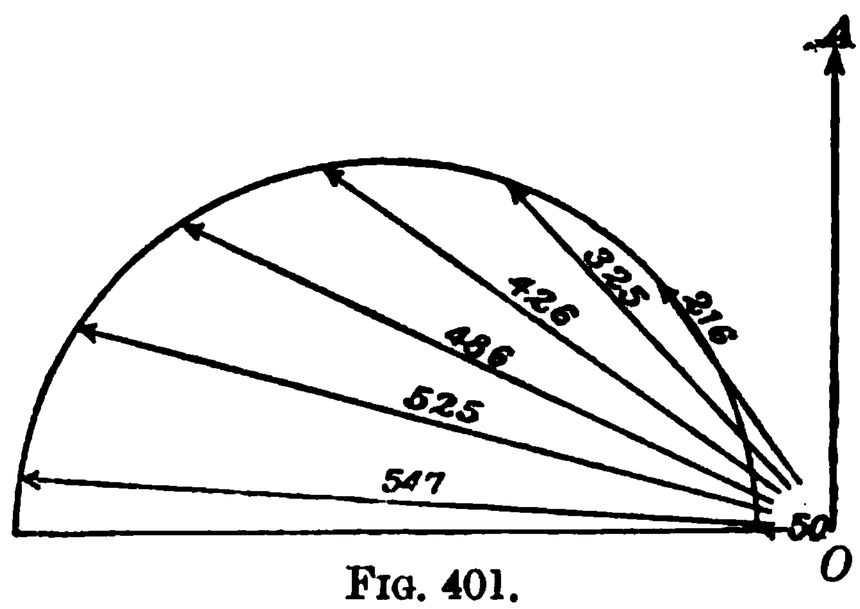


FIG. 401.

usefulness. We have now seen something of the labour of analysing the occurrences by means of ordinary vector diagrams.

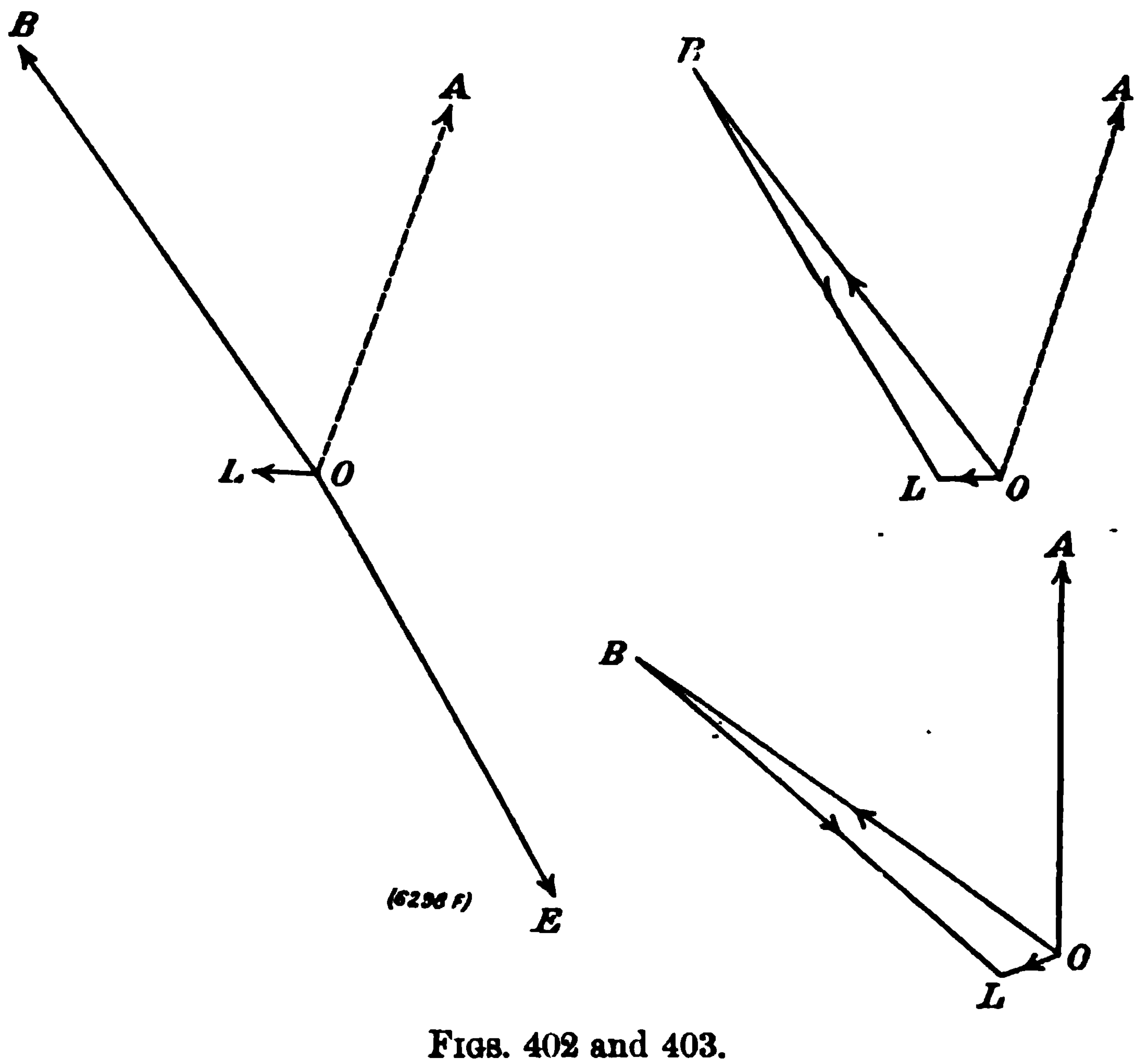
The analysis by means of the "circle diagram" is far more expeditious, and the significance of the results is more apparent.

Let us next, by means of the "circle diagram," investigate the relations of the secondary current in magnitude and direction.

We have seen that the vector resultant of primary and secondary magneto-motive forces must for all loads equal the practically constant magneto-motive force O L of the preceding Figs. 388 to 394, hence must be equivalent to that supplied by the primary current at no load—namely, 50 amperes per phase. Primary and second currents, O B and O E (Figs. 389 to 394), and their resultant O L, may, therefore, always be represented by a closed triangle, instead of by three lines radiating from O. Thus for the case of the diagram Fig. 391 for 400 secondary amperes, the triangle O B L at the right in Fig. 402 is a substitute for the

diagram O B L E at the left, and is a more convenient method of representation, since it brings the secondary current into the limits of the scheme of the "circle diagram." In the two diagrams of Fig 402, the primary voltage V_p is represented by the dotted line O A.

Now, it is furthermore convenient to always construct O A vertically, as in Fig. 403, letting O L take whatever position it will, instead of drawing O L horizontally, as in Figs. 388 to 394, and in the two diagrams of Fig. 402. It will be observed that for



Figs. 402 and 403.

no load O A and O L may be respectively vertical and horizontal (see Fig. 388), but that, as the load increases, the plan of constructing O A vertical results in making O L perform a short excursion below the horizontal, to the extent of a maximum angle attained at maximum load, after which it again progresses back towards the horizonal, and would, at standstill, regain that position were the primary and secondary windings devoid of resistance, as in the hypothetical case represented in Fig. 400.

In Fig. 404 the secondary current O D, and the resultant current O L of Figs. 388 to 394, have been reproduced as B L and O L respectively, the lines representing the primary current

having been omitted for clearness. Evidently the circumference of the same circle, serving as the locus of the primary current values in Fig. 401, is also the locus for the secondary currents. The locus of L is an arc of 50 amperes radius, drawn about O as a centre, and extending a short distance below the horizontal axis.

Obviously, the magnitude and phase of the secondary current for any load may be represented with but comparatively small error by chords from the right-hand extremity of the horizontal diameter of the circle of Fig. 401 to the intersection of the primary current vector with the circle—*i.e.* O L may, for most practical purposes, be taken as occupying a constant horizontal position 90° behind O A. For most practical purposes this construction is amply exact, and greatly facilitates approximate determinations of many other properties of the induction motor.

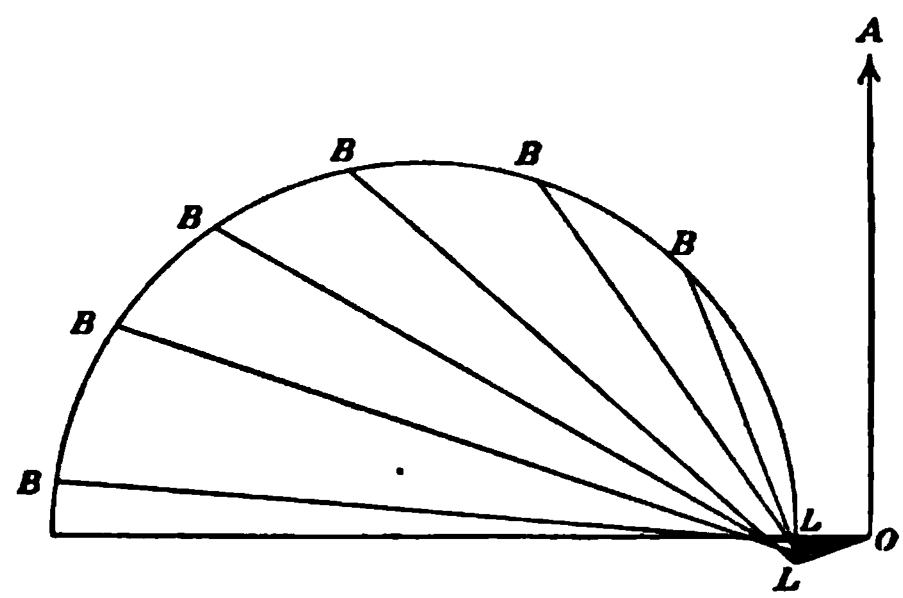


FIG. 404.

Now what determines the diameter of the circle? We see from Fig. 400 that it is the value of the current in a resistanceless secondary at standstill. This current is determined by the reactance of the primary and secondary windings. This we have estimated at ·294 ohm each, for, at standstill, the secondary periodicity is also 40 cycles per second. We have in Fig. 400 seen that the terminal voltage is, under these conditions, equal to the sum of the primary and secondary reactance voltages.

$$\text{Terminal voltage per phase} = 318 = H_p + H_s.$$
$$H_p = \cdot 296\ I_p.$$
$$H_s = \cdot 296\ I_s.$$
$$318 = \cdot 296\ (I_p + I_s).$$
$$= \cdot 296\ (2\ I_s + 50).$$
$$I_s = \frac{\frac{318}{\cdot 296} - 50}{2} = 513 \text{ amperes.}$$

Hence, if we wish to construct the diagram for a tested motor, we employ observed results for its reactance at standstill and for its magnetising current. For designing new motors we may predetermine these values as already explained. These predeterminations of the value of the inductance of the windings are but the roughest of estimates, and it is important to revise one's constants as often as possible from the results of actual tests. Although not contributing to any greater exactness in the final result, a much more convenient method is available. It is due, in the first instance, to Behrend. It is, however, not advisable to introduce it at this point, as it somewhat obscures the real occurrences, the graphical representation of which we shall next undertake.

From Figs. 395 and 397 it is seen that the secondary and primary currents corresponding to the motor's maximum load of

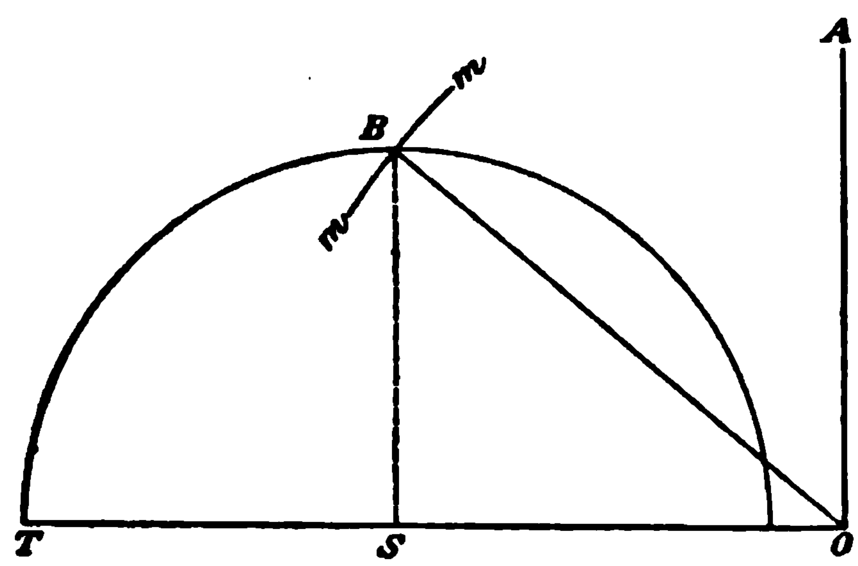


FIG. 405.

295 horse-power output, are 370 amperes and 390 amperes respectively. In Fig. 405 the circle has been redrawn, and the horizontal diameter has been produced to O. From O as a centre, the arc *m n* has been struck with a radius of 390, and the line O B, representing the primary current, drawn to the intersection of *m n* with the circumference. This, we observe, falls at the extremity of the circle's vertical radius S B, which represents the energy component of the primary current at the maximum load that the motor is capable of carrying.

In the same way for any other primary current—say, O B in Fig. 406—the line B U drawn from B perpendicular to the horizontal diameter L T, represents the energy component of the primary current U O, being the wattless component. Hence the energy input to the motor, for any load, is proportional to the corresponding length of the perpendicular B U, and if we calculate it for any one value of the total primary current input, we can

obtain it by construction for other inputs. Thus, for Fig. 405 we have determined the output, corresponding to the primary current O B, to be 295 horse-power, or 220,000 watts, the input being greater, by the total secondary I^2R loss, at the corresponding secondary current of 370 amperes—*i.e.* by $3 \times 370^2 \times \cdot 047 = 19{,}300$ watts. Hence the maximum input (*i.e.* the input corresponding to the maximum ordinate B S of Fig. 405) equals $220{,}000 + 19{,}300 = 239{,}300$ watts. This may be illustrated for the conditions corresponding to the diagram of Fig. 406, where B U is to the same scale as B S of Fig. 405, and is found to be 0·90 as great (*i.e.* $\text{B U} = 0{\cdot}90 \times \text{S B} = 0{\cdot}90 \times 250 = 225$ amperes); therefore at a primary current input of 300 amperes—($\text{O B} = 300$)—the energy input is $0{\cdot}90 \times 239{,}300 = 216{,}000$ watts.

We have stated that these diagrams are constructed to a scale of 100 amperes per centimetre, and 100 volts per centimetre.

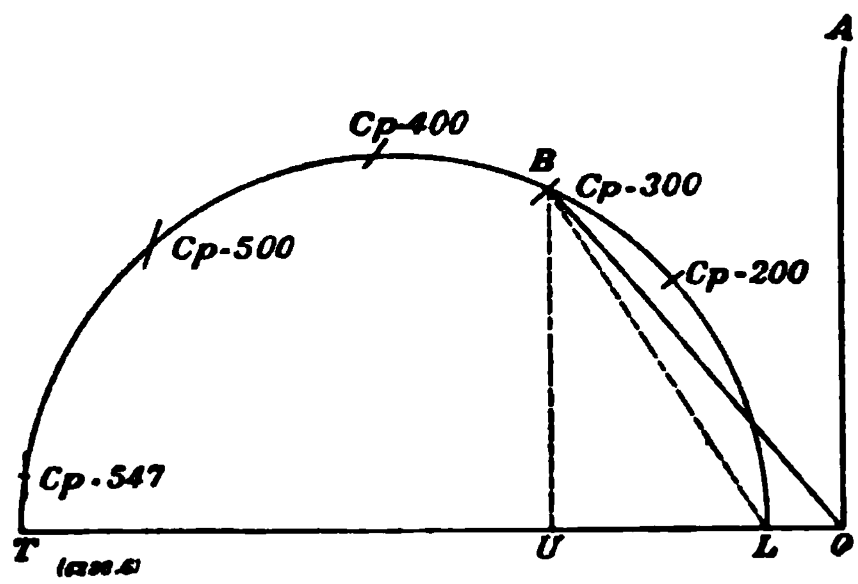


FIG. 406. (C_p corresponds to I_p in text.)

When, now, we wish to represent watts input by vertical ordinates, such as B S of Fig. 405, and B U of Fig. 406, we must first determine the scale. B S of Fig. 144 corresponds to the maximum input of $\frac{239{,}300}{3} = 79{,}500$ watts per phase, and is 2·5 centimetres long. Hence the ordinates represent watts input per phase to the scale of $\frac{79{,}500}{2{\cdot}5} = 31{,}800$ watts per centimetre. It is well to point out here that had the volts per phase been 1·0 instead of 318, we should have had an energy diagram to the scale of $\frac{31{,}800}{318} = 100$ watts per centimetre, as well as 100 amperes per centimetre, and 100 volts per centimetre. In other words, when volts and amperes are plotted to a certain scale per centimetre, then in such a diagram the watts per primary winding (*i.e.* per phase) are to the same scale per centimetre.

We can now obtain the watts input corresponding to any primary current input from the length of the vertical ordinate B U corresponding to that current. Neglecting the slight departure of O L from the horizontal at intermediate loads, we can also obtain the corresponding secondary current by taking the corresponding lengths B L, and, knowing the secondary resistance per phase to be ·047 ohm, we may derive the secondary I^2R loss ($3 \times {\cdot}047 \times$ B L), and subtracting this from the primary input we obtain the output from the motor (for which we continue for the present to assume zero primary resistance and no core loss or friction). We also obtain directly from the diagram the primary volt-amperes input from the products of the primary voltage per phase O A (constant at 318) and the total primary current per phase O B. This enables us to derive the power factor, which is the quotient of the watts input divided by the volt-amperes input, and also the efficiency as the quotient of output by input. This is carried out in Table LI., the values being readily scaled off from the diagram of Fig. 406 without its being necessary to draw any additional lines. The locations of B for the various amperes input (I_p) are, however, indicated on the diagram (Fig. 406).

TABLE LI.—RESULTS OBTAINED BY MEANS OF THE CIRCLE DIAGRAM METHOD.

Primary Current—I_p.	Primary Voltage—V_p.	Primary Volt-Amperes Input—$3\ I_p\ V_p$.	B U of Fig. 406 in Centimetres.	Primary Watts Input $(3 \times \text{B U} \times 16{,}900) = W_p$.	Cos. ϕ $(W_p \div 3\ I_p\ V_p)$ (Power Factor).	ϕ (Angle B O A) (Degrees).	Secondary Current—I_s.	Secondary I^2R Loss—$3\ I_s^2 \times {\cdot}047$.	Watts Output $= W_p - 3 I_s^2 \times {\cdot}047$.	Efficiency—$\frac{W_p - 3\ I_s^2 \times {\cdot}047}{W_p}$	"Apparent" Efficiency =	Horse-power Output.
50	318	47,700	0	0	0	90	0	0	0	0	0	0
200	318	191,000	3·30	157,500	·825	34·2	184	4,760	152,740	96·8	79·8	204
300	318	286,000	4·55	217,000	·759	40·5	275	10,800	206,700	95·2	72·3	277
400	318	382,000	5·00	238,000	·622	51·5	374	19,800	218,200	91·7	57·2	293
500	318	477,000	3·85	184,000	·385	67·5	462	30,200	153,800	83·4	32·1	206
547	318	522,000	·745	35,500	·068	86	508	35,600	0	0	0	0

The directness and ease of the constructions by means of the circle diagram, which have led to the results set forth in Table LI., are in striking contrast to the laborious constructions in the vector diagrams employed in obtaining the results of Table L.;

while the latter constructions ultimately show more clearly the phenomena involved, the circle diagram is the really practical method which, by its directness, permits of investigations which would be very laborious by any other means.

The "apparent efficiency," given in Table LI., expresses the ratio of output to volt-amperes input, and is a term frequently used, because in continuous current motors one is so accustomed to think of the watts input being identical with the volt-amperes input, that such a ratio came to be considered to also have its meaning with reference to induction motors. But, of course, this term of "apparent efficiency" has but a limited significance. The

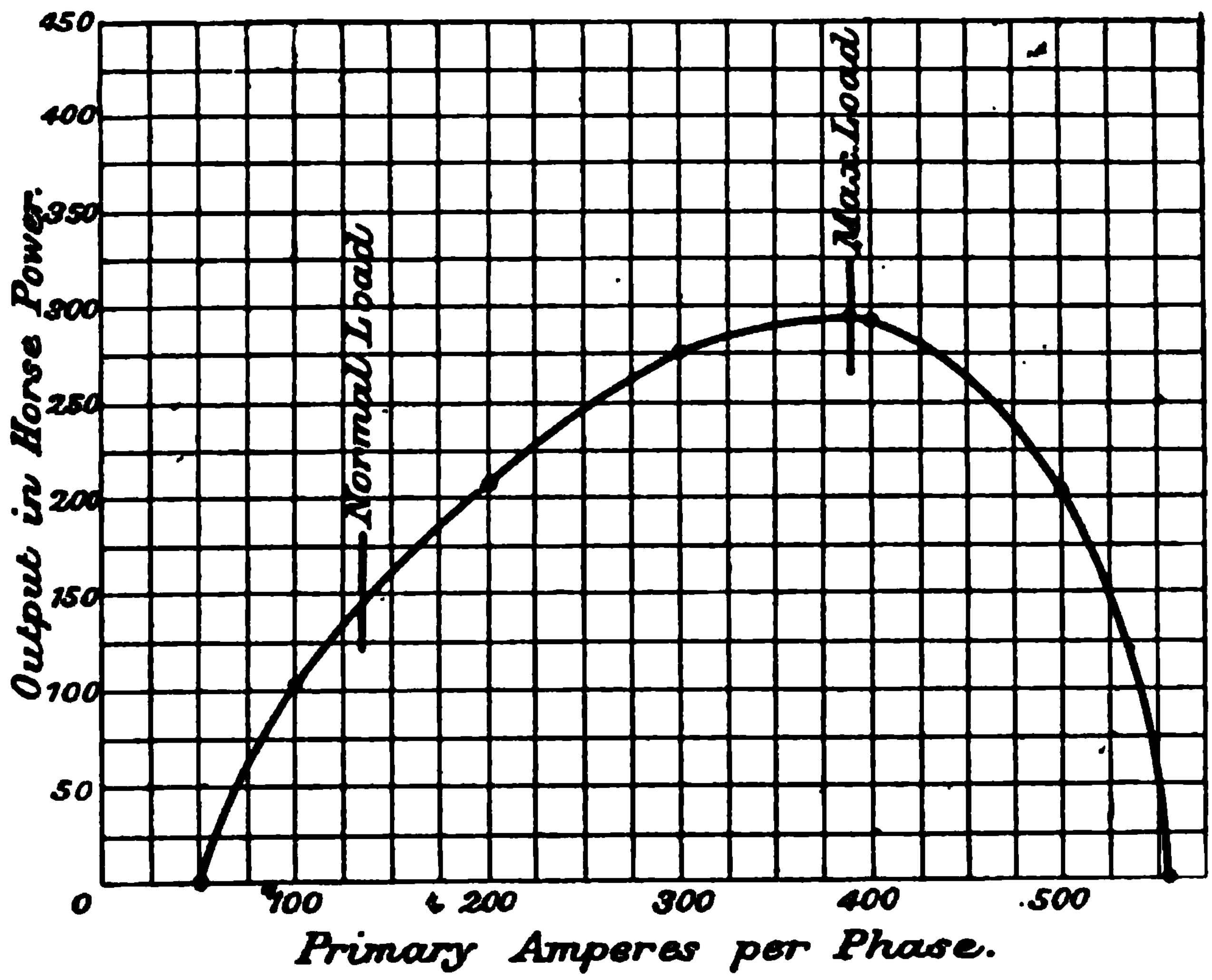


FIG. 407.—Output.

true efficiency is higher, and may be obtained from the "apparent efficiency," by dividing by the power factor. In Figs. 407 to 409, horse-power output, power factor, and efficiency have been plotted in terms of the primary current for the values in Table LI.

A word must be said as to the point of maximum power factor. This occurs at that value of the primary current for which B falls at the point of tangency of the line drawn from 0 tangent to the circle. This is, for the case in hand, shown on the diagram in Fig 410, where the primary current O B is found to be equal to 165 amperes, its energy component B U to 137 amperes, and the power factor to $\frac{\text{BU}}{\text{BO}} = \frac{137}{165} = \cdot 83$. The power factor is equal to the cosine of the angle of lag of the primary current O B behind the

terminal voltage O A. Now the cosine of an angle is larger the smaller the angle becoming equal to 1·00° for 0°, *i.e.* for the case

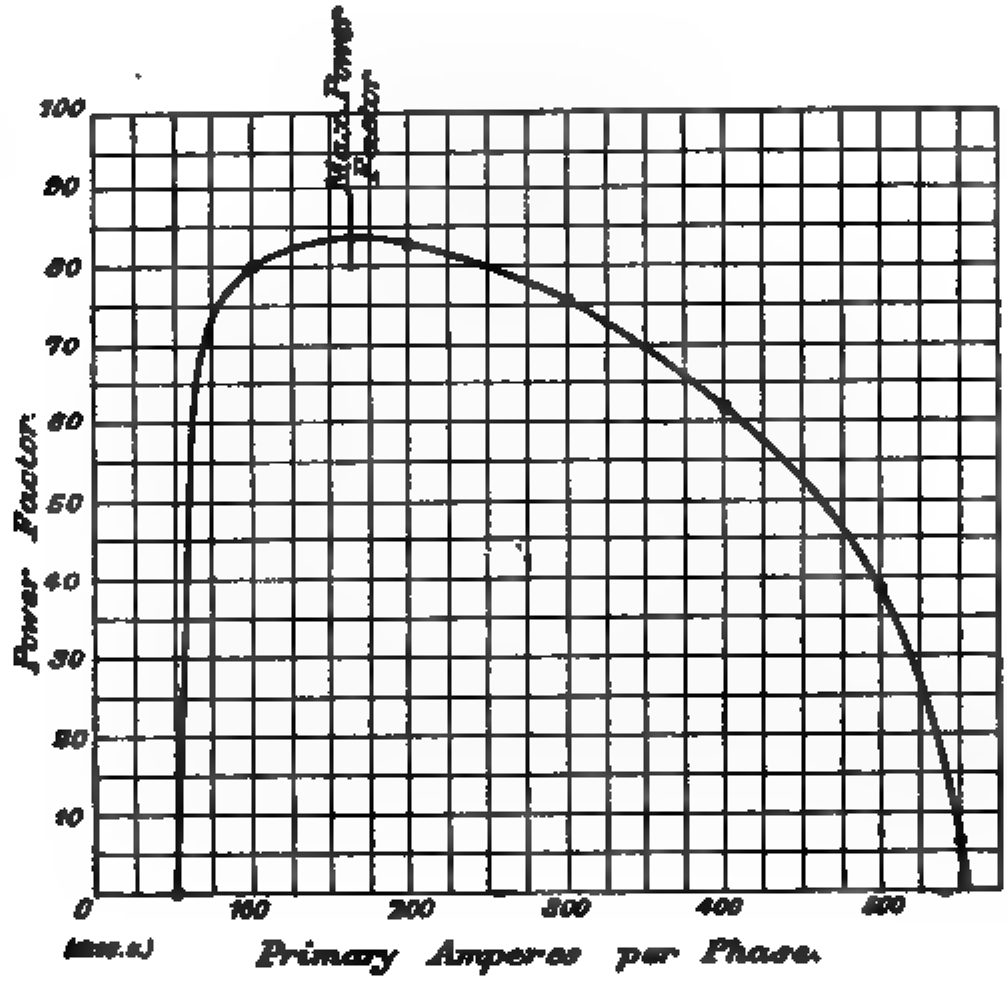


FIG. 408.—Power Factor.

FIG 409.—Efficiency.

of a current in phase with its voltage, hence the maximum power factor will correspond to that value of the current whose vector O B makes the smallest angle with O A. This is obviously the

case when O B is tangent to the circle. For less and greater values of the primary current, the power factor is lower. The maximum value of the power factor corresponds to the diagram of Fig. 410, and is then 0·83, the primary current being 165 amperes, and this again appears as the maximum in the curve of power factor values given in Fig. 408.

§ 15. **The Dependence of "Slip" upon the Secondary I^2R Loss.**—This important consideration must next be taken up. The discussion may be prefaced by asserting that the "slip" expressed in percentage of the synchronous speed equals the secondary I^2R loss expressed in percentage of the total input to the secondary, which, for our motor with zero primary resistance, is also equal to the total input to the primary. That the percentage "slip" is equal to the percentage secondary I^2R loss, may be seen by turning back to the calculations by which we obtained the

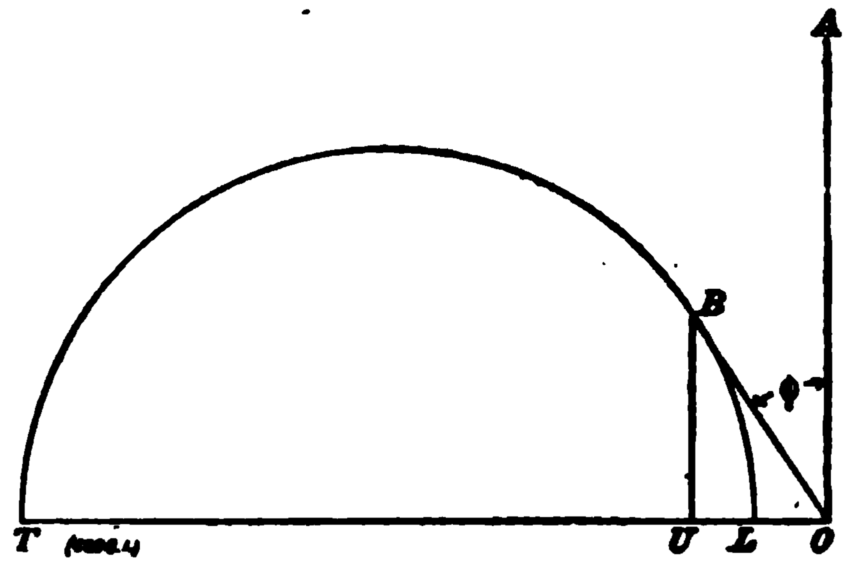


FIG. 410.

diagrams in Figs. 388 to 394. There we saw that we required sufficient secondary "slip" to generate in the secondary windings just enough voltage to drive the secondary current per phase through the ohmic resistance of the windings of one phase. Obviously then, were the magnetic flux M_s, which is linked with the secondary windings, to remain constant, the required "slip" would be proportional to the secondary current. But from Fig. 396, page 349, we see that M_s falls off with increasing values of the secondary current, at first slowly, and ultimately very rapidly, and the "slip" must increase more rapidly than the current, to the extent of the rate of decrease in the flux M_s. This, however, was taken into consideration, step by step, and the values obtained for the "slip" were plotted in terms of the secondary current in Fig. 399, page 351. The secondary I^2R loss which has been already given in Table LI. is repeated in Table LII., in which are also given the watts input, and the secondary I^2R loss expressed

as a percentage of the watts input. The percentage "slip," from Fig. 399, is given in the last column.

TABLE LII.—SECONDARY I^2R LOSS, WATTS INPUT, AND PERCENTAGE SLIP.

Primary Amperes Input—I_p.	Secondary Amperes—I_s.	Secondary I^2R Loss (from Table XI.).	Watts Input —W_p.	Secondary I^2R Loss as Percentage of W_p.	Per Cent. "Slip" (from Fig. 138).
50	0	0	0	0	0
200	184	4,760	157,500	3·02	3
300	275	10,300	217,000	4·75	4·75
400	374	19,800	238,000	8·3	8·3
500	462	30,200	184,000	16·4	16·4
547	503	35,500	35,500	100	100

The agreement between the values in the two last columns is in accordance with the proposition that the percentage "slip" is equal to the secondary I^2R loss expressed in percentage of the total input to the rotor—*i.e.* of the total energy transmitted by induction to the secondary circuits. How can this be best represented in our diagram? At standstill the "slip" is 100 per cent., and the secondary I^2R loss is equal to the input to the rotor. For the motor we are considering, the secondary I^2R loss is, at standstill, 35,500 watts, and there being no output at standstill, this is 100 per cent. of the input. At no load the slip is zero, thus, of course, zero per cent. of the input. We shall consider these, and the intermediate loads in Table LII., and examine them with special reference to the "slip," and to its graphical representation on the diagram. Now, for purposes of illustration, let us take the values in the above table, corresponding to 300 primary amperes per phase, and construct the diagram shown in Fig. 411 (see page 363), in which the only novelty introduced consists in the extension of B U to V, and in letting B V constitute the hypotenuse of the right-angled triangle B V L. From this construction we have: $B\,U : B\,L = B\,L : B\,V$, $B\,V = \frac{B\,L^2}{B\,U}$. Expressing lengths in centimetres, we have: Secondary current $= 100 \times B\,L$; secondary I^2R loss per phase $= 10{,}000 \times B\,L^2 \times {\cdot}047 = 470 \times B\,L^2$ $\therefore$ $B\,L^2 = {\cdot}00213 \times$ secondary I^2R loss per phase. Watts input per phase $= 31{,}800 \times B\,U$ $\therefore$ $B\,U = {\cdot}0000315 \times$ watts input per phase.

$$B\,V = \frac{B\,L^2}{B\,U} = \frac{{\cdot}00213 \times \text{secondary } I^2R \text{ loss per phase}}{{\cdot}0000315 \times \text{watts input per phase}} = 67{\cdot}5 \times \frac{\text{Secondary } I^2R \text{ loss per phase}}{\text{Watts input per phase}}.$$

But we have already seen that the percentage "slip" is equal to $100 \times \frac{\text{Secondary } I^2R \text{ loss per phase}}{\text{Watts input per phase}}$. Hence B V $= \cdot 675 \times$ percentage slip, and percentage slip $= 1{\cdot}48$ B V, B V $= 3{\cdot}2$ centimetres, $\therefore$ percentage slip $= 1{\cdot}48 \times 3{\cdot}2 = 4{\cdot}75$ per cent., which agrees with the value 4·75 per cent. already obtained in Table LII. by calculation. For the value corresponding to the diagram in Fig. 411, a full scale construction gives convenient proportions, and this would generally be the case throughout the rated capacity of most

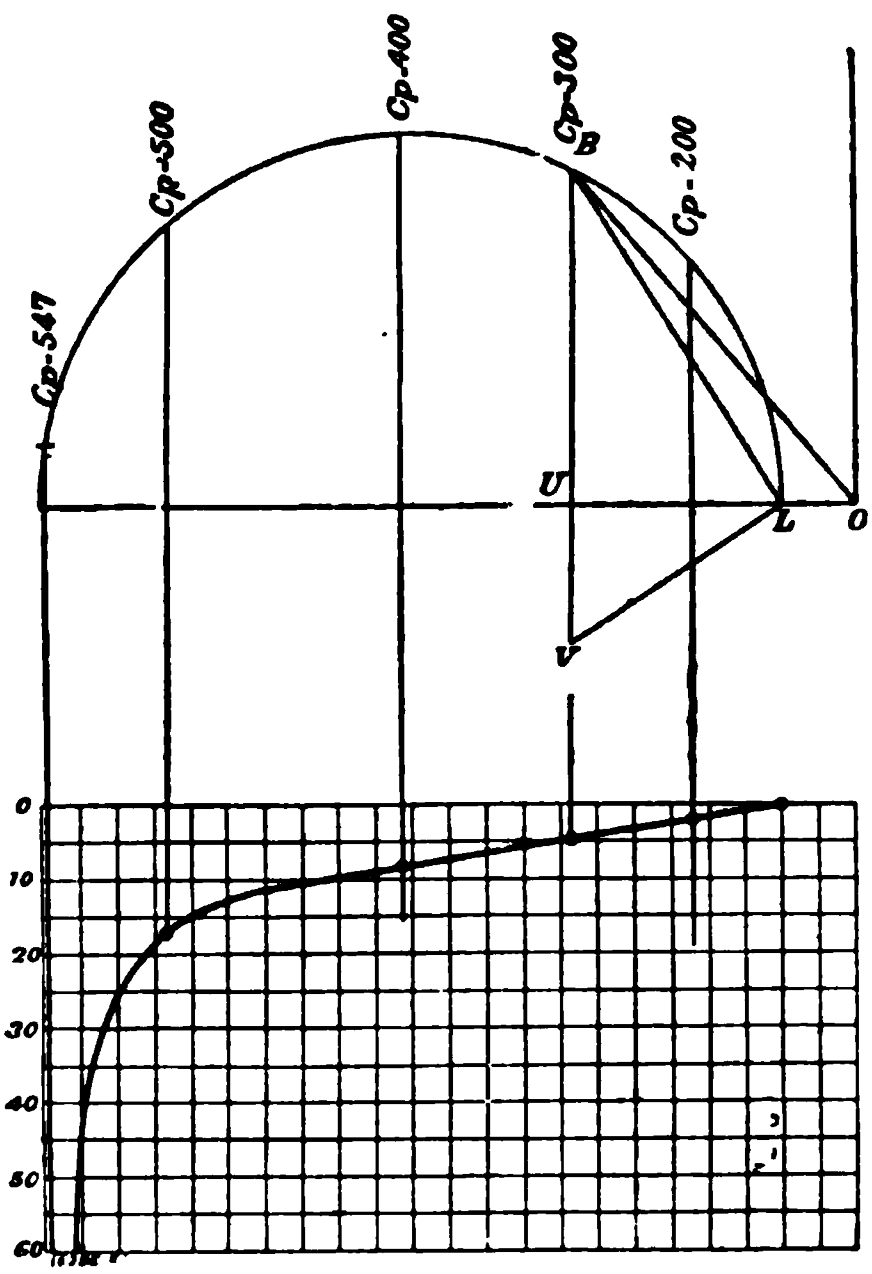


FIG. 411.—Diagram showing Percentage of Total Energy Dissipated in the Secondary Circuit, this being also the Percentage Slip. (C_p in figure corresponds to I_p in text.)

induction motors, but for the determination of the slip within these limits a graphical construction has no special interest. Its chief value lies in studying the phenomena for much higher current values, and for such values the diagram assumes unsuitable proportions. It is hence desirable to plot the values of the slip to a much smaller scale, vertically downward below the diagram, as shown in Fig. 411, where a scale of ·5 millimetre per 1 per cent. slip is used.

It will now be understood that by increasing the secondary resistance, either of the windings or of the resistances in series

with them, the percentage slip may be proportionately increased for a given load. This is almost always very undesirable from the standpoint of the performance of the motor when running normally, but it is a consideration of great importance as regards the performance of the motor when first switched upon the circuit, and during acceleration. This is the next matter to be taken up, and it requires a consideration of the torque of the motor under different conditions.

§ 16. **The Torque of Induction Motors.**—"Torque," as used in this article, denotes the turning moment in kilogramme-metres. Thus a rotor exerts a torque of 100 kilogramme-metres when, at the extremity of a radius of 1 metre, it exerts a force equal to the weight of a mass of 100 kilogrammes; or when, at a radius of 2 metres, it exerts a force equal to the weight of a mass of 50 kilogrammes, etc. If we multiply the force in kilogrammes by the distance through which the point of application of the force moves per second ($2\pi \times$ radius in metres $\times$ revolutions per second), the product will be *power* in kilogramme-metres per second. Thus, if a motor is exerting a torque of 100 kilogramme-metres at 600 revolutions per minute, its load is $2\pi \times 100 \times \frac{600}{60} = 6280$ kilogramme-metres per second. Since 1 British horse-power equals 76 kilogramme-metres per second, this is equal to $\frac{6280}{76} = 82{\cdot}5$ horse power. Now, were the speed of an induction motor perfectly constant, the torque exerted by the rotor conductors, and available from the rotor shaft, would be proportional to the watts output plus the rotor's core loss and friction, which, for the present, we are taking equal to zero. But the torque increases more rapidly than in proportion to the watts output to the extent that the speed falls with the load. By first calculating the torque for some one load, we may readily derive it for other loads by taking it proportional to the change in load, modified by this ratio of the change in speed. Thus, from Fig. 407, page 359, we find that our motor, at a primary input of 100 amperes, has an output of 101 horse-power, and (the slip being 1·4 per cent.) a speed of $600 \times {\cdot}986 = 592$ revolutions per minute $= 9{\cdot}86$ revolutions per second; the corresponding torque $= \frac{101 \times 76}{2\,\pi \times 9{\cdot}86} = 124$ kilogramme-metres.

At 200 amperes we have the torque $= \frac{204}{101} \times \frac{{\cdot}986}{{\cdot}970} \times 124 = 254$

kilogramme-metres. By similar calculations for other values we obtain Table LIII.

TABLE LIII.—TABULATION OF TORQUE VALUES FOR INDUCTION MOTOR.

Primary Amperes Input per Phase.	Horse-power Output (from fig. 407).	Speed in Revolutions per Minute.	Torque in Kilogramme-metres.
50	0	600	0
100	101	592	124
200	204	582	254
300	272	571	347
400	293	551	385
500	204	502	294
547	0	0	58

The last value in Table LIII. for the torque corresponding to 547 primary amperes could not be obtained in this way, for the motor is then at rest. Hence it is desirable to ascertain some other method of deriving the relative values of the torque. The torque is, in fact, proportional to the ordinate B U drawn from the intersection B of the vector representing primary current, with the circumference of the circle. That this is the case is evident from Table LIV., in which the last two columns give respectively the length of B U in centimetres, and the torque in kilogramme-metres per centimetre, the latter being derived by dividing the values of the torque already calculated above, by the length of B U in centimetres. The torque is thus seen to be proportional to B U to the scale of 155 kilogramme-metres per centimetre.

TABLE LIV.—LENGTHS OF B U, AND TORQUE IN KILOGRAMME-METRES (FIG. 411).

Primary Amperes Input per Phase.	Torque in Kilogramme-metres.	Length of B U in Centimetres.	Torque in Kilogramme-metres per Centimetre on the Basis of its Proportionality to B U.
50	0	0	0
100	124	0·80	155
200	254	1·65	155
300	347	2·25	155
400	385	2·49	155
500	294	1·90	155
547	(58)	0·375	155

We have already seen that the watts input are also proportional to B U. Hence, for zero primary resistance and zero core loss, the torque is proportional to the watts input, and this is con-

sistent, for the watts input to the motor are in this case equal to the watts input to the rotor, there being no stator losses, and the speed falls off proportionately to the percentage which the secondary I^2R loss bears to the total watts input to the rotor, *i.e.* by just the amount by which the torque increases in greater proportion than the load on the motor, thus leaving the torque exerted by the rotor proportional to the watts input. Were there a resistance loss in the primary windings and a stator core loss, these would require to be first deducted from the watts input *to the motor*, and the total torque exerted by the rotor would be proportional to the watts input *to the rotor*. That would be the torque *exerted by the rotor conductors*. The torque finally available from the rotor shaft or pulley is the less amount obtained after deducting the bearing and friction losses and the small core loss in the rotor.

Let us now again examine the diagram in Fig. 394, which represents the conditions in the motor at standstill, with 318 volts per phase at its primary terminals. The diagram was drawn to represent the conditions arrived at after carrying the motor through the cycle of loads from running free, to normal load, maximum overload, and finally to standstill. But, of course, it just as truly also represents the state of affairs at the instant the motor is switched upon the circuit. Hence this motor, if switched upon the circuit with its secondary windings closed upon themselves—*i.e.* without external resistance—would, for an instant, absorb 547 amperes per phase, and would start off with a torque of 58 kilogramme-metres and at a rate of acceleration corresponding to the momentum of its rotor and of the load upon it. Now we have seen that the primary current input at full load is 135 amperes, and we find by scaling it off from the diagram in Fig. 406, page 357, that for O B = 135 amperes, B U = 1·105 centimetres. Hence the torque exerted by the motor at rated full load equals $1{\cdot}105 \times 155 = 171$ kilogramme-metres.

Therefore, as we switch the motor directly upon the line with its secondary windings short circuited, it absorbs $\frac{547}{135} = 4{\cdot}05$ times full load current, and develops only $\frac{58}{171} = 0{\cdot}34$ of full load torque.

The motor develops a "specific" torque of but $\frac{58}{547} = 0{\cdot}106$ kilogramme-metres per ampere input. But by inserting resistance in the secondary circuits (either by the employment of slip rings, as indicated in Fig. 302, page 252, or by arranging the resist-

ances within the rotor spider, as indicated diagrammatically in Fig. 300, page 251), a much higher value may be obtained, and with a lower consumption of current; the specific torque—*i.e.* the torque per ampere—being very greatly improved in consequence. We will now proceed to show that this is the case: In the diagram of Fig. 412 there is a resistance inserted in each phase of the secondary circuit, sufficient in amount to limit the primary current at starting to 450 amperes, it being assumed that a terminal voltage of 318 volts per phase is maintained constant at the primary terminals. We find B U equal to 2·35 centimetres, $\therefore$ torque $2{\cdot}35 \times 155 = 365$ kilogramme-metres. This is $\frac{365}{171} = 2{\cdot}14$ times full

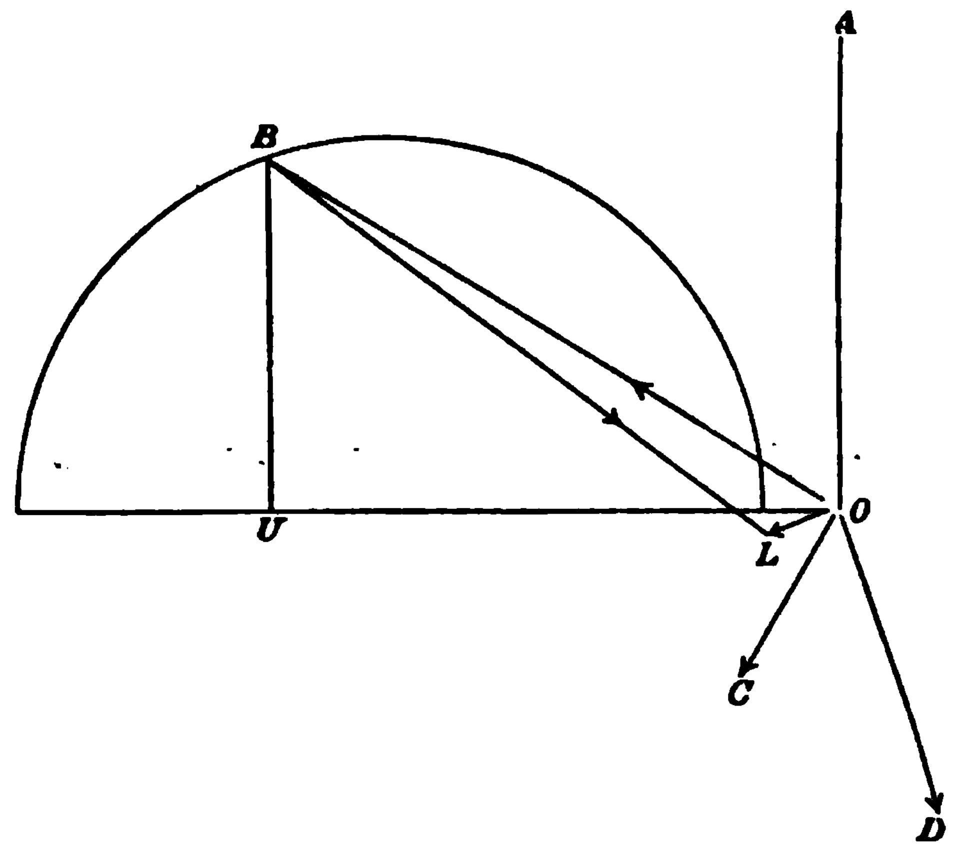


FIG. 412.—Diagram of Resistance inserted in Secondary Circuit.

load torque, and $\frac{450}{135} = 3{\cdot}34$ times full load current, and the torque per ampere—*i.e.* the "specific" torque—is $\frac{365}{450} = 0{\cdot}81$ kilogramme-metres per ampere, nearly eight times as great as the specific torque obtained (0·106 kilogramme-metres per ampere) when the secondary was short circuited. We must ascertain the *amount* of the increase in secondary resistance which has brought about this improvement. Lay off (Fig. 412) the primary reactance voltage O C equal to $450 \times {\cdot}294 = 132$ volts, and thus obtain O D as the component, which with O C balances the terminal voltage O A. At right angles to O D lay off O L equal to 50 amperes. L B, equal to 419 amperes, is the secondary current. But letting R_s = secondary resistance per phase, we have (from p. 362), watts

input $= I^2R_s = 31,800 \times$ B U $= 31,800 \times 2{\cdot}35 = 74,700$; $R_s = \frac{74700}{4192} = 0{\cdot}426$ ohm—*i.e.* to the resistance of the windings per phase, 0·047 ohm, has been added ·379 ohm, bringing the total resistance per phase up to $\frac{0{\cdot}426}{0{\cdot}047} = 9{\cdot}05$ times the value on short circuit.

The maximum obtainable torque corresponds to the maximum value of B U, hence to such a resistance as shall require 390 primary amperes per phase, this being the current corresponding to the motor's maximum load. The diagram for this is shown in Fig 413, in which—

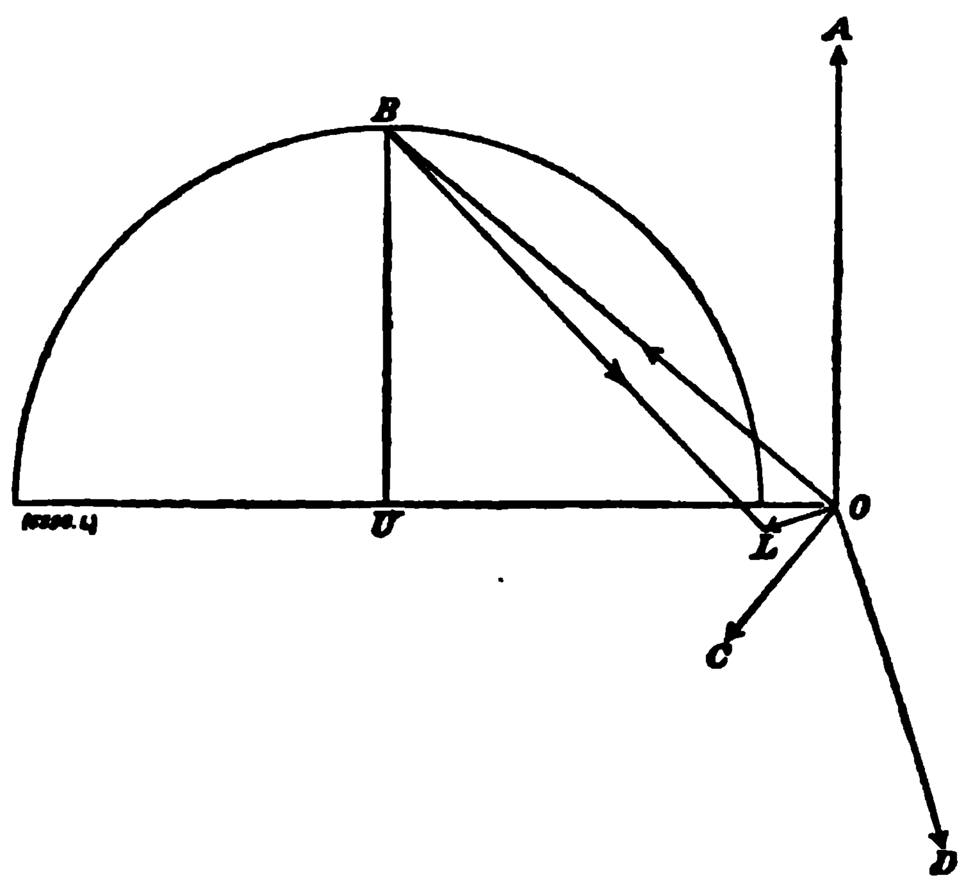


Fig. 413.—Diagram of Resistance inserted in Secondary Circuit.

H_p $= 390 \times {\cdot}294 = 115$ volts = O C.
I_s $= 376$ amperes = B L.
B U = 2·5 centimetres.
$376^2 \times R = 31,800 \times 2{\cdot}5 = 79,500$.
$R = \frac{31,800 \times 2{\cdot}5}{376^2} = {\cdot}590$ ohm, or $\frac{{\cdot}590}{{\cdot}047} = 12{\cdot}6$ times the resistance on short circuit.
Torque $= 155 \times 2{\cdot}5 = 387$ kilogramme-metres, or $\frac{387}{171} = 2{\cdot}27$ times full load torque.

This is the maximum torque which this motor is capable of exerting at normal primary voltage (318 volts), and the required amperes input are $\frac{390}{135} = 2{\cdot}9$ times full load current. The "specific" torque is $\frac{387}{390} = 1{\cdot}0$ kilogramme-metre per ampere.

At 250 primary amperes input per phase we have the diagrams of Fig. 414.

$H_p = 250 \times \cdot 294 = 73\cdot 5$ volts.
$I_s = 235$ amperes.
B U = 2·0 centimetres.
Watts input per phase = 31,800 × 2·0 = 63,600 watts.

$R = \frac{63,600}{235^2} = 1\cdot 16$ ohms, which is $\frac{1\cdot 16}{\cdot 047} = 27\cdot 6$ times the resistance on short circuit.

Torque = 155 × B U = 155 × 2·0 = 310 kilogramme-metres, the torque already being on the decrease, and only equal to $\frac{310}{171} = 1\cdot 82$ times full load torque for $\frac{250}{135} = 1\cdot 85$ times full load current.

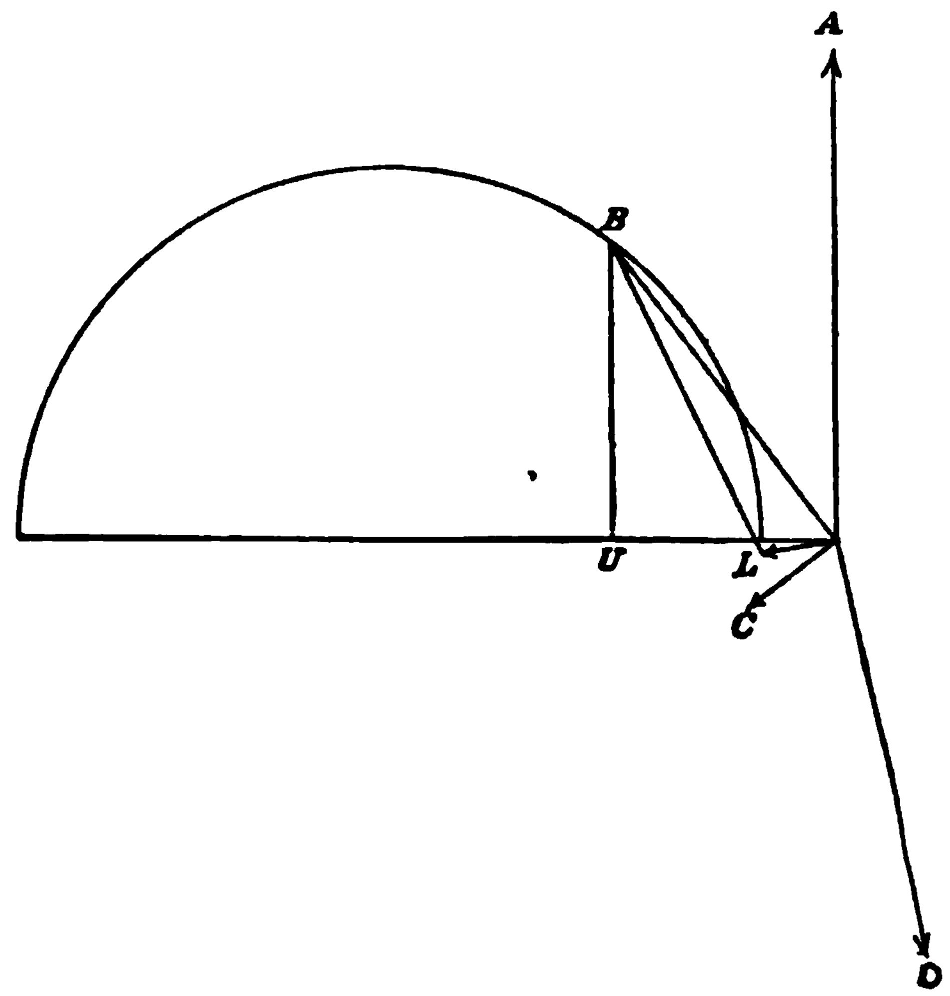


FIG. 414.

The "specific" torque equals $\frac{310}{250} = 1\cdot 25$ kilogramme-metres per ampere, this still being on the increase.

If such a resistance is employed as to require, at starting, the primary current corresponding to rated full load, 135 amperes per phase, we shall have the diagram of Fig. 415 with—

$H_p = 135 \times \cdot 294 = 39\cdot 6$ volts.
$I_s = 120$ amperes.
B U = 1·1 centimetres = 35,000 watts input per phase, and also equal to 1·1 × 155 = 171 kilogramme-metres torque.

$R = \frac{35,000}{120^2} = 2\cdot 44$ ohms, or $\frac{2\cdot 44}{\cdot 047} = 52$ times the resistance on short circuit.

Thus, with a secondary resistance of 2·44 ohms per phase, the motor starts with full load torque and requires full load primary current.

The "specific" torque at full load current is equal to $\frac{171}{135}=$ 1·27 kilogramme-metres per ampere, this "specific" torque having passed through its maximum value of 1·29 kilogramme-metres per ampere at 165 primary amperes, the value corresponding to the maximum power factor. From that point the specific torque falls off, *rapidly* with respect to the decreasing primary current, and *slowly* with respect to the increasing resistance.

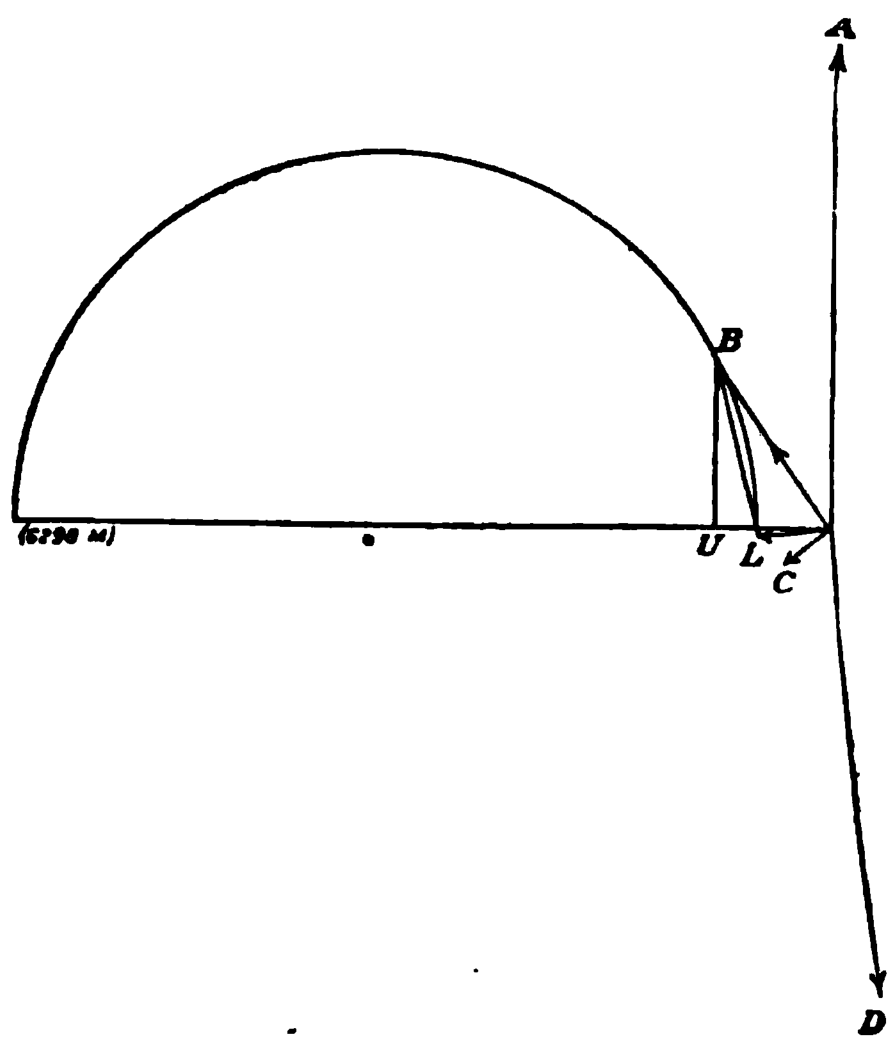


FIG. 415.

At 90 amperes input (Fig. 416, page 371) 4·35 ohms per phase are required, for—

I_p = 90 amperes.
H_p = $90 \times 294 = 26{\cdot}4$ volts.
I_s = 70 amperes.
B U = 0·67 centimetre.
$R = \frac{0{\cdot}67 \times 31{,}800}{70^2} = 4{\cdot}35$ ohms.
Torque $= 0{\cdot}67 \times 155 = 104$ kilogramme-metres.

The "specific" torque is now but $\frac{104}{90} = 1{\cdot}16$ kilogramme-metres per ampere. At 50 primary amperes input, corresponding

to the magnetising current, the specific torque has become zero, and the resistance infinite—*i.e.* the open circuited secondary.

We have thus traced through the cycle of values for the torque from short circuited to open circuited secondaries.

The values corresponding to Fig. 394, page 346, and to Figs. 412 and 413, pages 367 and 368, and Figs. 414 to 416, are brought together in Table LV., together with others readily deduced therefrom.

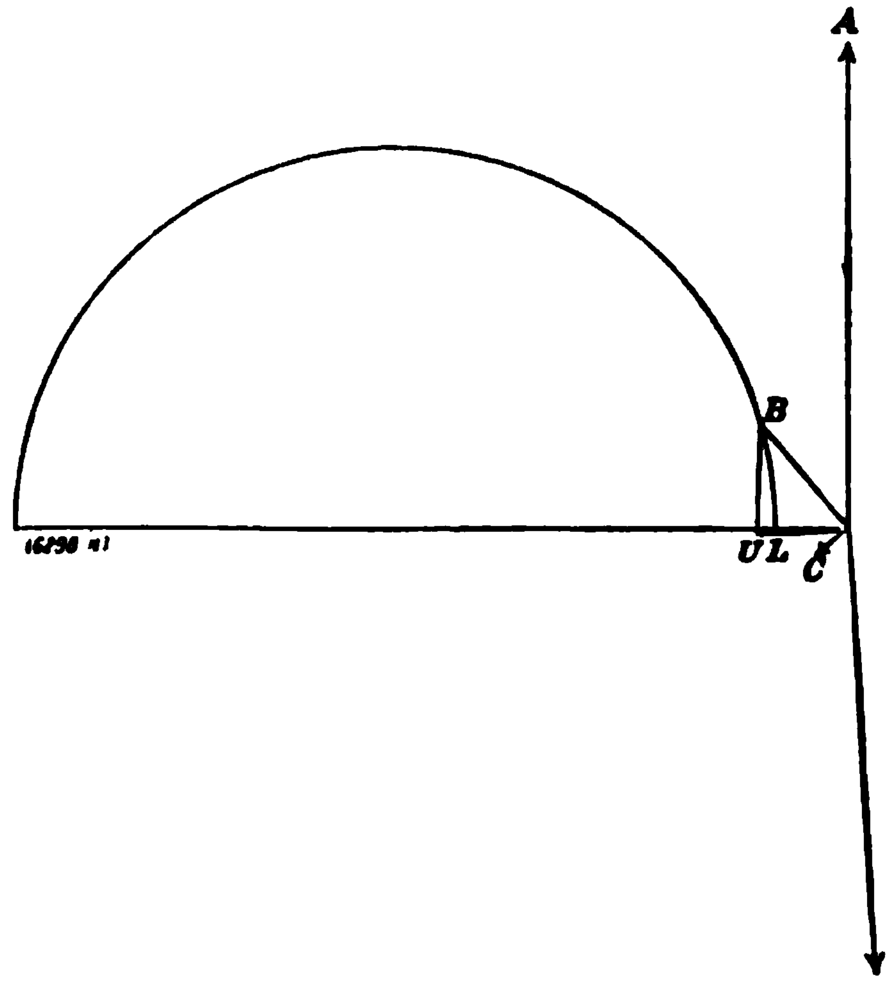


FIG. 416.

In Figs. 417 to 421 are plotted curves from the values in Table LV.

TABLE LV.—DATA DEDUCED FROM DIAGRAMS FIGS. 394 AND 412 TO 416.

Figure Number for Diagram.	Primary Amperes per Phase—I_p.	Secondary Amperes per Phase—I_s.	Secondary Resistance in Ohms per Phase R_s.	Watts Input per Phase (also equals Secondary $I^2 \cdot R$ per Phase) equals $31{,}800 \times B\,U$.	Torque in Kilogramme-metres $= 155 \times B\,U$.	"Specific" Torque—Kilogramme-metre per Primary Ampere.	Ratio of Torque Obtained to the Full Load Torque of 171 Kilogramme-metres.	Ratio of Primary Amperes to the Full Load Primary Current of 135 Amperes.	Power Factor (cos. ϕ).	ϕ in Degrees.	Tan ϕ_s (equals $\frac{\cdot 294}{R_s}$)	ϕ_s in Degrees.
394	547	500	·047	11,800	58	0·106	0·34	4·05	·068	86	6·3	81
412	450	419	·426	74,700	365	0·81	2·14	3·34	·52	59	0·69	35
413	390	367	·590	79,500	389	1·00	2·28	2·90	·64	50	0·50	27
414	250	235	1·16	63,600	311	1·25	1·82	1·85	·80	37	0·254	14
415	135	120	2 87	35,000	171	1·27	1·00	1·00	·82	35	0·103	6
416	90	70	4·35	21,300	104	1·15	0·61	0·67	·745	42	0·068	4

From these curves we observe that the use of a comparatively small additional resistance in series with the secondary windings is accompanied by a great increase in starting torque (see Fig. 418), but with only a very slight decrease in the primary current required (see Fig. 417). With a total resistance of about 0·6 ohm per phase in the secondary circuit—*i.e.* with about thirteen times the resistance on short circuit—the maximum torque is reached.

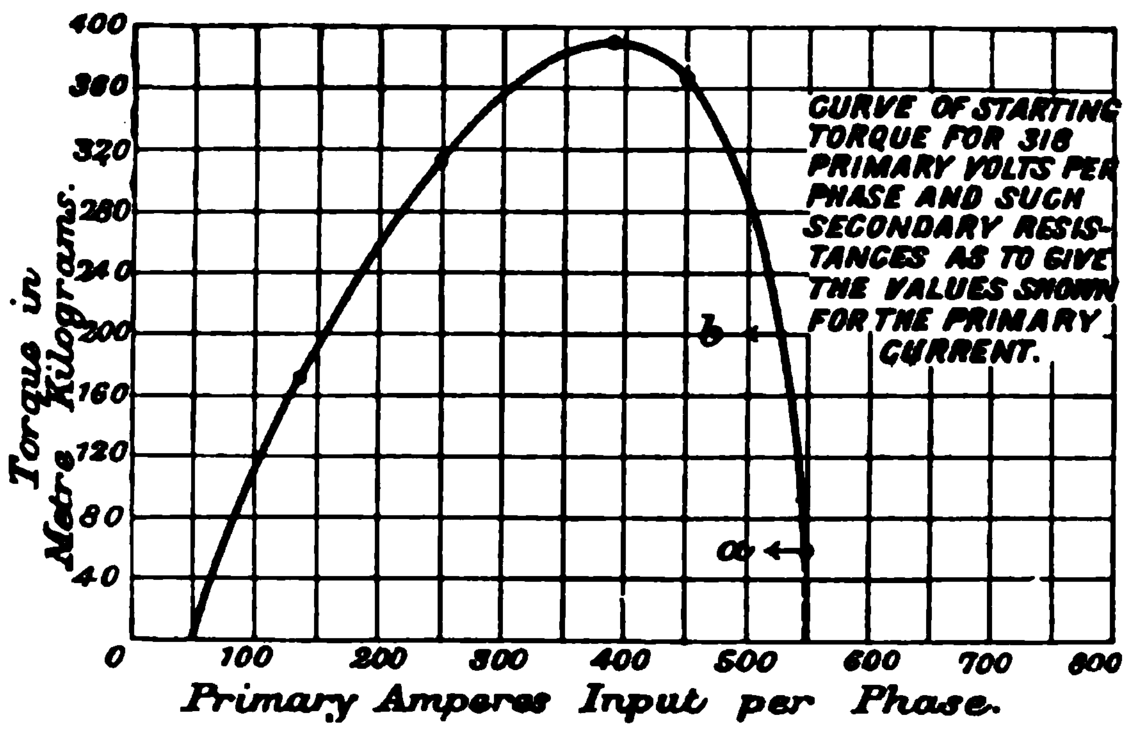


FIG. 417.—Starting Torque and Primary Current.

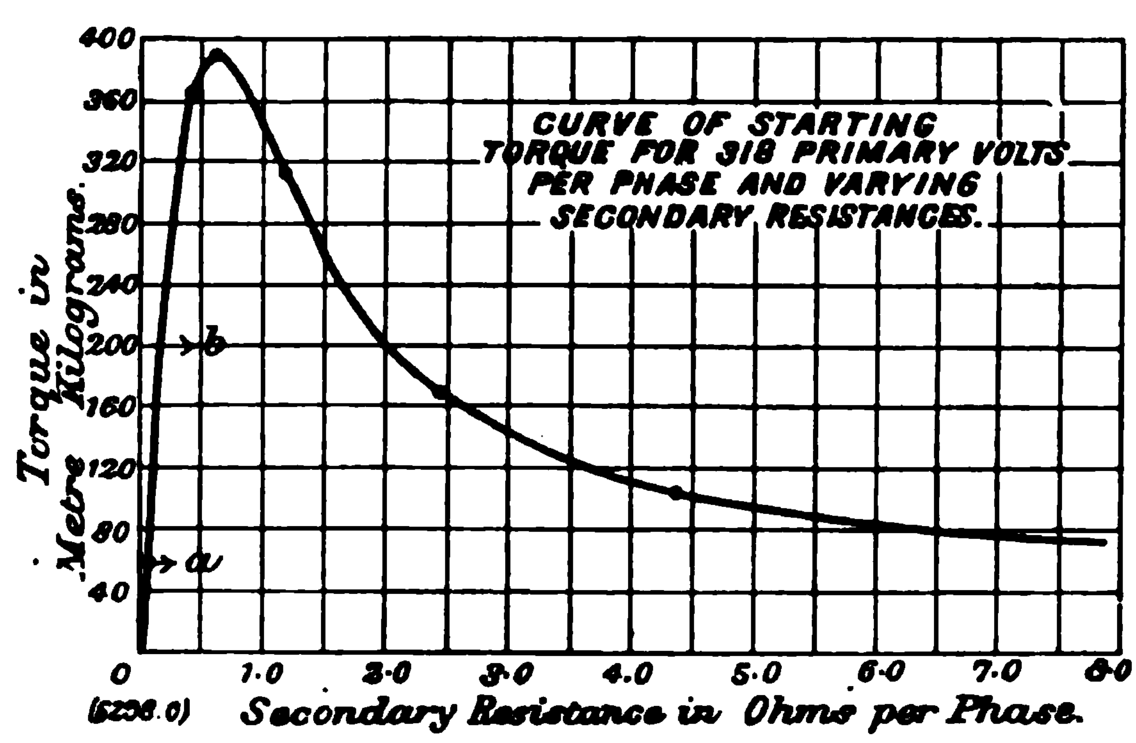


FIG. 418.--Starting Torque and Secondary Resistance.

Hence for cases in which, irrespective of the amount of primary current, it is important to obtain the maximum starting torque, or the maximum rate of acceleration for a given load, the resistance should be adjusted to 0·6 ohm per phase for this particular motor. The motor will then, at starting, be capable of exerting $\frac{390}{171} = 2{\cdot}3$ times full load torque, but will require $\frac{390}{135} = 2{\cdot}9$ times full load current. Generally it is desired that the motor shall

have a high "specific" torque—*i.e.* that it shall start with a maximum, or at any rate a *fairly* high torque, per ampere absorbed during starting. From Fig. 419 we see that the maximum

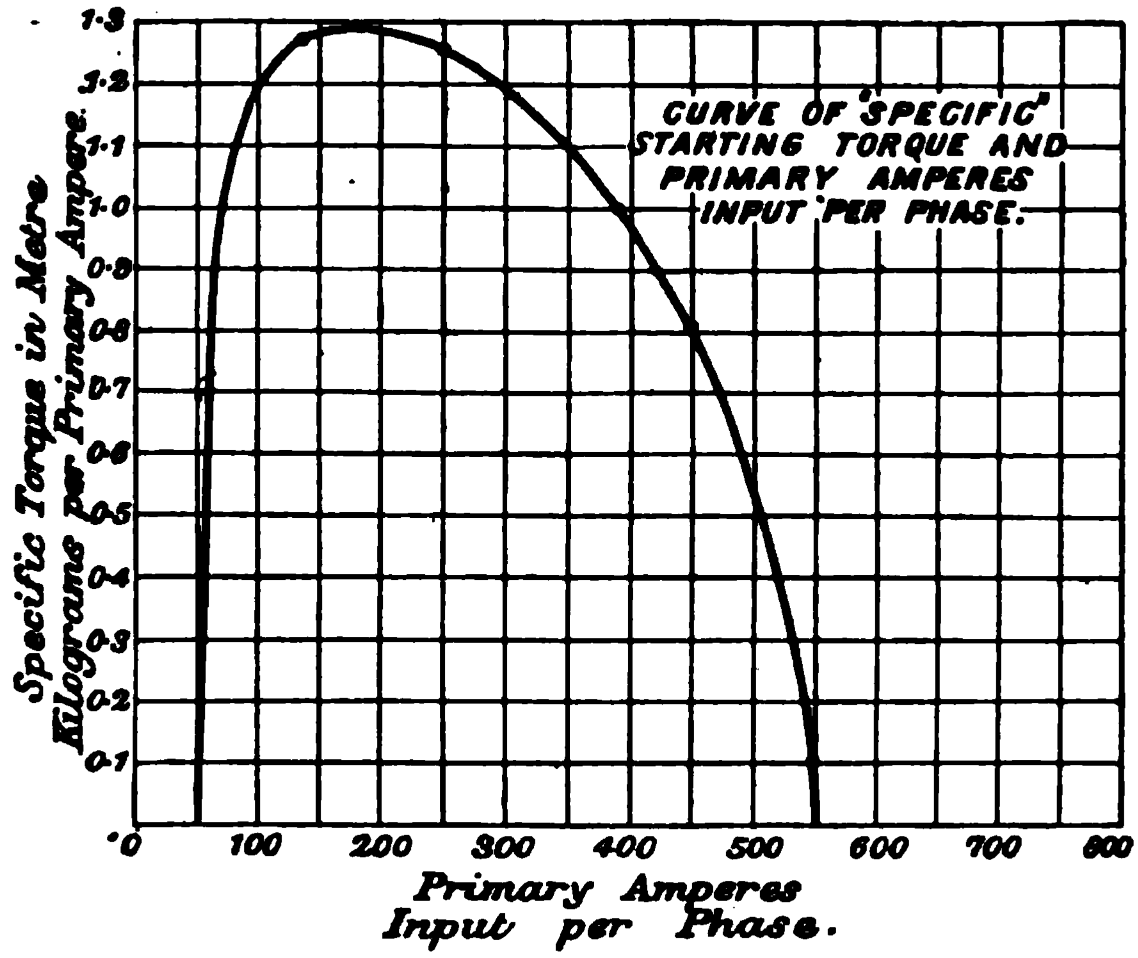


FIG. 419—Specific Starting Torque and Primary Current.

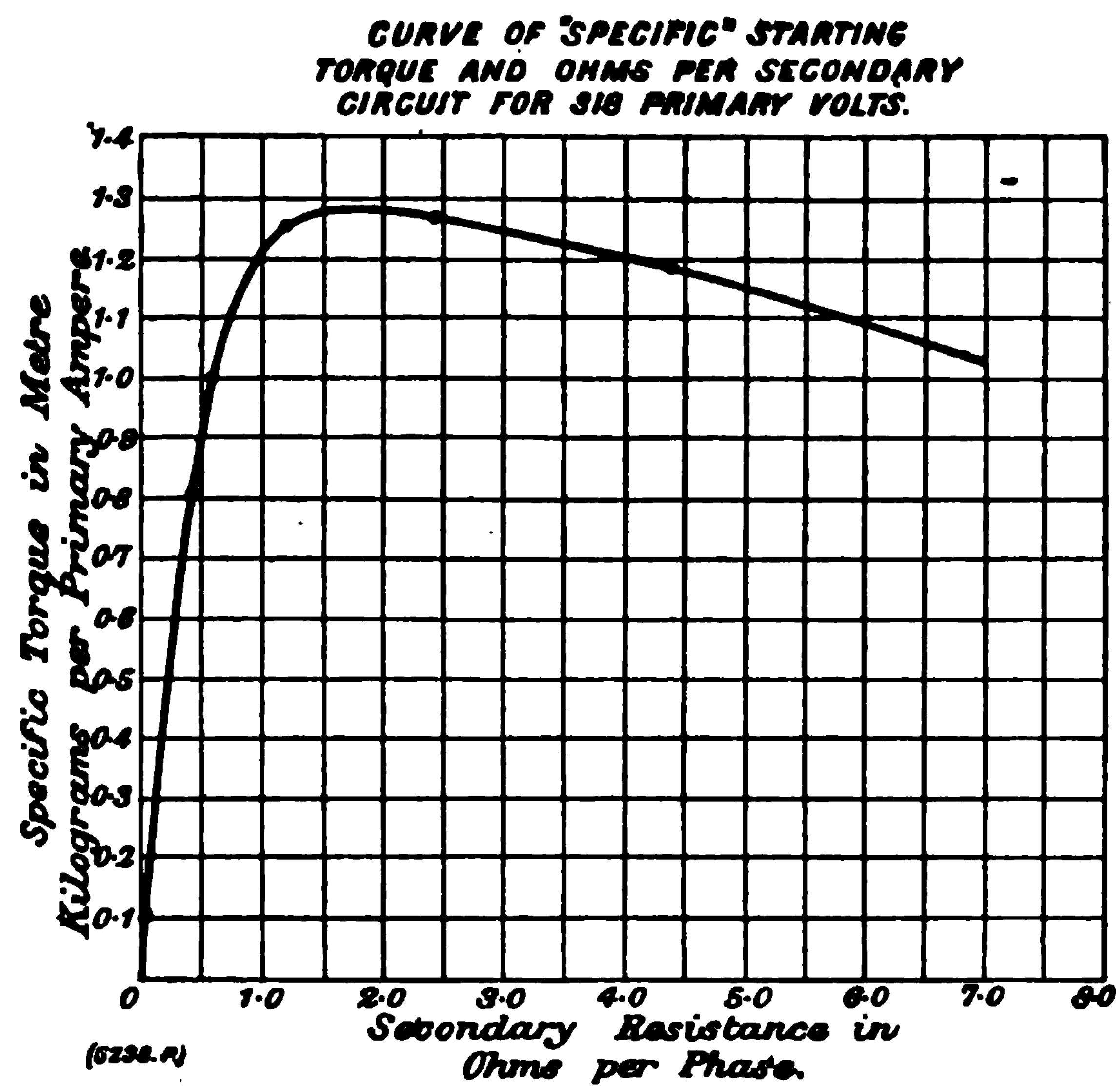


FIG. 420.—Specific Starting Torque and Secondary Resistance.

value of the specific torque is 1·29 kilogramme-metres per ampere, and that it occurs at an adjustment with about 3·5 ohms per phase (see Fig. 420), and requires about 165 amperes input—*i.e.* 1·23

times full load amperes, and gives a torque of $165 \times 1{\cdot}29 = 212$ kilogramme-metres, or $\frac{212}{171} = 1{\cdot}23$ times full load torque.

Of course, we can so adjust the resistance as to restrict the starting current to very small values. Thus, for a starting current of 90 amperes (two-thirds of full load current), we should want 4·35 ohms per phase, and should obtain a starting torque of 104 kilogramme-metres (61 per cent. of full load torque).

We have fairly covered the subject of torque in the explanations leading up to the curves of Figs. 417 to 421.

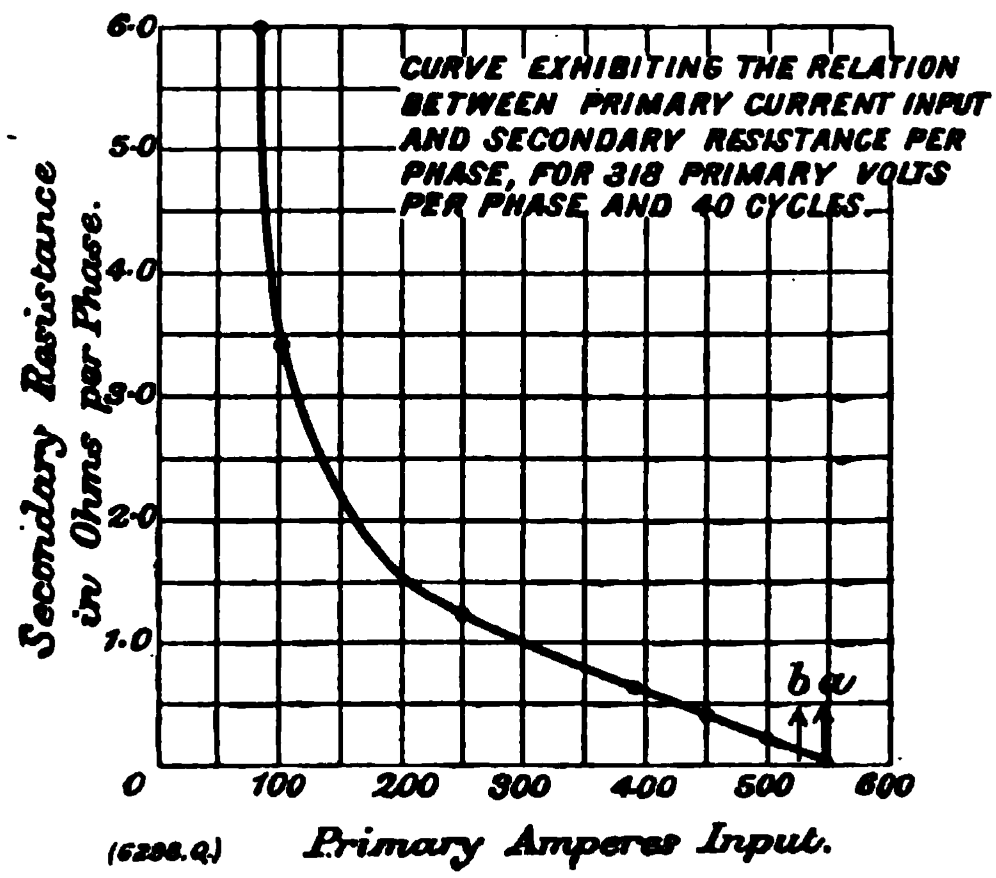


FIG. 421.—Secondary Resistance and Primary Current.

For practical purposes the calculations described are those most useful in this connection. But a clearer insight into the inner workings will be obtained by examining the relations of the residual flux M_s, secondary current I_p, and the torque resulting from these two quantities. From Figs. 394 and 412 to 416, we obtain the values for E_g, Torque, and ϕ, in Table LVI., in which are also shown the values deduced therefrom for M_g and for M_s ($M_s = M_g \cos. \phi_s$). From these the product $M_s\ I_s$ is obtained, and then the ratio of torque to $M_s \times I_s$.

It is evident, then, that the torque is proportional to the product of the secondary current I_s, and the flux actually penetrating the secondary windings M_s, and is equal to $0{\cdot}66 \times$ flux in megalines $\times$ current in amperes.

TABLE LVI.—VALUES OF E_g, TORQUE, ϕ_s, M_g AND M_s.

Figure Number.	C_p.	C_s.	E_g (100×(O D).		ϕ_s (from Table XLVIII.) (Degrees).	Cos. ϕ_s.	M_s ($M_s = M_g$ cos. ϕ_s).	$M_s \times I_s$.	Torque Kgm.: metres (Table LV.).	$\frac{\text{Torque}}{M_s \times I_s}$.
394	547	500	149	1·12	51	·156	0·176	88	58	0·66
412	450	419	215	1·61	35	·819	1·32	558	365	0·66
413	390	367	240	1·80	27	[illegible]	1·60	[illegible]	389	0·66
414	250	235	276	2·07	14	·970	[illegible]	472	311	0·66
415	136	120	290	2·17	6	·994	2·16	260	171	0·66
416	90	70	300	2·25	4	·998	2·25	158	104	[illegible]

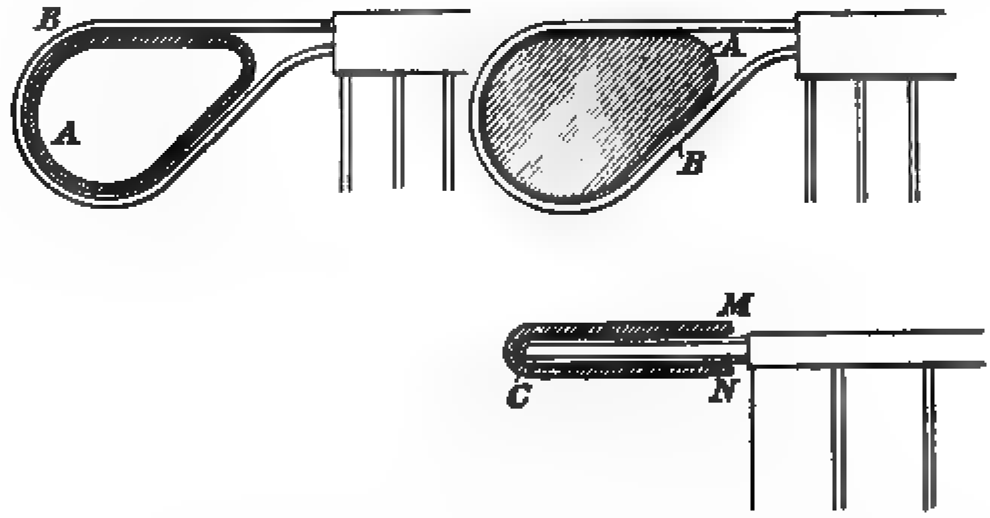
The constant 0·66 is dependent upon the geometrical dimensions of the motor, such as active length of secondary conductor, and the "breadth coefficient" of the winding. M_s being the flux actually penetrating through the secondary windings, it follows that torque when expressed in terms of M_s and I_s is independent of ϕ_s. But $M_s = M_g$ cos. ϕ_s, and $M_g = K\ E_g$ (K being a constant) $\therefore$ $M_s = K\ E_g$ cos. ϕ_s, and $\therefore$ from Torque$=K\ M_s\ I_s$ we obtain Torque$=K^1\ E_g\ I_s$ cos. ϕ_s. E_g is a function of E_p and H_p, and these are properties of the primary winding.

However, ϕ_s is a property of the secondary winding, and is equal to the angle whose tangent is the ratio of its reactance to its resistance—*i.e.* $\phi_s = \tan^{-1}\dfrac{\text{secondary reactance}}{\text{secondary resistances}}$, and as $\cos.\ \phi_s = \cos.\left(\tan^{-1}\dfrac{\text{secondary reactance}}{\text{secondary resistance}}\right)$, it follows that, for a given secondary current and a given resistance, the starting torque is less, the greater the reactance of the secondary winding.

The writer has proposed (British Patent No. 17,641 of 1901) to make use of this circumstance to improve the starting properties of a motor with a wound secondary. The arrangement is illustrated in principle in Figs. 422 to 425, and is seen to consist in the provision of closed circuits within inductive range of the end connections of the secondary windings, which may be either upon the stator or rotor. At starting, and during acceleration, the periodicity in the secondary windings is high, and these auxiliary closed circuits are very effective in reducing their inductance. At normal running speed the periodicity is small, being the small percentage of the primary periodicity corresponding to the per-

centage slip, and the currents induced in these auxiliary circuits are practically negligible, hence also the losses therein, during normal running. Such a device could not be applied to the *primary* winding of an induction motor, as this continuously carries currents of the full periodicity, and the auxiliary circuits would be the seat of a large and permanent expenditure of energy. The maximum starting torque obtainable is, however, also increased by diminishing the primary inductance.

§ 17. **Reducing the Current at Starting.**—The best means of reducing the secondary inductance is by the employment of a squirrel cage rotor, as the end connections and their inductance are then practically eliminated. Moreover, the winding may be considered as having a "breadth factor" equal to 1·00; hence, in



FIGS. 422–425.—Suggested Method for reducing the Inductance at Starting, of the Secondary Windings of Induction Motors.

The shaded cross section *A* within the loop formed by the end connections *B* is that of an annular ring of copper or aluminium. The outer shields in Figs. 424 and 425 are connected at the core end by conducting ligaments running from *M* to *N*.

Torque $= K \times I_s \times M_s$ not only would K be $\frac{1{\cdot}00}{{\cdot}95} \times 0{\cdot}66 =$ 0·695, but M_s would be greater because of the lower inductance (*i.e.* ϕ_s would be less, and cos. ϕ_s greater, so that M_s would more approach to M_g in magnitude). In a squirrel cage rotor, however, whatever resistance is adopted for the winding must remain unchanged for starting and running, and if made sufficiently great for effective starting, the large slip and excessive secondary I^2R loss are very objectionable for normal running conditions. The squirrel cage motor should preferably be proportioned for correct running conditions, and arrangements should be made for starting a motor free, and afterwards applying the load. Where such

arrangements are permissible, the results obtained in all other respects are not to be equalled by any type of "wound" motor. There nevertheless frequently arise occasions where squirrel cage rotors may preferably be wound with, say, one-half or one-third as great total cross section of face conductors as is provided in the primary winding. This doubles and trebles respectively the full load "slip," and the secondary I^2R loss, decreases the efficiency, and increases the heating. Such small increase in resistance (two to three times that of a normally proportioned winding) does not much decrease the primary current, as we can see at a glance from the limited range between *a* and *b* in Fig. 421, page 374, where primary current is plotted against resistance from the values in Table LV. But from the curve in Fig. 417 we see that the *torque* is doubled and trebled in the two cases, although the primary current is only decreased from 547 to 525 amperes—*i.e.* by but 5 per cent. Thus, recognising that for a good squirrel cage motor with only moderate rotor heating and efficiency, we cannot, at starting, appreciably reduce the current input to the motor itself by any practicable increase of resistance, we must resort to the use of a compensator, as already diagrammatically shown in Figs. 286 and 298, on pages 241 and 249, and reduce the current drawn from the line. This means, however, reducing the voltage at the motor's primary terminals. If we tap from the middle point of the compensator, M_s and I_s will both be reduced to about one-half, and the torque being proportional to their products, will be but 25 per cent. of its value at full voltage. But the current, drawn from the line at the moment of starting, is one-half of the value absorbed by the motor, or 25 per cent. of the value which the motor absorbed when subjected directly to the line voltage, hence the torque per ampere primary line current, the "specific" torque, remains unchanged, although the maximum torque attainable from the motor has been reduced to one-fourth, as also the maximum current which it will take from the line. The purpose of the compensator is, then, to reduce the line current at starting, for cases where less torque suffices, than the maximum torque of which the motor is capable. This maximum obtainable torque is proportional to the square of the terminal voltage. Except for the additional current consumed by the compensator, the torque per ampere consumed from the line is the same, whether a compensator is used or not.

Another way of reducing the current at starting is to have the primary windings "delta" connected for normal running, and

change them to Y connection for starting. The voltage per phase is thus but ·577 as great at starting as when running, and the current also is correspondingly smaller than it would be were the "delta"-connected primary windings switched directly upon the line. The torque is but $\cdot 577^2 = \cdot 333$ of that which could be obtained by applying full voltage per phase. The torque per ampere drawn from the line remains, however, unchanged, for with, say, 100 amperes per winding, we have, with delta-connected primary, 173 amperes per line to the motor, and if the torque is then 100 kilogramme-metres, the specific torque is $\frac{100}{173} = \cdot 577$ kilogramme-metres per ampere from line. Now, when the windings are Y-connected, the motor takes but 57·7 amperes per winding, and also per line, and develops a torque of only 33·3 kilogramme-metres ($33\cdot 3 = \sqrt{\cdot 577} \times 100$), the same *specific* torque of $\frac{33\cdot 3}{57\cdot 7} = \cdot 577$ kilogramme-metres per ampere from line.

This method, as practically carried out by the British Schuckert Company, has been described and illustrated on page 248, and in Fig. 298. A similar method, for the purpose of employing but a single double-throw switch, has been devised by Mr H. S. Meyer. It is employed with the standard motors of the British Thomson-Houston Company. As arranged for Y starting and Δ running of three-phase motors, the connections are as shown diagrammatically in Fig. 426. The four-bladed switch shown at the right connects the three primary windings in Y when thrown into the upper position for starting. When sufficient acceleration has been attained, the switch is thrown down to the lower position, connecting the primary windings in "delta." For large motors an oil switch is employed; for smaller motors the switch is of the air break type.

Fig. 427 illustrates an analogous arrangement patented by Mr Meyer, designed for starting two-phase squirrel cage motors. Here the motor is wound with two circuits in parallel, which, during starting, are connected in series. This arrangement, therefore, corresponds to a compensator with half-voltage taps, reducing torque and current to one-quarter the value at voltage full. It is particularly useful for Corporation supply systems, where it is sometimes required that the starting current shall not exceed one and a quarter times full load value. This is a cheap and reliable arrangement; the switch quite does away with the need for a compensator.

SWITCH DIAGRAM FOR CHANGING FROM Y TO Δ CONNECTION.

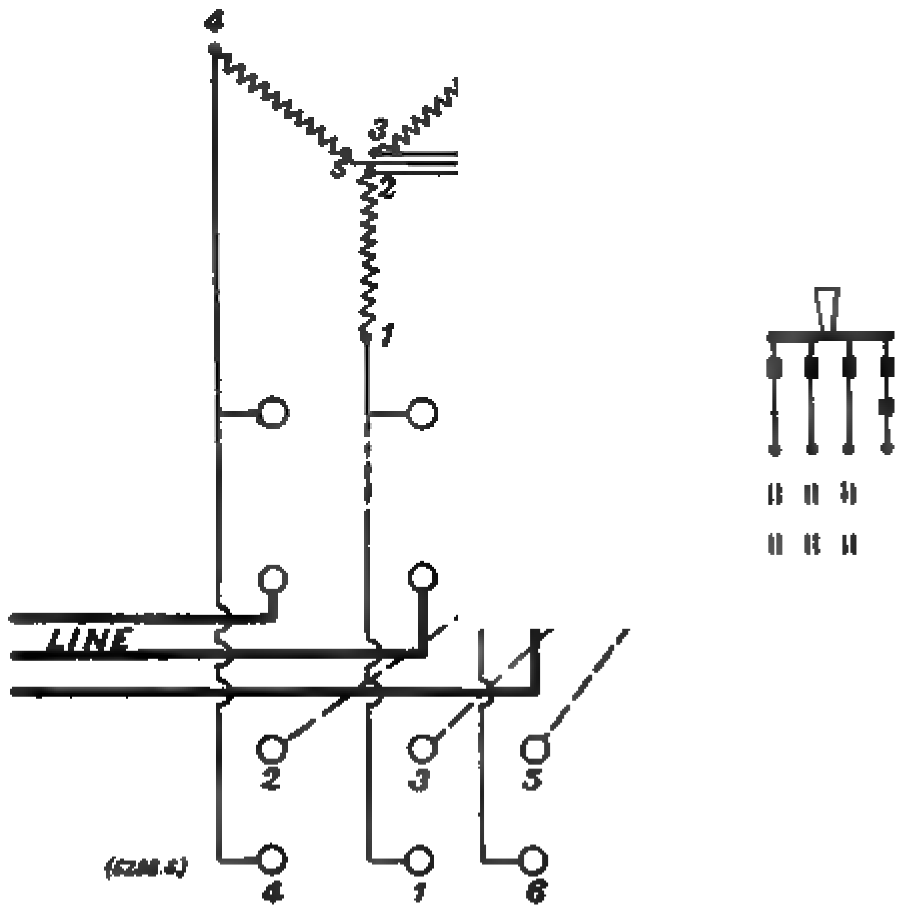


FIG. 426.

SWITCH DIAGRAM TO CHANGE FROM HALF VOLTAGE TO FULL VOLTAGE IN TWO PHASE MOTORS.

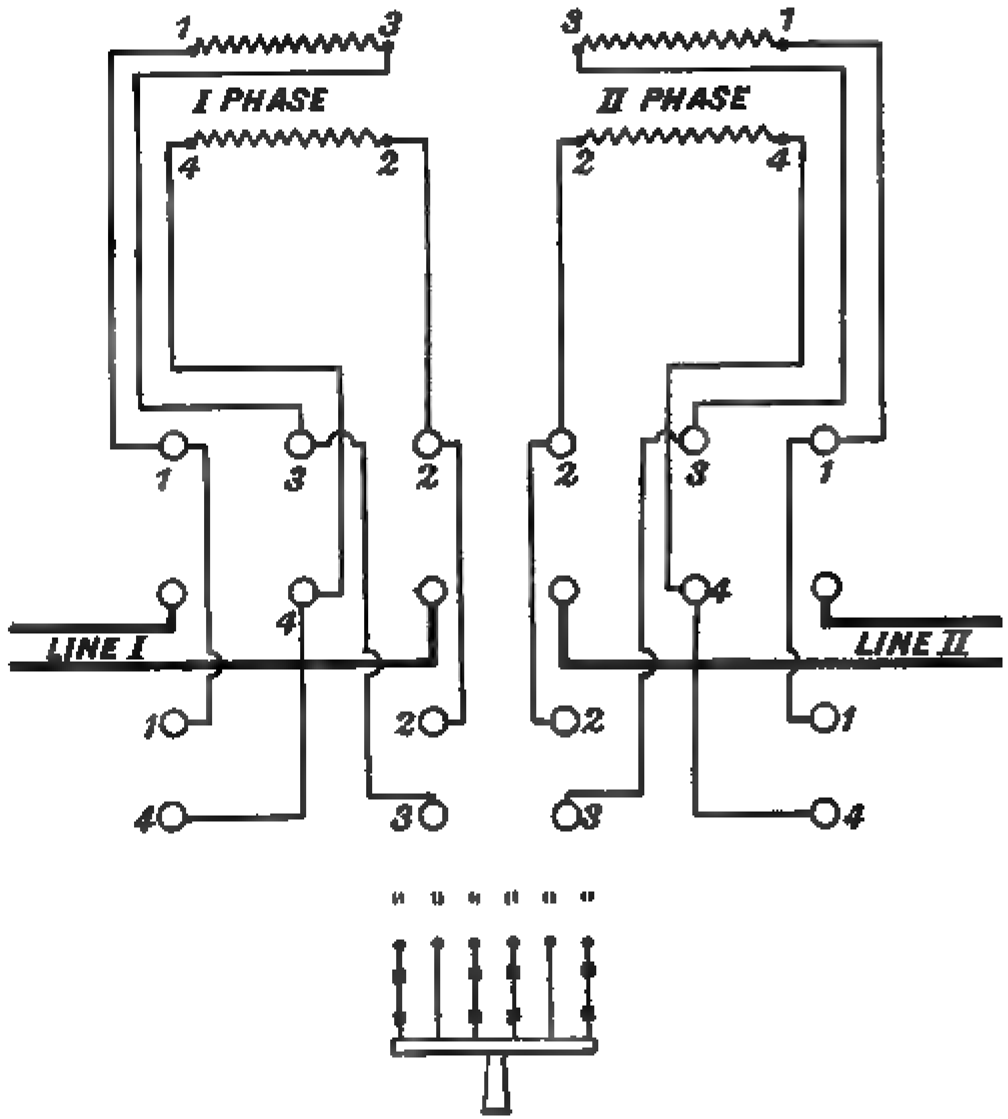


FIG. 427.

§ 18. **Greater Flexibility of the Compensator Method of Starting.**—Methods such as those just described avoid the additional expense of a compensator, but are less flexible, as but one fixed ratio, 1·73 : 1, is at our disposal, whereas with a compensator it is customary to provide several sets of terminals at different distances along the compensator's windings, and use the set best adapted to this particular motor *and its work*, it frequently being desirable not to attempt to assign the precise proportion beforehand. Obviously the line is best protected from heavy currents by using at the motor terminals the lowest voltage practicable for starting it, and for maintaining a suitable acceleration against whatever load it is required to carry.

In Fig. 421, page 374, the primary amperes input at starting has been plotted against the secondary ohms per phase, and from this curve Table LVII. has been derived, showing in the third column the impedance of the motor at standstill for various values of the secondary resistance.

TABLE LVII.—IMPEDANCE OF INDUCTION MOTOR AT STANDSTILL.

Primary Voltage per phase.	Primary Amperes Input.	Impedance of Motor in Ohms per Phase.	Secondary Resistance in Ohms per Phase.
318	90	3·57	5·0
318	120	2·65	2·8
318	150	2·12	2·0
318	200	1·59	1·50
318	300	1·06	1 00
318	400	0·80	0·57
318	450	0·71	0·40
318	500	0·64	0·20
318	547	0·58	0·047

The motor impedance at standstill, and the secondary resistance per phase—*i.e.* the values in the two last columns of Table LVII.—are plotted in Fig. 428. It is the value of this impedance at the point where the curve cuts the axis of abscissæ (*i.e.* at zero secondary resistance per phase) which gives us the reactance of the motor, in this case ·592 ohm. The quotient of the volts per phase, divided by this value, gives us $\frac{318}{\cdot 592}=537$ amperes, the value of the primary current input at standstill. This, minus the no-load running current, $537-50=487$ amperes, is *approximately* the diameter of the circle which we use for constructing our diagrams and analysing the motor's performance. (The exact

diameter of the circle is greater, corresponding to the assumption of inductanceless secondary.)

§ 19. **Testing a Motor's Performance.**—Whereas, in endeavouring to predetermine a motor's performance, we arrive at these quantities by the methods already described for estimating, first, the magnetising current, and, secondly, the inductance; on a *completed* motor these quantities are measured, the first by running the motor unloaded, and the second by observing the primary current input at standstill at known terminal voltage, and deducing the impedance from these observed values. The circle can then at once be constructed, and all the other properties of the motor under various conditions of service (except the heating

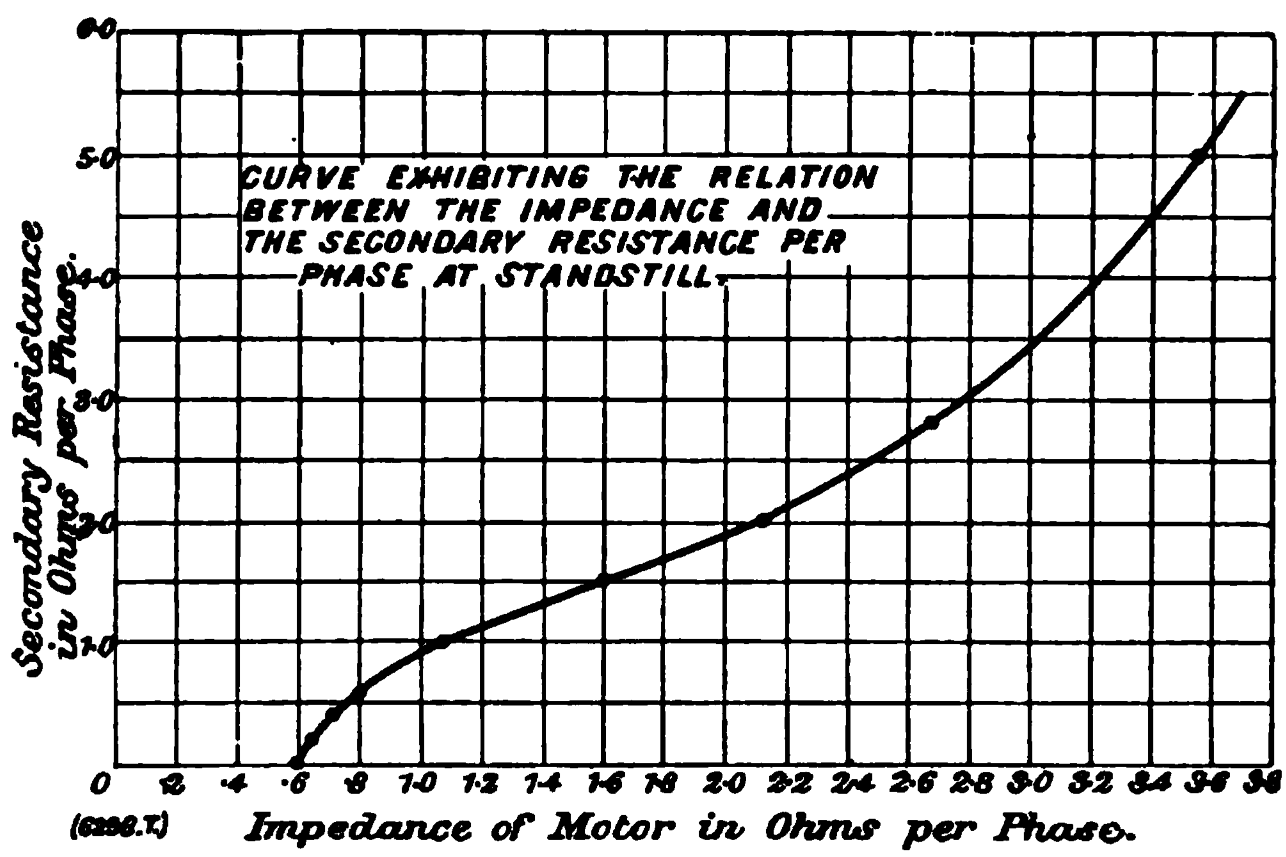


FIG. 428 —Curve between Impedance and Secondary Resistance at Standstill.

and load losses) deduced fairly approximately by means of graphical constructions without need for further tests.

If we want the motor to start with a certain current, we can find the corresponding resistance which must be given to the secondary winding from the curve of Fig. 421, page 374. Thus, if we want the starting current to be equal to full load current, 135 amperes, we find from the curve of Fig. 421 that we must have 2·35 ohms per phase. This is $\frac{2{\cdot}35}{{\cdot}047}=50$ times the resistance on short circuit. $\frac{1}{50}=0{\cdot}020$, and from Fig. 374, H., page 322, we note that 2 per cent. is the per cent. slip at full load. In fact, this offers another way of deducing the per cent. slip for a given rotor resistance per phase, or, having given the per cent. slip

corresponding to a given current, to deduce the resistance required per phase to limit the current at starting to this value.

To show more convincingly that this is the case, Table LVIII. has been prepared, in which the values of the "slip" are taken from the curve of Fig. 399, page 351, and from these values are deduced in the adjacent columns of the table the values of the resistance required to limit the current at starting to the values at running corresponding to the slip. These will be found to agree with Fig. 421.

TABLE LVIII.—VALUES OF "SLIP," AND OF LIMITING RESISTANCES REQUIRED.

Primary Current.	Corresponding per Cent. Slip on Normal Running with Short Circuited Rotor with ·047 Ohm per Phase. Values taken from Curve of Fig. 399.	100 × Reciprocal of Per Cent. Slip.[1]	Factor of Preceding Column × ·047. This equals the required Resistance per Phase.
100	1·4	72·	3·38
135	2·0	50·	2·35
150	2·2	46·	2·16
200	3·0	33·	1·55
300	4·8	21·	9·9
400	8·3	12·1	·57
450	11·6	8·6	·41
500	16·4	6·1	·287

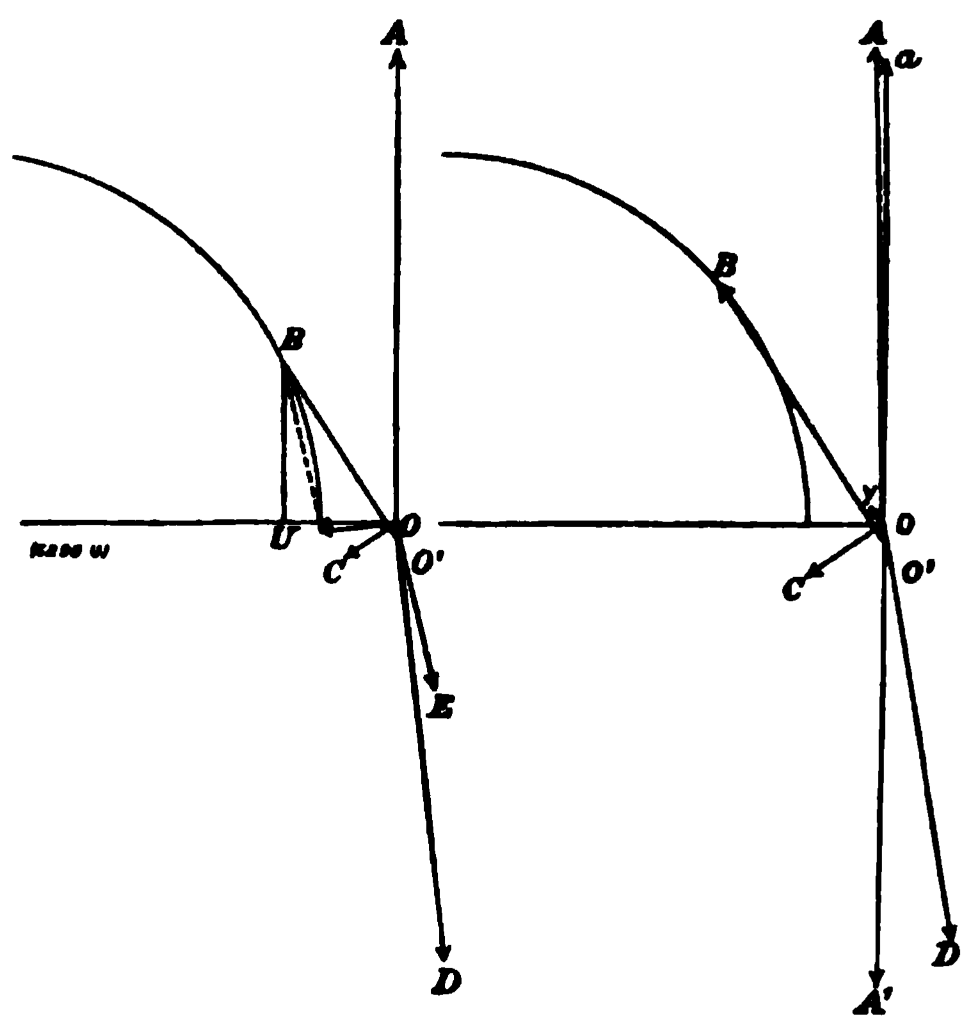
We have now fairly covered the more important possibilities of the typical "circle diagram" with the assumption of zero primary resistance, and no core losses or friction losses. It remains to describe the required modifications in order to take into account the core loss and friction loss, and the resistance of the primary windings.

§ 20. **The Motor Used in Describing the Diagrams.**— The motor of which we have made use in describing the diagrams had a magnetising current of 50 amperes. Its core loss equalled 3600 watts, and its friction loss may be estimated at 2200 watts; the total energy input at no load was, therefore, $3600+2200=5800$ watts. Hence—energy current input at no load equals $\frac{5800}{3\times318}=6{\cdot}1$ amperes. Then, still holding to the assumption of

[1] This is the factor by which one must multiply short circuited resistance (·047 ohm per phase) in order to obtain total resistance per phase at starting in order to limit primary current at starting to the values given in the first column of this table.

zero primary resistance, the diagram for, say, 135 amperes input becomes that shown in Fig. 429, the origin from which the primary current is drawn being O^1 instead of O, the length O O^1 being, to the scale of 100 amperes per centimetre, $\frac{6 \cdot 1}{10} = 0 \cdot 61$ millimetre.

We note that this leads to a slight decrease in the angle by which the primary current O^1B lags behind the terminal voltage O^1A, and hence that the power factor (cos. ϕ) is slightly higher for a given load than when this no load energy component is neglected. But it is obvious that the difference is but slight, and that it may



Figs. 429 and 430.—Diagrams for 135 Amperes and 200 Amperes Input per Phase.

be considered not to materially affect the understanding that we have gained as to the general occurrences in an induction motor under various conditions of operation.

Where, however, we have heretofore taken the watts input as equal to the secondary I^2R loss plus the output from the motor, we should in practice make the more precise analysis, and take the watts input to the rotor equal to secondary I^2R loss + friction + output from motor, and the watts output from the motor as corresponding, *not* to the watts output corresponding to the product of the torque exerted by the rotor windings and the speed, but to this quantity less the friction of the motor. These changes in no wise affect in principle the interesting and valuable con-

clusions we have deduced as to the inter-relations of all these quantities, but merely mean that for a precise analysis still more care must be bestowed on the subject. It will be agreed that it has been a somewhat laborious task already to trace through these relations, and that the introduction of these slight modifications would have obscured the essential points which it was desirable to emphasise. The diagrams will not be repeated with the further modifications required for taking these losses into account, as they do not greatly affect the conclusions as to the motor's performance, further than with regard to input and output, consequently to commercial efficiency, and for efficiency estimates, one would, with the induction motor, proceed on practically the same lines as for a continuous current motor.

These remarks also hold true with regard to the primary I^2R loss. Its inclusion is, of course, essential for efficiency determinations, but does not radically affect the diagrams for the general performance of the motor.

The diagram for Fig. 430 is plotted for the value of 200 amperes input per phase, and with due regard to the actual primary resistance of 0·036 ohm per phase.

Primary I R voltage per phase $= 200 \times {\cdot}036 = 7{\cdot}2$ volts. The counter electro-motive force O A^1 is now no longer equal and opposite to the terminal voltage O A, but to the smaller value O a, which latter and O r, the primary I R voltage, are the two components into which the terminal voltage O A may be resolved. It may be further explained that in this diagram the vectors, in direction and magnitude, are as follows:—

O a equal and opposite to the primary counter electro-motive force O A^1.
O A the terminal voltage $V_p = 318$ volts.
O r the primary I R voltage $= I_p R_p = 200 \times {\cdot}036 = 7{\cdot}2$ volts.
O B the primary current $= 200$ amperes.
O C = the primary reactance voltage $= H_p = 200 \times {\cdot}294 = 59$ volts.
O A^1 = the primary counter electro-motive force $= 312$ volts.
O D = imaginary component voltage $E_g = 283$ volts.

It is thus seen that taking the primary I R voltage into account slightly reduces the value obtained for the angle of lag of primary current O B behind primary terminal voltage O A, and consequently improves the motor's power factor, cos. ϕ, but it will be observed that the effect of taking this into account is but slight, even at this value of 200 amperes (1·48 times full load current), and that one will not go far wrong as to general conclusions by disregarding this I R component in the diagram of the circle.

§ 21. **The Advantages of the Circle Diagram.**—We may thus sum up to the effect that the diagram of the circle affords a very valuable and practicable method of quickly determining the performance of an induction motor under various conditions of starting and running; that it assists us greatly in obtaining a clear picture of the phenomena taking place; that, even neglecting the primary I R voltage and the core and friction losses, it leads to quite an accurate determination of the values of the primary and secondary currents, of the torque and of the slip, and of the phase relations of the currents, voltages, and magnetic fluxes; that the additional exactness required for taking primary I R voltage and core and friction losses into account presents no difficulty for cases where the modification is worth while, but that, as they affect chiefly the efficiency, which can better be determined by ordinary methods, it is in practice generally not worth while to introduce these modifications in the diagrams.

§ 22. **The Dispersion Co-efficient of the Induction Motor.**—The diagram of the circle affords a valuable means of investigating questions relating to the design of induction motors for various periodicities and speeds, and it is to this branch of the subject that we shall now proceed.

The treatment is capable of being made much simpler than by the strict use of the methods so far described. For explanatory purposes it has been desirable to proceed step by step, and to avoid the introduction of unnecessarily difficult conceptions. The time has, however, now come to introduce the so-called "dispersion coefficient" of the induction motor, generally denoted by σ.

If in Fig. 440 *A*, page 397, we let

A = the distance O T; *i.e.* from origin O to left hand end of diameter,

B = the length O L, from origin O to right hand end of diameter,

σ = dispersion coefficient;

Then,

$$\sigma = \frac{B}{A-B}.$$

A is the primary current which, for resistanceless windings, would, at standstill, be absorbed by the motor when normal rated voltage is applied at its terminals, the secondary (rotor) circuits being, of course, short circuited. B is the primary current which would flow under the same conditions were the secondary windings on open circuit, and (neglecting hysteresis and friction) B is also the

"magnetising current"—*i.e.* the current flowing when the motor is running free and without load.

The "dispersion coefficient" (σ) is an extremely important conception, as we shall see, and one must estimate its value at the very outset of the preparation of a design. It takes into account the inductance of the motor, and obviates the necessity for estimating this quantity from the lines per ampere turn per centimetre of length, as, for explanatory purposes, we have done in our illustrative example.

Behrend has proposed the simplest of all formulæ for estimating the value of the dispersion coefficient σ. His formula is

$$\sigma = C \frac{\Delta}{\tau}.$$

C is a constant stated by Behrend to depend "upon the shape and size of the slots and upon a great many other conditions of which we are still profoundly ignorant." ("The Induction Motor," by B. A. Behrend, page 36. Published by *New York Electrical World and Engineer.*)

Δ is the radial depth of the air gap in centimetres.

τ is the polar pitch at the air gap in centimetres.

λ is the net length of core.

It has seemed to the writer that the chief fault in Behrend's otherwise very excellent treatise consists in his overlooking altogether the influence of the inductance of the end connections, which may readily be of preponderating amount, and is in all cases of very appreciable influence upon the total inductance.

To overcome this difficulty, Table LIX. has been prepared, in which values of C for wide open and for completely closed slots are given in terms of $\frac{\lambda}{\tau}$, that is, in terms of the ratio of net core length (λ) to polar pitch (τ). These values are plotted in the two curves of Fig. 431, and results obtained by the use of these curves (or of Table LIX.) will be consistent with those derived by the more tedious method already set forth, starting from the basis of the constants in Table XLVIII., page 333, and in accordance with which the inductance and the magnetising current had each to be separately estimated in order to arrive at the ratio $\frac{B}{A-B}$, or σ. By the use of σ, while one still requires ultimately an estimation of the magnetising current B, one loses sight of the reactance as such (the reactance is, of course, $\frac{\text{Primary voltage per phase}}{A}$), its

influence being indirectly taken into account, and merged in that of the "dispersion coefficient" σ.

FIG. 431.—Behrend's Constant for wide open and for closed Slots.

By Fig. 410, reproduced here, the following relation was shown: Maximum Power Factor $=\dfrac{BU}{BO}$ where BO is a line drawn from O tangent to the circle, and BU is a normal from the point

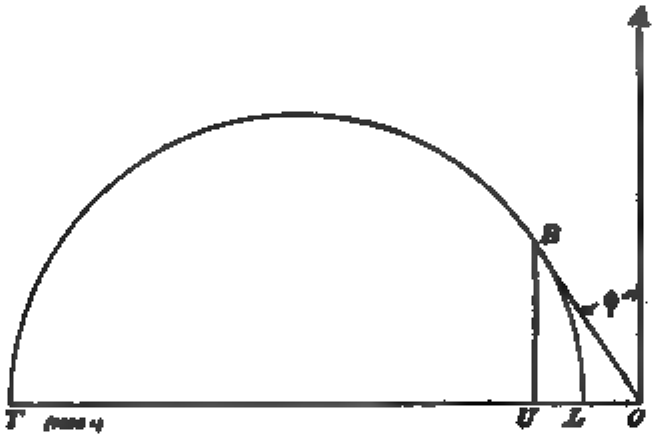


FIG. 410.—Circle Diagram.

of tangency to the horizontal diameter of the circle. $BU : BO = \dfrac{A-B}{2} : \dfrac{A-B}{2}+B$; $\therefore$ maximum power factor $=\dfrac{A-B}{A+B}$.

Now $A-B=\dfrac{B}{\sigma}$, and $A+B=B\ (1+2\ \sigma)$; $\therefore$ maximum power factor $\dfrac{1}{1+2\ \sigma}$.

The short circuit current for resistanceless windings is equal to A, as $\frac{B}{A-B}=\sigma$; $B=\sigma A-\sigma B$; $A=\frac{B(1+\sigma)}{\sigma}$.

B, the magnetising current, has to be calculated in accordance with the method already set forth.

As σ is generally such a small quantity, $\frac{1+\sigma}{\sigma}$ may often conveniently be taken as approximately equal to $\frac{1}{\sigma}$, making $A=\frac{B}{\sigma}$.

TABLE LIX.—VALUE OF BEHREND'S CONSTANT, C, FOR WIDE OPEN AND COMPLETELY CLOSED SLOTS.

Net Length (λ) of Core between Flanges in Per Cent. of Pitch (τ).	Value for C in Behrend's Formula for $\sigma = C\frac{\Delta}{\tau}$.	
	Wide Open Slots.	Completely Closed Slots.
150	6·0	12·5
140	6·2	12·6
130	6·4	12·7
120	6·7	12·9
110	7·1	13·1
100	7·5	13·4
90	8·2	13·8
80	8·8	14·3
70	9·5	14·8
60	10·3	15·4
50	11·3	16·1
40	12·2	16·8
30	13·5	17·7

These values and the curves in Fig. 431 give fair approximations in most cases.

§ 23. **Zigzag Dispersion.**—As stated on page 333, the so-called "zigzag" dispersion is inversely proportional to Δ, the radial depth of the air gap, and to H, the average number of slots per pole for stator and rotor. The values in Table LIX. are fair approximations in the case of designs where $\Delta \times H$ lies between 1·4 and 2·0. For designs lying outside of these values a correction must, in exact work, be applied, which is greater the greater the deviation of $\Delta \times H$ from this range of values.

In the first illustrative examples which follow, the brief method of obtaining σ by the use of the curves of Fig. 431 alone will be used. For later examples an additional curve correcting for the "zigzag" dispersion will be introduced, as leading to still greater precision in the determination of σ.

§ 24. **Breadth Factor.**—By "breadth factor" is denoted a term taking into account the distribution of the armature winding. For customary three-phase windings, for which the winding of each phase is distributed over one-third of the pole pitch, the breadth factor is 0·96, and in quarter phase windings, where the winding per phase is distributed over one-half of the pitch, the breadth factor is 0·91. The breadth factor for any other winding may be found in the following table:—

Percentage of Polar Pitch covered by one side of the Armature Winding.	"Breadth Factor."
20 per cent.	0·99
40 "	0·94
60 "	0·86
80 "	0·76
100 "	0·64

§ 25. **Values of σ in Motors by Various Manufacturers.**—Estimated and observed values for σ in some fifty-seven motors by eight different manufacturers have been tabulated on pages 452 and 453, Table LXV.

CHAPTER XVI

EXAMPLES OF INDUCTION MOTOR DESIGN

§ 1. **Brown, Boveri & Co.'s 25 H.P. Induction Motors.**—Particulars of the designs of a number of modern induction motors have been placed at the writer's disposal by the courtesy of Messrs Brown, Boveri & Co., of Baden, Switzerland. The general mechanical construction is, for the slip-ring type, shown in the outline drawings and photographs of Figs. 432 to 435 and for the squirrel cage type in Figs. 436 to 438 (Plate 22).

We shall first analyse the data of Messrs Brown, Boveri & Co.'s 6-pole, 25 horse-power design for a no-load speed of 1000 revolutions per minute at a periodicity of 50 cycles per second. The design is for a wound rotor with slip rings, and of the general type shown in Figs. 432 to 435.

SPECIFICATION FOR BROWN, BOVERI THREE-PHASE INDUCTION MOTOR.

Rated horse-power	25
Number of poles	6
Periodicity in cycles per second	50
Synchronous speed in revolutions per minute	1000
Voltage	240
External diameter of stator punchings, centimetres ...	46
Internal diameter of stator punchings, " ...	32·15
External diameter of rotor punchings, " ...	32
Radial depth of air gap, centimetres (Δ)	·075
Internal diameter of rotor punchings, centimetres ...	22
Circumference at air gap, centimetres	101
Polar pitch at air gap, centimetres $=(\tau)=$	16·8
Gross length between flanges, centimetres $=(\lambda_g)$	20

As no ventilating ducts are employed in the motor, the net length of laminations, λ_n, differs from the gross length only by the 10 per cent. allowance for insulating varnish on the core plates.

Net length lamination between flanges, in centimetres—λ_n 18·0

$$\frac{\lambda_n}{\tau}=\frac{18{\cdot}0}{16{\cdot}8}=1{\cdot}07$$

Fig. 432.—Brown Boveri Slip Ring Induction Motor, Longitudinal Section.

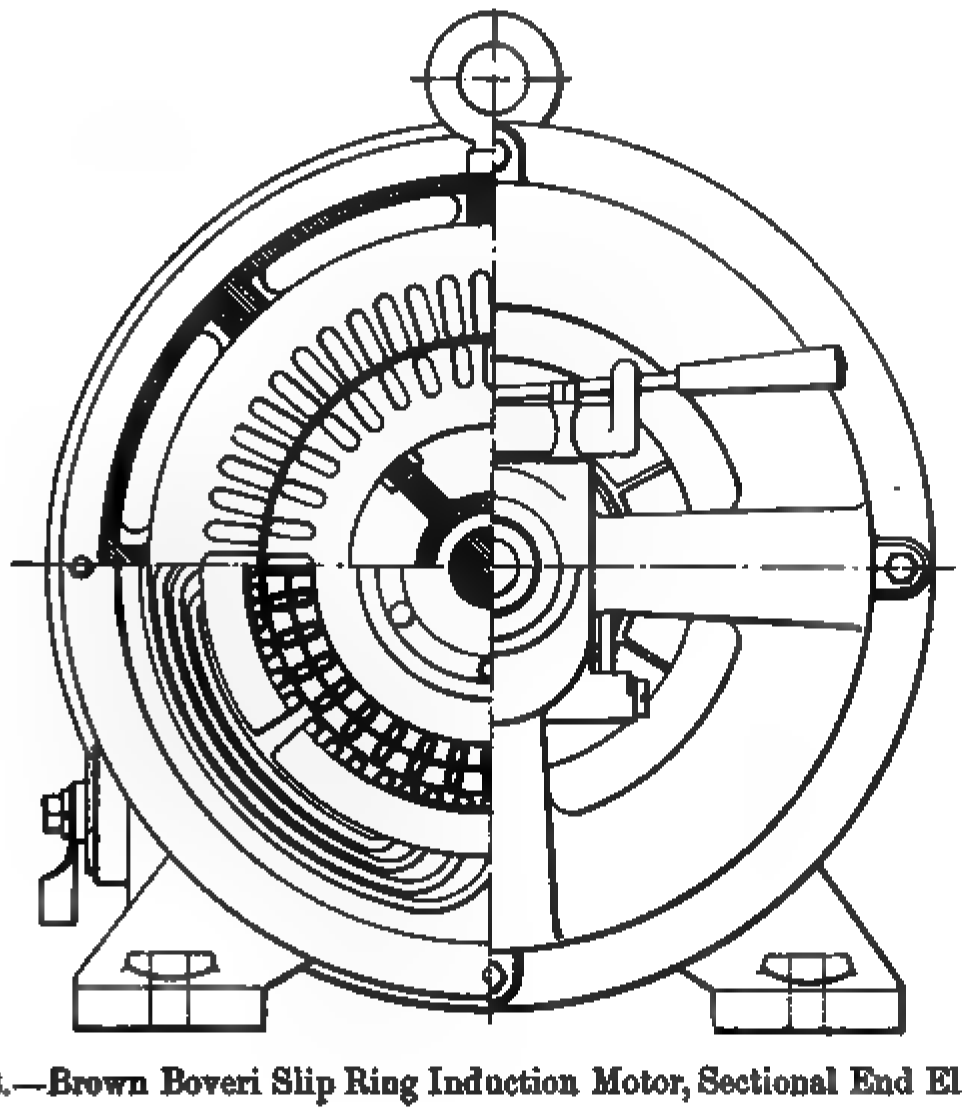


Fig. 433.—Brown Boveri Slip Ring Induction Motor, Sectional End Elevati

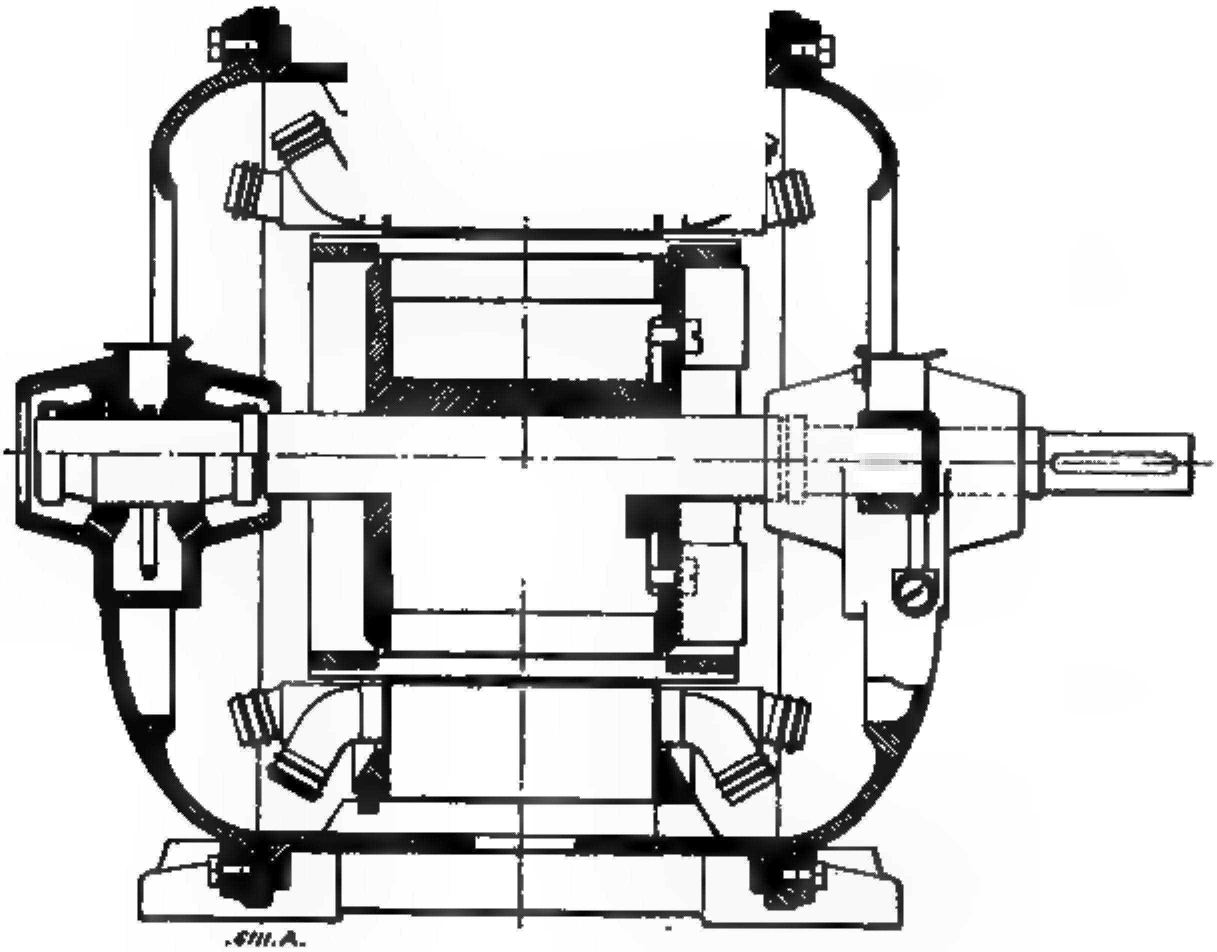


FIG. 436.—Brown Boveri Squirrel Cage Induction Motor, Longitudinal Section.

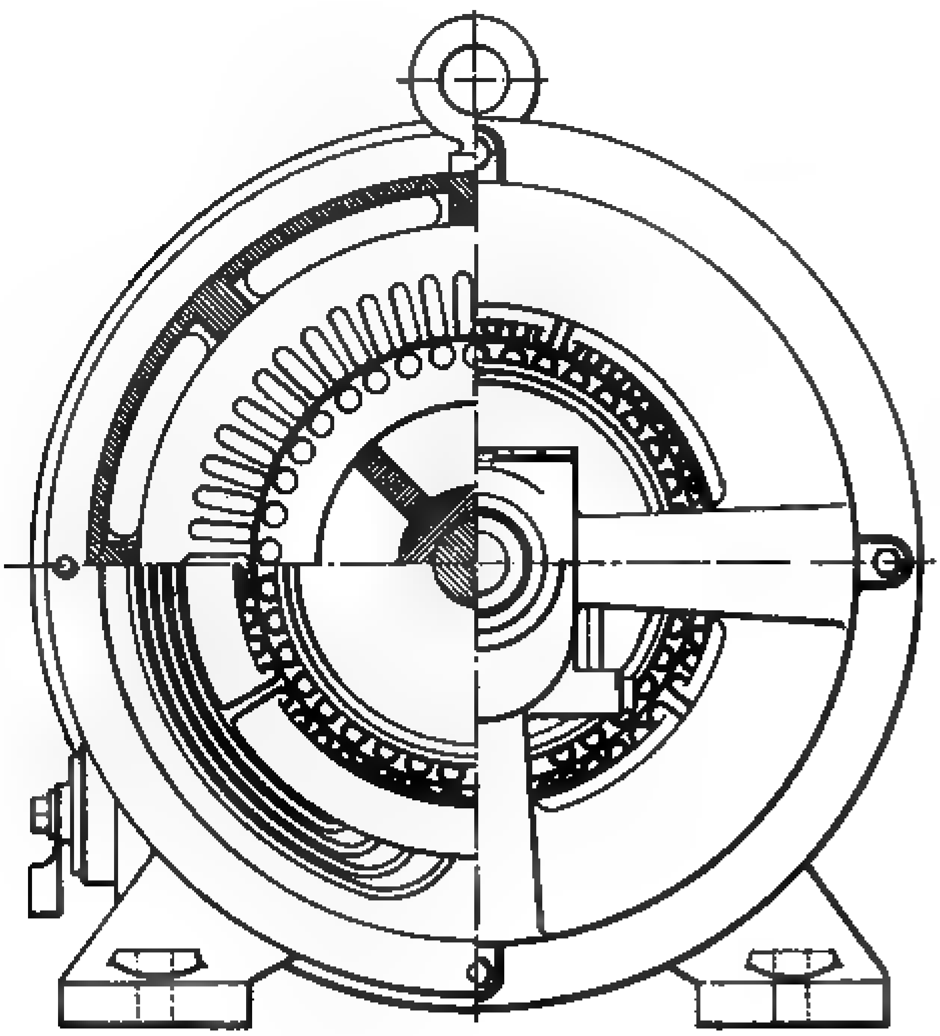


FIG. 437.—Brown Boveri Squirrel Cage Induction Motor, Sectional End Elevation.

FIG. 434.—Slip Ring Induction Motor, by Brown, Boveri & Co. (see page 390).

FIG. 438.—Squirrel Cage Induction Motor, by Brown, Boveri & Co. (see page 390).

FIG. 435.—Slip Ring Induction Motor, by Brown, Boveri & Co. (see page 390)

The slots and the arrangement of the conductors in the slots are as shown in Fig. 439, and are seen to be nearly closed. From this, and from the value of 1·07 for $\frac{\lambda_n}{\tau}$, we obtain from Fig. 431 the value of 12·5 for C in Behrend's formula for the "leakage factor" (page 387).

$$\sigma = C\frac{\Delta}{\tau} \therefore \sigma = 12{\cdot}5 \times \frac{{\cdot}075}{16{\cdot}8} = 0{\cdot}056$$

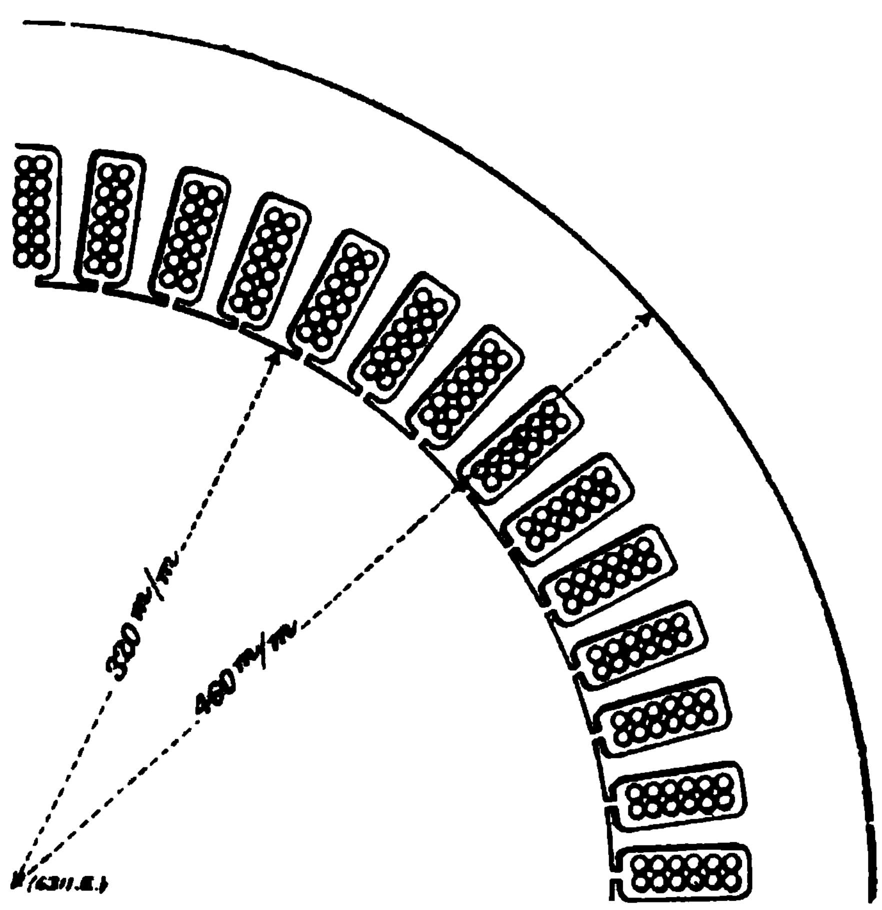


Fig. 439.—Arrangement of Conductors and Slots, Brown Boveri 25 H.P. Motor.

The designers at Brown, Boveri's works quote 0·055 as the value for σ for this design, as determined from the formula—

$$\sigma = \frac{\text{No load current}}{\text{(Theoretical) amperes at standstill} - \text{no load current}},$$

or, in the nomenclature we have adopted, $\sigma = \frac{B}{A-B}$.

The agreement in this instance happens to be far closer than would generally be the case.

It has been shown on page 387 that—

$$\text{Maximum power factor} = \frac{100}{1+2\sigma} = \frac{100}{1{\cdot}112} = 89{\cdot}3 \text{ per cent.}$$

But this, as was explained, involved certain assumptions, which combine to give a result sligntly lower than the true value.

For the subsequent calculation the maximum power factor will

be taken as cos. $\phi = 0{\cdot}90$, and this is also approximately the value at the rated load of 25 horse-power.

Note.—The continental horse-power is used, having 736 watts.

Watts output at full load $= 25 \times 736 =$	18,400
Preliminary assumed efficiency, per cent.	89·6
Watts input at full load	20,500
Volt-amperes input at full load (*i.e.* 3 × one phase) $\frac{20{,}500}{0{\cdot}90}$	22,700
Ditto per phase	7,570
Connection of stator windings	Delta
Volts per stator winding (*i.e.* per phase)	240
Full load amperes per stator winding	31·5
Energy component of full load amperes per stator winding ($31{\cdot}5 \times {\cdot}90$)	28·4
Full load line amperes ($31{\cdot}5 \times 1{\cdot}73$)	54·6
Diameter of bare conductor on stator, millimetres ...	4·4
Diameter of insulated conductor on stator, millimetres	5
Cross section of stator conductor, square centimetre ...	0·152
Current density at full load $= \frac{31{\cdot}5}{0{\cdot}152} =$ (amperes per square centimetre)	208
Depth of stator slot, millimetres	36
Width of stator slot, millimetres	14
Number of conductors per stator slot	12
Arrangement of conductors in stator slot	2 × 6
Number of stator slots	54
Total number of stator conductors (54×12)	648
Total number of stator turns $\left(\frac{648}{2}\right)$	324
Ditto per phase $\left(\frac{324}{3}\right)$	108
Number of stator turns per phase $\left(\frac{324}{3}\right)$	108
Number per pole per phase $\left(\frac{108}{6}\right)$	18
Approximate internal voltage per phase	230
Megalines flux per pole M ($E = 4{\cdot}2\ TNM \times 10^{-8}$) ...	1·02
Width stator slot opening, millimetres	3
Stator tooth pitch	18·6
Per cent. exposed iron at stator surface	84
Per cent. exposed iron at rotor surface	86
Mean for stator and rotor (k)	85
Exposed cross section iron at air gap, square centimetres ($\tau \times \lambda \times k$)	256
"Spreading coefficient"	1·20
Corrected air gap cross section	307
Gap density, average $\left(\frac{1{,}020{,}000}{307}\right)$	3,320
Maximum gap density ($1{\cdot}7 \times 3320$)	5,640

Ampere turns for gap ($0{\cdot}8 \times 0{\cdot}075 \times 5640$)...	338
Total ampere turns ($338 \times 1{\cdot}12$) ...	378
Ditto per phase $\left(\frac{378}{2}\right)$ ...	189
Maximum current at no load $\left(\frac{189}{18}\right)$ ...	10·5
R.M.S. current at no load $\left(\frac{10{\cdot}5}{\sqrt{2}}\right)$ ...	7·4
R.M.S. current in per cent. of full-load current $\left(\frac{7{\cdot}4 \times 100}{31{\cdot}5}\right)$	23·5
Total radial depth of stator punchings, centimetres ...	6·93
Total radial depth above slot, centimetres ...	3·27
Magnetic cross section of stator core (*i.e.* above slot), square centimetres ...	$2 \times 59{\cdot}0$
Stator core density ...	8,600
Width of tooth at narrowest point, millimetres ...	5·5
Cross section per pole at narrowest point, sq. cm. ...	89
Maximum density at any stator tooth at narrowest point $= \frac{1{,}020{,}000}{89} \times 1{\cdot}7$ ...	19,400
$\frac{DN}{100}$ for stator $\left(\frac{8{\cdot}6 \times 50}{100}\right)$ ...	4·3
Watts stator core loss per kilogramme (from fig. 384, page 331) ...	7·3
Net weight stator punchings, kilogrammes ...	85
Net weight rotor punchings, kilogrammes ...	50
Core loss in stator iron (estimated) ...	620

The stator core loss quoted by the manufacturers is but 310 watts. This is an exceptionally good result for this density and periodicity, and is considerably lower than the average. As indeterminate stray losses to a considerable amount occur at load in induction motors, it is preferable to estimate core losses on a very conservative basis.

Core loss in rotor iron ...	60
Total core loss ...	680
Embedded length of one stator turn, centimetres ...	36
Free length of one stator turn, " ...	79
Mean length of one stator turn, " ...	115
Total length of each stator winding (*i.e.* of winding of one phase), centimetres (115×108) ...	12,400
Resistance per phase at 60° Cent., ohm $\left(\frac{12{,}400 \times {\cdot}0000020}{0{\cdot}152}\right)$	0·163
Total stator I^2R loss, watts ($3 \times 31{\cdot}5^2 \times 0{\cdot}163$) ...	480
Total rotor I^2R loss, " ...	580
Total core loss (stator + rotor), watts ...	680
Allowance for friction loss, watts ...	400
Total of losses, watts ...	2,140
Output in watts ...	18,400
Input in watts ...	20,540
Full load efficiency, per cent. ...	89·6

Weight of stator copper, kilogrammes	51
Weight of rotor copper, "	30
Total weight of copper conductors, kilogrammes ...	81
Total weight laminations, kilogrammes	135
Cost copper at 2s. per kilogramme	162
Cost laminations at 0·6s. per kilogramme	81
Cost net effective material, shillings	243
Cost per horse-power for net effective material, shillings...	9·7
Weight complete motor, kilogrammes	550
Ditto per horse-power, kilogrammes	22·0
Ratio of weight of net effective material to total weight of motor	0·40

Letting A = the ideal short circuit current for this motor, and B = the magnetising current per winding, we have $B=7{\cdot}4$. The quoted value is 6·7. The estimated value (7·4 amperes) will be used.

And $\sigma\ (=0{\cdot}056)=\dfrac{B}{A-B}\ \therefore\ 0{\cdot}056=\dfrac{7{\cdot}4}{A-7{\cdot}4}$.

$$A=\frac{7{\cdot}4}{0{\cdot}056}+7{\cdot}4=132+7{\cdot}4=139=\text{ideal short circuit current.}$$

The motor's impedance per phase $=\dfrac{240}{139}=1{\cdot}73$ ohms.

The circle diameter $=A-B=132$.

From the ideal diagram (*i.e.* the diagram neglecting losses) of Fig. 440 A, drawn to a scale of 40 amperes per centimetre, we may obtain the power factors for various values of the primary current as the ratio of the ordinates at the intersections of the primary current values with the circumference of the circle to the primary current values. These are given in Fig. 440, *B*, page 397.

We know, futhermore, that these ordinates are proportional to the total input. The ordinate corresponding to full load current of 31·5 amperes per phase is found to measure 0·715 centimetres, and as this corresponds to 25 horse-power output, we have, at 89·5 per cent. efficiency, $\dfrac{25}{0{\cdot}895}\times 736=20{,}540$ watts input. The diagram is thus to the scale of $\dfrac{20{,}540}{0{\cdot}715}=28{,}800$ total watts per centimetre. (This must, of course, be the case, for the diagram is plotted to the scale of 40 amperes per centimetre, and as there are three phases and 240 volts per phase, there must be an input of $3\times 240\times 40=28{,}800$ total watts per centimetre.) This enables us to plot the total watts input in terms of the primary current. This is done in Fig. 440, *C*.

In Fig. 440 *D*, the secondary current is plotted in terms of the primary current. The "equivalent" secondary current is considered—*i.e.* the ratio of transformation is taken at 1 : 1.

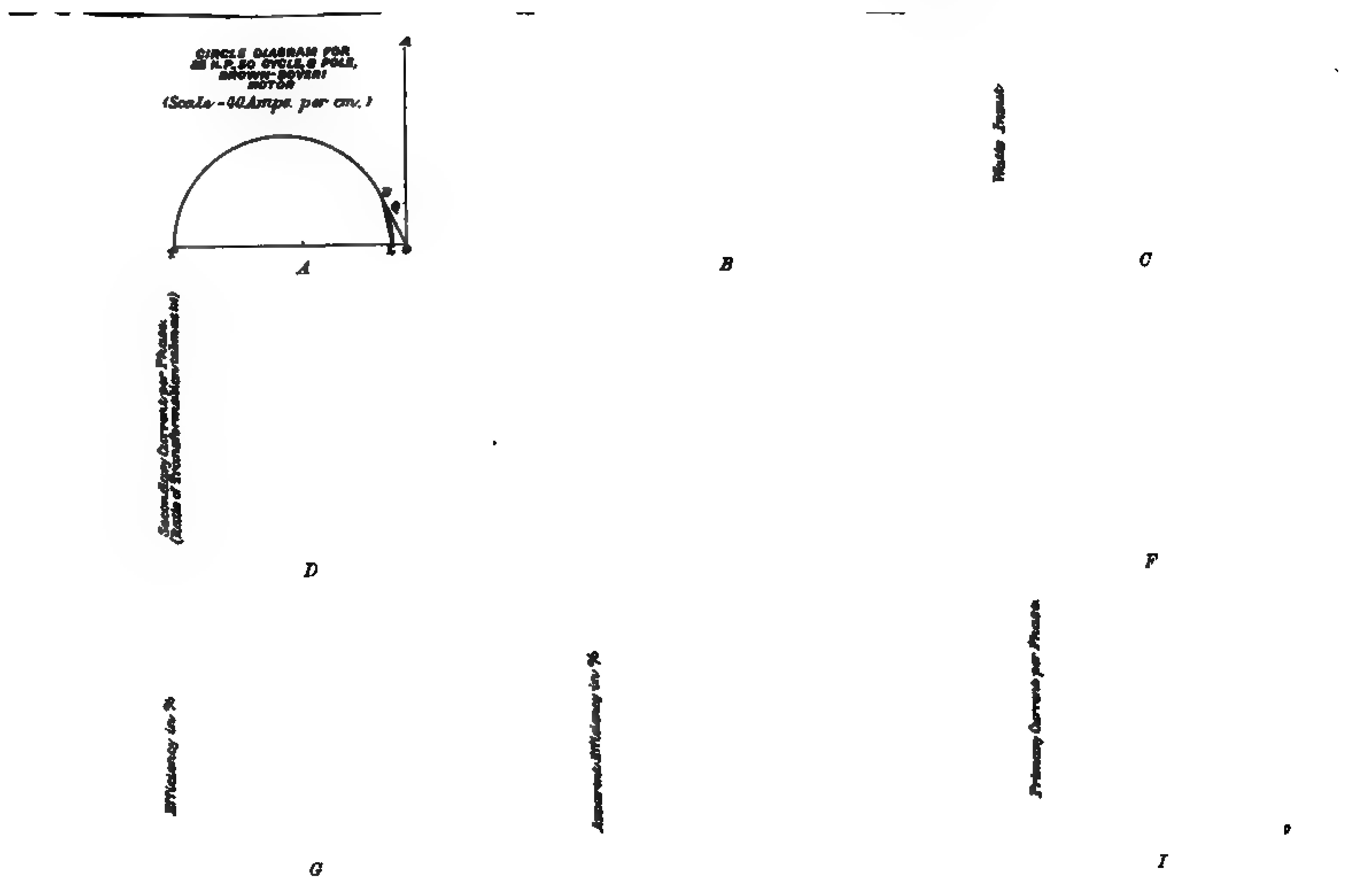


FIG. 440.—Curves of Brown Boveri 25 H.P., 50 Cycle, Three-Phase Motor.

The secondary currents are taken from the diagram of Fig. 440 *A*, *L* being assumed to be their origin.

From corresponding values of the primary and secondary current, and of the total core loss (which may be assumed constant at 680 watts), the component and total losses have been plotted in curve *E*; and from these and the input values of curve *C*, the output curve *C* is derived. The ratio of the input and output curves gives us the efficiency curve of Fig. 440, *F*.

In Fig. 440, *H*, the curve of "apparent" efficiency (product of "true" efficiency and power factor, cos. ϕ) has been plotted from the values in curves *G* and *B*, Fig. 440.

In curve D, Fig. 440, the primary current is plotted as a function of the output in horse-power.

§ 2. **Brown, Boveri & Co.'s 8 H.P. and 75 H.P. Motors.**—Two other Brown Boveri designs, relating respectively to an 8-pole, 75 horse-power motor, running at a synchronous speed of 750 revolutions per minute at a periodicity of 50 cycles per second, and to a 4-pole, 8 horse-power motor for 1,500 revolutions per minute and 50 cycles per second, will next be considered, the calculations being arranged in parallel columns. The former has a wound rotor, and the latter a squirrel cage rotor.

SPECIFICATION FOR BROWN BOVERI THREE-PHASE INDUCTION MOTORS.

	Rated Horse-power, 75.	Rated Horse-power, 8.
Number of poles	8	4
Periodicity in cycles per second	50	50
Synchronous speed in revolutions per minute ...	750	1,500
Voltage	500	250
External diameter of stator punchings, cms. ...	70	33
Internal diameter of stator punchings, „ ...	50·2	21·1
External diameter of rotor punchings, „ ...	50·0	21·0
Radial depth of air gap, Δ centimetres	·10	·05
Internal diameter of rotor punchings, cms. ...	35	14
Circumference at air gap, centimetres	157·5	66·4
Polar pitch at air gap (τ), centimetres	19·7	16·6
Gross length between flanges, λ_g	28	12
(There are no ventilating ducts)		
Net length laminations between flanges, λ_n ...	25·2	10·8
$\frac{\lambda_n}{\tau}=$	1·28	·65
Type of slots employed	Nearly closed	
C in Behrend's formula (from fig. 431, page 387)	12	14
$\sigma=C\frac{\Delta}{\tau}$	·061	·042
Experimentally observed values of σ	·06	·045

	Rated Horse-power, 75.	Rated Horse-power, 8.
Maximum power factor	·891	·918
Watts output at full load	55,100	5,890
Full load efficiency	93·8	87·2
Watts input at full load	59,500	6,750
Total volt-amperes input at full load	66,500	7,350
Volt-amperes input at full load per phase ...	22,166	2,450
Connection of stator windings	Δ	Y
Volts between terminals	500	250
Volts per phase	500	144
Full load amperes per stator winding	45·3	17
Full load line amperes	78·2	17
Diameter of bare conductor on stator	·29	·34
Diameter of insulated conductor on stator ...	·35	·40
Cross section of one stator conductor	·066	·091
Number of conductors in parallel	4	1
Cross section of all parallel conductors	·264	·091
Current density at full load, amperes per square centimetre	172	187
Depth of stator slot	4·5	3·0
Width of stator slot	1·6	1·0
Number of conductors per slot	42	12
Arrangement of conductors in stator slot ...	4 × 11	2 × 6
Number of stator slots	72	48
Total number of stator conductors (parallel conductors taken as one)	756	576
Total number of stator turns	378	288
Number of stator turns per phase	126	96
Number of stator turns per pole per phase ...	15·75	24
Approximate internal voltage per phase (E) ...	490	138
Megalines flux per pole M ($E = 4{\cdot}2\ TNM \times 10^{-8}$)	1·85	·686
Width of stator slot opening	0·3	0·2
Stator tooth pitch	2·18	1·38
Per cent. exposed iron at stator surface	85	85
Per cent. exposed iron at rotor surface	85	100
Mean for stator and rotor, per cent.	85	93
Exposed cross section iron at air gap	420	164
Spreading coefficient	1·2	1·2
Corrected air gap cross section	505	197
Gap density, average	3,640	3,480
Maximum gap density	6,200	5,900
Ampere turns for gap	495	235
Total ampere turns	565	270
Ampere turns per pole per phase	282	135
Maximum current at no load	17·9	5·62
R.M.S. current at no load	12·6	3·96
R.M.S. current in per cent. of full load current	30·1	23·2
Total radial depth of stator punching	9·9	5·95
Total radial depth of stator punching above slot	5·35	2·95

	Rated Horse-power, 75.	Rated Horse-power, 8.
Magnetic cross section of stator core (*i.e.* above slot)	2 × 136	2 × 32
Stator core density	6,800	10,700
Width of tooth at narrowest point	·66	·45
Cross section per pole at narrowest point ...	150	58·5
Maximum density at any stator tooth at narrowest point	20,900	19,900
$\frac{DN}{100}$ for stator	3·4	5·35
Watts stator core loss per kilogramme	5·4	9
Nett weight stator punchings	265	29
Nett weight rotor punchings	1,430	260
Core loss in stator iron (from fig. 384, page 331)	1,430	260
Core loss in rotor iron	190	30
Total core loss	1,620	290
Embedded length of one stator turn	50·4	21·6
Free length of one stator turn	89·6	70·4
Mean length of one stator turn	140	92
Total length of each stator winding (*i.e.* winding of one phase)	17,700	8,800
Resistance per phase at 60° Cent. (ohms) ...	0·134	0·194
Total stator I^2R loss	828	170
Total rotor I^2R loss	1,000	250
Total core loss (stator + rotor)	1,620	290
Allowance for friction loss	650	180
Total of losses (watts)	4,098	890
Output in watts	55,200	5,900
Input in watts	59,298	6,790
Full load efficiency, per cent.	93·1	87·0
Weight of stator copper, kilogrammes	126	21·6
Weight of rotor copper, „	62	6·0
Total weight of copper conductors, kilogrammes	188	27·6
Total weight laminations, kilogrammes	410	48
Cost copper at 2s. per kilogramme, shillings	376	55·2
Cost laminations at 0·6s. per kilogramme, „	246	28·8
Cost of net effective material, „	622	84
Cost per horse-power for net effective material, shillings	8·3	10·5
Weight of complete motor, kilogrammes ...	1,450	175
Weight per horse-power, „	19·4	21·9
Magnetising current, B	12·6	3·96
$\sigma =$	·061	·042
Ideal short circuit current $= \frac{B}{5} + B$	220	98
Diameter of the circle, $A - B$	207	94
Ratio of weight of net effective material to total weight of motor	0·62	0·43

The circle diagram and the curves of the performance of these motors as determined from the above data are given in Fig. 441,

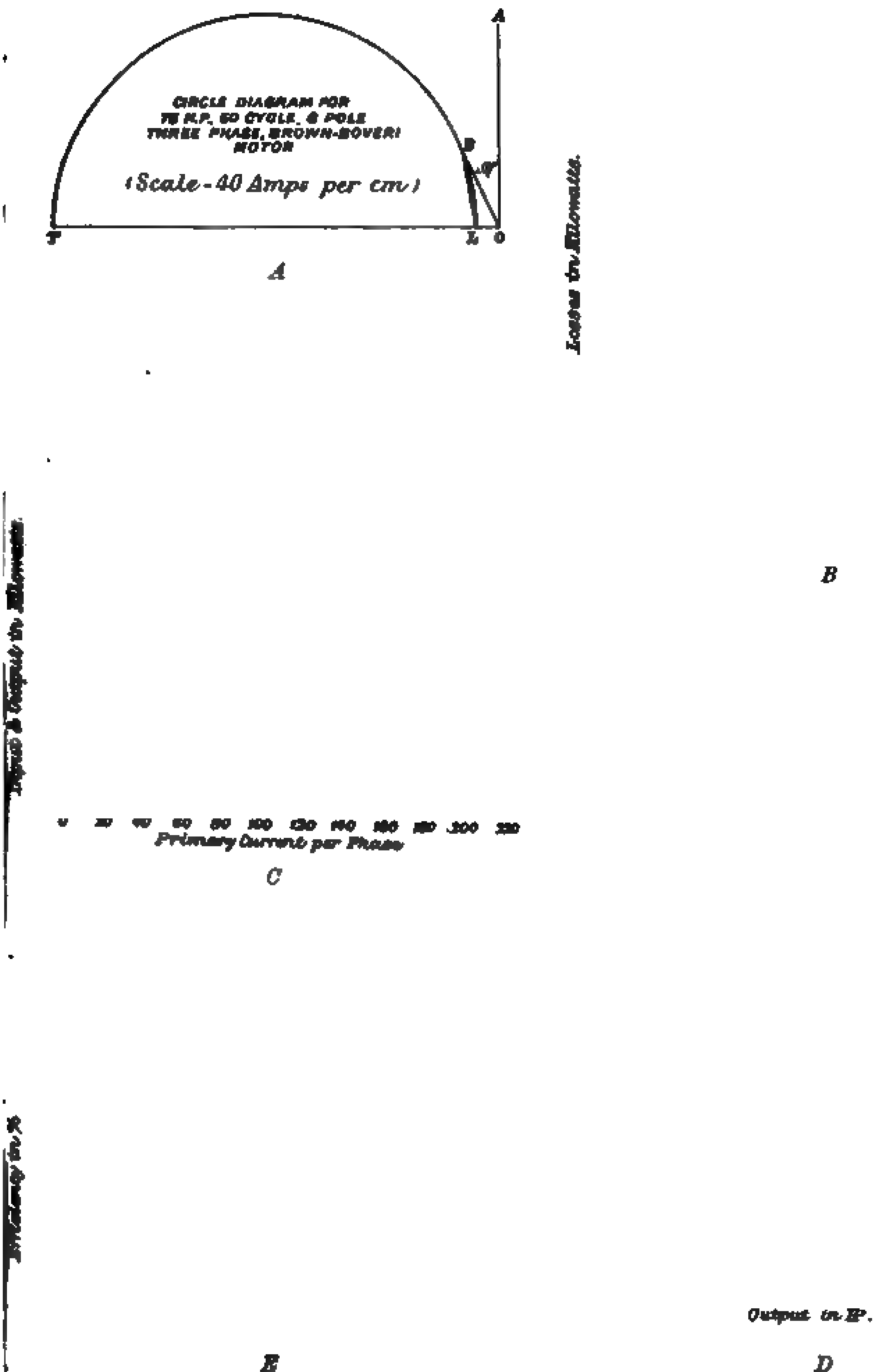


Fig. 441.—Curves of Brown Boveri 75 H.P., 50 Cycle Three-Phase Motor.

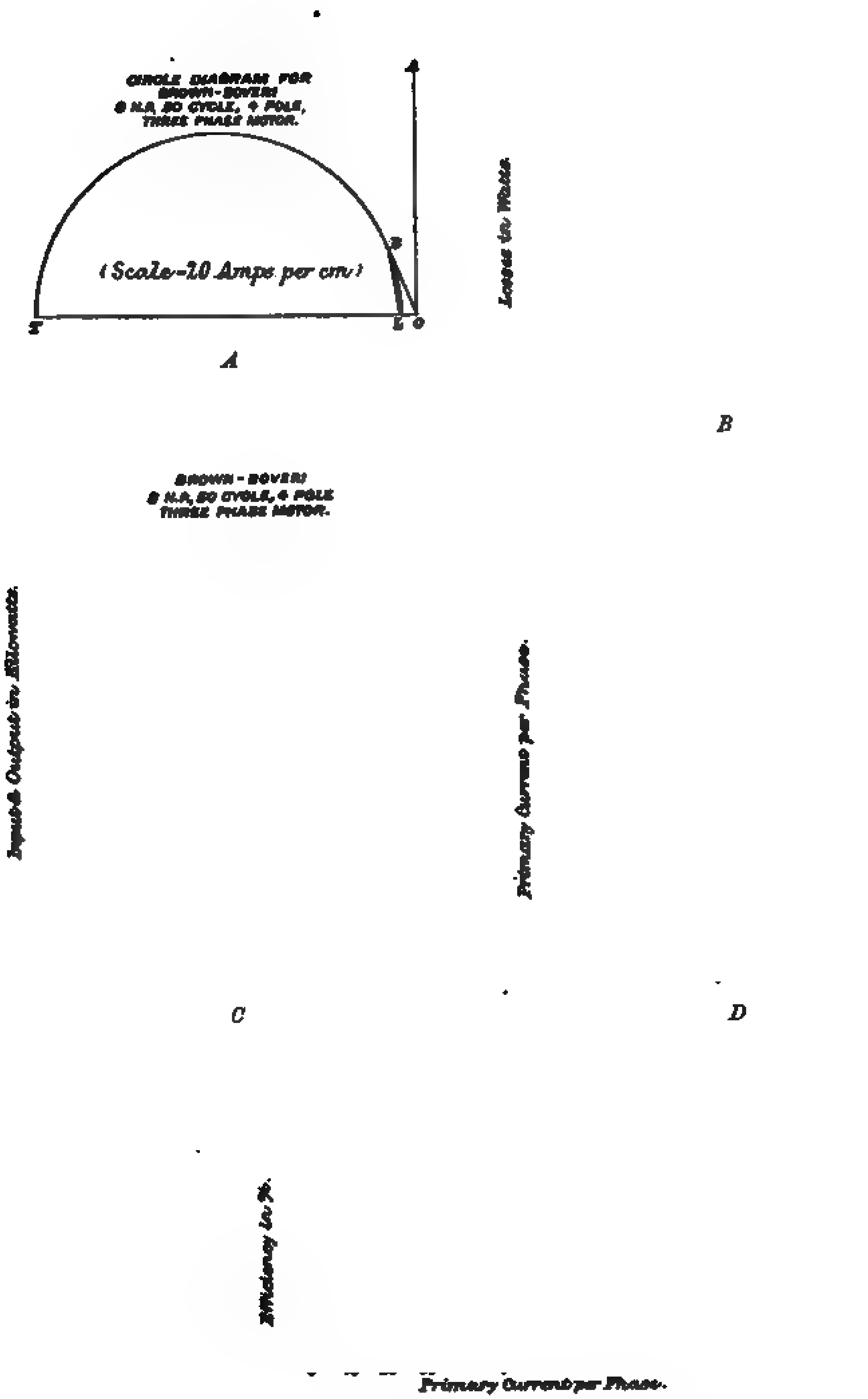


FIG. 442.—Curves of Brown Boveri 8 H.P., 50 Cycle Three-Phase Motor.

page 401, for the 75 horse-power motor, and in Fig. 442, page 402, for the 8 horse-power motor.

In Figs. 443 to 445 are given curves plotted from test results of Brown Boveri motors similar to, but not quite identical with, those above analysed.

The 8 horse-power, 4-pole Brown Boveri design, which we have described in detail, had a squirrel cage winding, and considerable

Fig. 443.—Test Results 25 H.P. Motor.

Fig. 444.—Test Results 60 H.P. Motor.

"slip" at full load. The "slip" at full load, as percentage of the synchronous speed, may be obtained, as we have seen in a previous article, from the ratio of the rotor I^2R to the total input to the rotor.

Total input to stator at full load	6790	watts
Stator I^2R loss + stator core loss = 170 + 260	430	"
Input to rotor	6360	"
Secondary I^2R loss at full load	250	"
Per cent. "slip" at full load $= \dfrac{250 \times 100}{6360}$	3·9	

The motor thus runs at full load at a speed of $\cdot96 \times 1500 = 1440$ revolutions per minute, as shown in the speed curve of Fig. 445.

In this instance we have taken for the rotor I^2R loss the value of 250 watts given by the manufacturers.

§ 3. **Copper Losses in Squirrel Cage Motors.**—This is an opportune occasion for investigating the question of the copper losses in squirrel cage motors.

Fig. 446 gives a diagrammatical representation of a squirrel cage winding for 2 poles and 36 face conductors. These are represented by 36 radial lines. The inner and outer circles represent the end connections. The magnetic flux set up by the stator

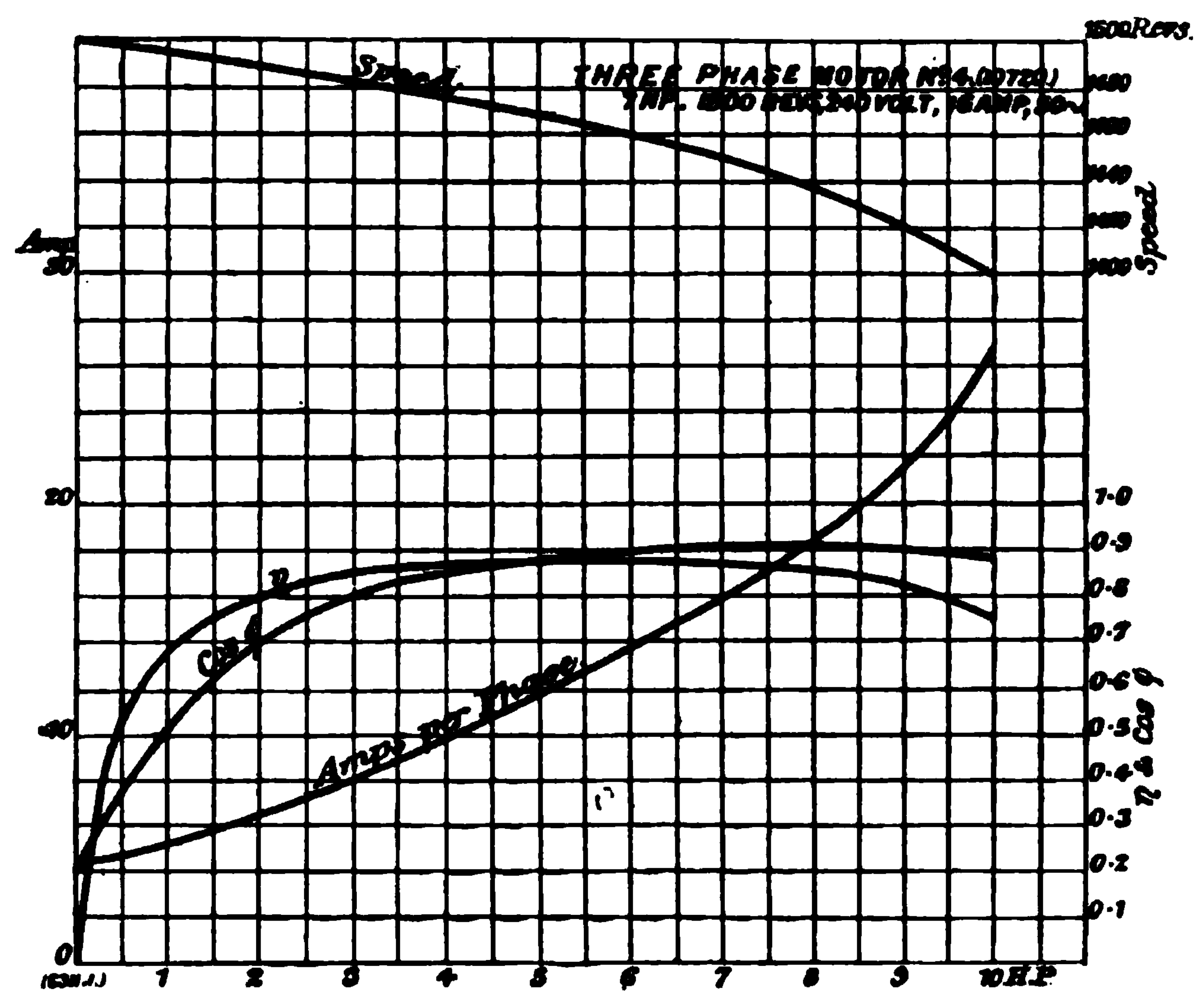


FIG. 445.—Test Results 7 H.P. Motor.

ampere-turns, as we have seen in a previous section, varies in its shape between two limits, curves A and B, Fig. 380, page 326, but it is amply exact to take the equivalent sine curve C (same fig.) as the basis of the calculation. The electro-motive forces occasioned by the relative movement of the conductors in the field vary in their magnitude according to their angular position, and, of course, are also to be taken as sinusoidally distributed. It can be shown by a more exact treatment of this subject that the currents produced by these electro-motive forces are also sinusoidally distributed (see G. Roessler, *Elek. Tech. Zeit.*, 1898, Nos. 45 and 46).

In Fig. 446 every conductor has the value of the current written against it, on the assumption that the maximum current

in any of the conductors is equal to 1·00. We see that the current in conductor A is zero, whilst conductor B carries the maximum current 1·00; the current in conductor C, opposite conductor A, is zero again.

The losses in one conductor can now be found very easily, because every conductor will carry successively those values written against them in Fig. 446, and as these are ordinates of a sine curve, the loss in each conductor will be the root-mean-square of the current, multiplied by the resistance R of a conductor. If Z denotes the number of conductors, and I_s the secondary R.M S. current, then the losses in the conductors themselves are Z I_s^2R.

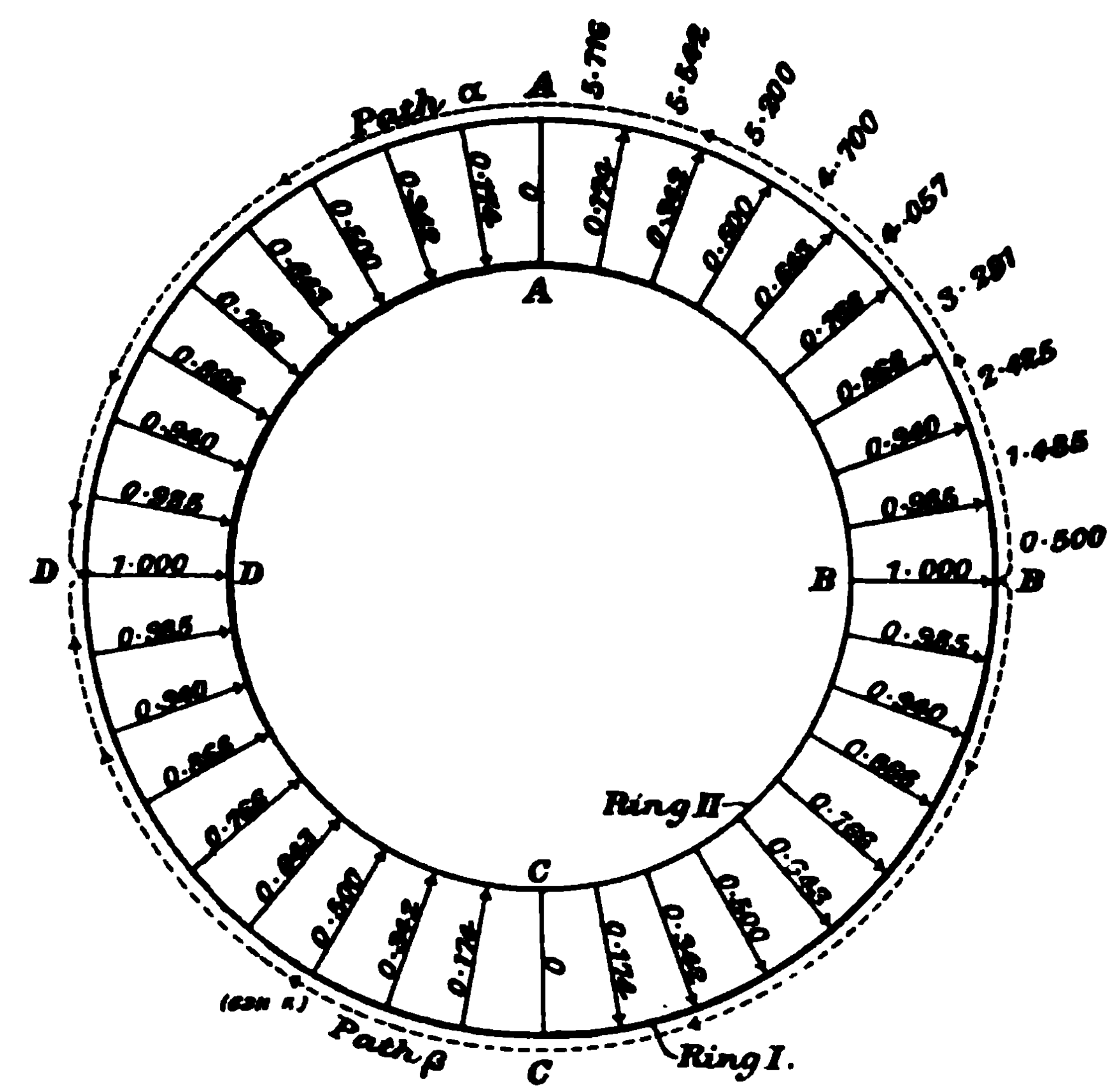


FIG. 446.—Diagram of Squirrel Cage Winding, 2 Poles, 36 Conductors.

The current flowing in the end rings depends upon the number of face conductors per pole and upon the secondary current I_s. As the conductors of Fig. 446 are symmetrically distributed, it follows that the current from the conductors lying between A and B will flow into the ring I, and, following path a, will return into the conductors lying between A and D, whilst the conductors lying between B and C send their current along path β into the conductors lying between C and D.

The current in the end rings will therefore increase in value from point B up to A, and also from B up to C; the conductor B itself sends half its current in the one direction and half in the other direction. The adjacent conductors on either side increase

the current to 1·485, and so on, as shown in Fig. 446, where the values of the current in the outer end ring (ring I), have been stated from B to A.

The maximum current is found to be 5·72 at the points A and C respectively, and from there the current decreases again. The distribution of the current in the ring II is exactly the same as in ring I, the direction being reversed.

It will be seen that the values of the current in successive parts of the rings follow a sine curve, and, as the conductors revolve, each part of the end ring carries a current varying in its amplitude from $+5{\cdot}72$ to $-5{\cdot}72$ and sinusoidally.

It is not very difficult to ascertain how the value 5·72 has been obtained. All the conductors between A and B send their current through path a, therefore the maximum current in the end rings in path a must be equal to the sum of the currents in those conductors, but this is equal to the number of conductors between A and B multiplied by the average current in those conductors.

As we have altogether 36 conductors, we have 9 conductors between A and B, and as the average of the ordinates of a sine wave $= \frac{2}{\pi} \times$ maximum ordinate, we have $9 \times \frac{2}{\pi} \times 1 = 5{\cdot}72$, coinciding with the value in Fig. 446. Max. ordinate $=$ R.M.S. $\times \sqrt{2}$.

Let $Z_1 =$ number of conductors per pole, and $I_s =$ R.M.S. current in one conductor; then the maximum current in the end rings $= \frac{2}{\pi} \frac{Z_1}{2} I_s \sqrt{2}$. Therefore R.M.S. currrent in the end rings $= \frac{2}{\pi} \frac{Z_1}{2} I_s$.

The resistance of that part of one ring connecting two adjacent conductors may be taken as R_1; then the loss of energy in it is $\left(\frac{2}{\pi} \frac{Z_1}{2} I_s\right)^2 R_1$, and the total loss in two rings $= 2\,Z \left(\frac{2}{\pi} \frac{Z_1}{2} I_s\right)^2 R_1 = Z\,I_s^2 \cdot \frac{2\,Z_1^2}{\pi^2} R_1 = Z\,I_s^2 \times 0{\cdot}2\,Z_1^2\,R_1$.

The total copper loss in the rotor is, therefore, $Z\,I_s^2\,(R + 0{\cdot}2\,Z_1^2\,R_1)$.

The same result has also been obtained by Fischer-Hinnen (*Z.f. E.*, Wien, 1900, No. 33), though by a considerably more complicated method than that given here. Osnos (*E. T. Z.*, 1901, No. 8), who compares the results obtained by Roessler with those of Fischer-Hinnen, shows that they disagree only by a fraction of 1 per cent.

We may use this formula for estimating the I^2R loss in the squirrel cage rotor of the 8 horse-power, 50-cycle 4-pole motor.

The rotor has 36 bare conductors of 8·5 millimetres diameter, lying in slots of 11 millimetres diameter. These dimensions are chosen so as to agree with the I^2R loss given by the manufacturers, as the actual dimensions of the squirrel cage conducting system were not given. The length of a conductor is 16 centimetres, and, therefore, its resistance $R = \frac{\cdot 000002 \times 16}{\cdot 57} = 0{\cdot}0000562$.

The secondary current, I_s, can be taken from Fig. 442, *A*, as 16·5 amperes, for a 1 : 1 ratio of transformation.

As there are 576 conductors in the stator, and 36 conductors in the rotor, the ratio of transformation is $\frac{576}{36} = 16$, and therefore the R.M.S. current in each conductor is $16{\cdot}5 \times 16 = 264$.

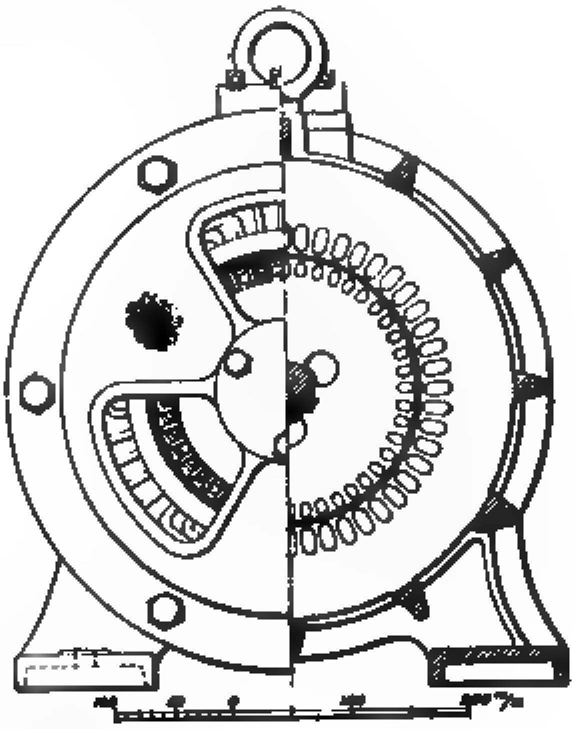
The end rings have a diameter of 19·5 centimetres and a cross section of 1·30 square centimetres. The resistance R_1, of one end ring between two adjacent conductors, is therefore $\frac{\cdot 000002 \times 19{\cdot}5\pi}{1{\cdot}30 \times 36}$ $= \cdot 00000262$.

Here we have 9 ($= Z_1$) conductors per pole, the total I^2R loss in the rotor, according to our formula, is $36 \times 264^2 (0{\cdot}0000562 + 0{\cdot}2 \times 9^2 \times 0{\cdot}00000262 = 142 + 108 = 250$ watts), this being distributed in the proportion of 57 per cent. in the peripheral conductors and 43 per cent. in the end rings. It is instructive to note the high percentage loss occurring in the end rings, in spite of their relatively large cross section.

§ 4. **An Analysis of Heating Tests on an Induction Motor.**—Up to this point the performance of the motor has only been considered with reference to its ability to give the required output, and to its efficiency and general performance at various loads. It is necessary in designing a motor to give careful attention to the question of heating. Very elaborate tests regarding heating have been published by Emil Ziehl in *E. T. Z.*, 1902, Heft 12, and an analysis of these tests in a condensed form will now be given.

The motor, manufactured by the Berliner Maschinenbau A.-G. vormals L. Schwartzkopf, had a rated output of 10 H.P. at 1500 R.P.M., 190 volts and 50 periods. The diameter of the rotor was 22·4 cm., its length 11 cm. Three horizontal ducts were provided in the rotor, but no vertical interlaminar ducts. The outer cylindrical surface of stator laminations was exposed directly to the air, so as to increase the radiation. In the semi-enclosed form the end shields had three large ventilating apertures, whilst in the

totally-enclosed form these apertures were omitted. The general construction of the motor is shown in Figs. 447 and 448.



Figs. 447 and 448.—Sectional Elevations of 10 H.P. Motor by Berlinger M. A.-G.

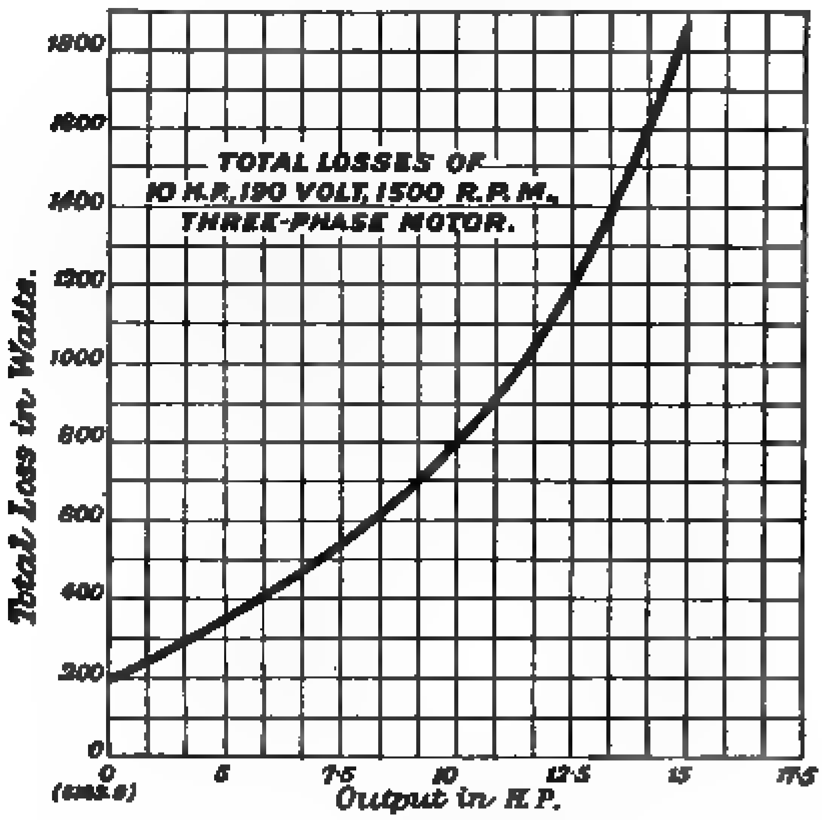


Fig. 449.—Total Losses of 10 H.P. Motor.

The losses have been estimated from the experimental data given, and have been plotted in Fig. 449 with the output as abscissae.

The motor was loaded successively with 5, 7·5, 10, 12·5 and 15 H.P., and the curves in Figs. 450 to 452 show the temperature increase for the semi-enclosed and for the totally-enclosed motor.

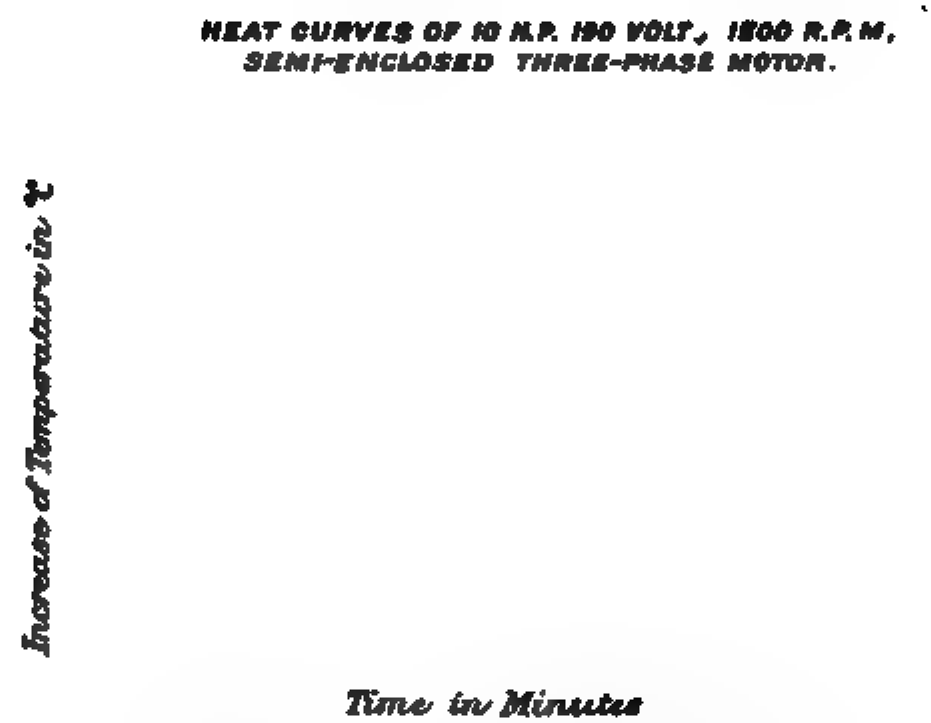


Fig. 450.—Heating Tests on Semi-Enclosed Motor.

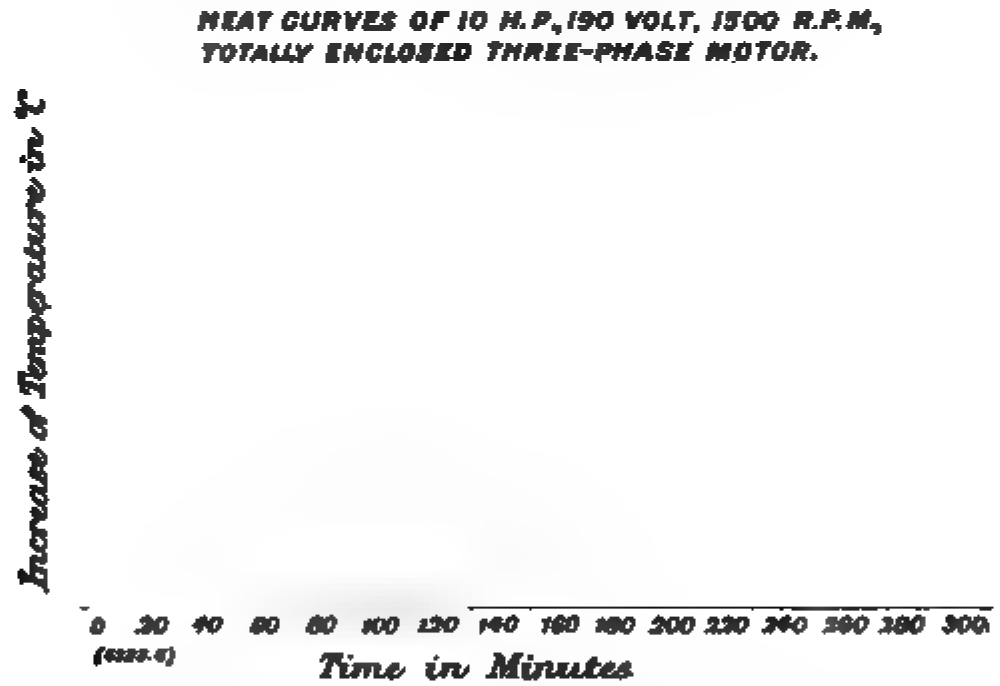


Fig. 451.—Heating Tests on Totally-Enclosed Motor.

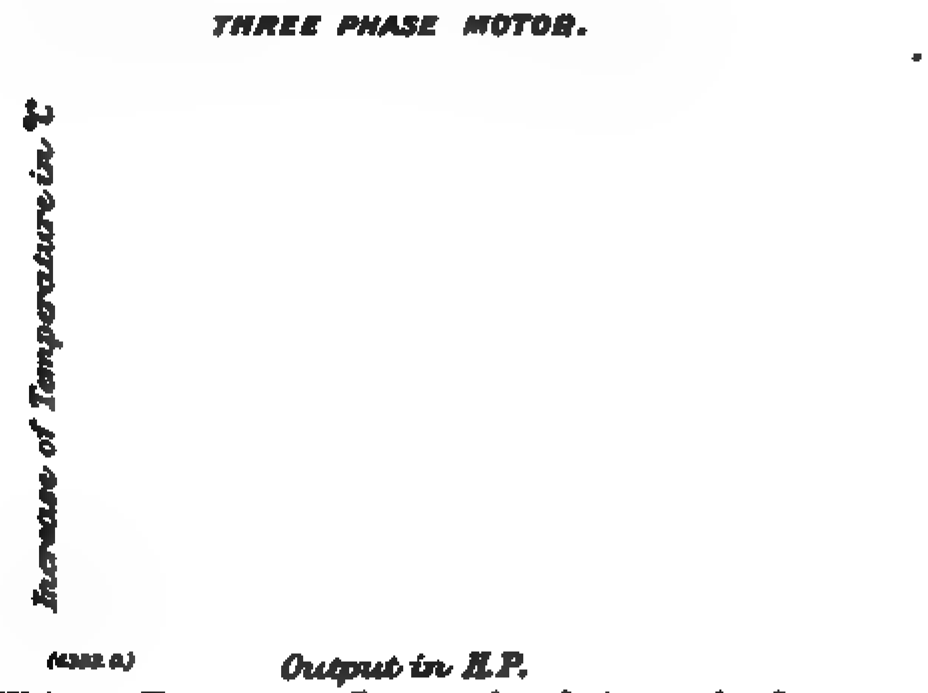


Fig. 452.—Ultimate Temperature Increase in relation to the Output.

After the motor had been heated up, the rate of cooling was observed first at standstill, then with the motor running at no load. The respective curves are given in Figs. 453 and 454.

An analysis of these tests leads to exceedingly interesting

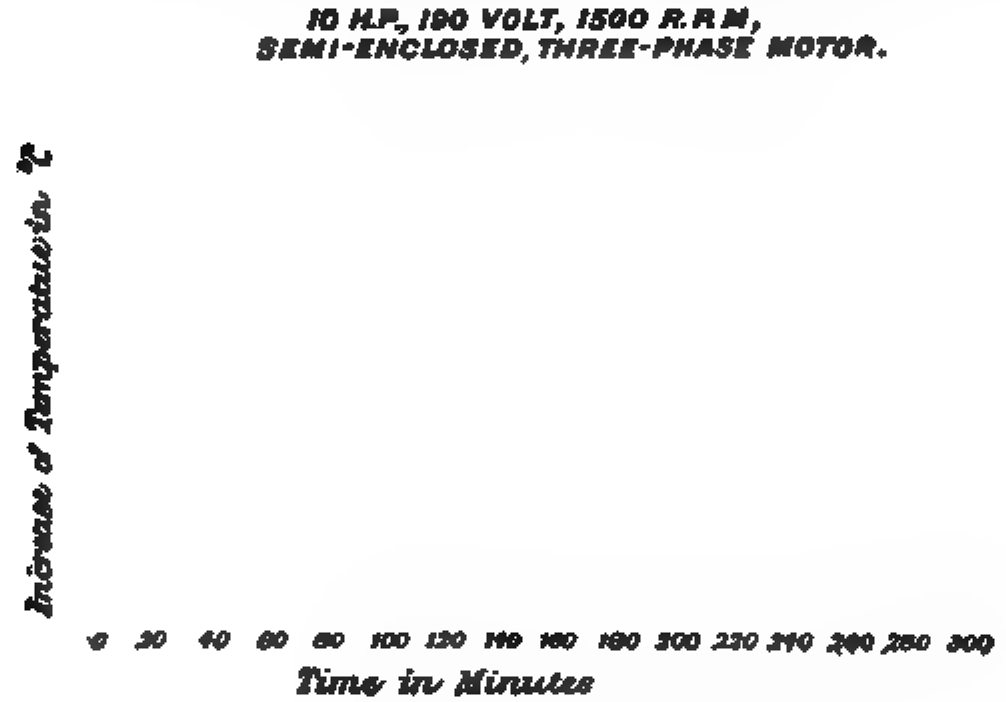


FIG. 453.—Cooling Tests on Semi-Enclosed Motor.

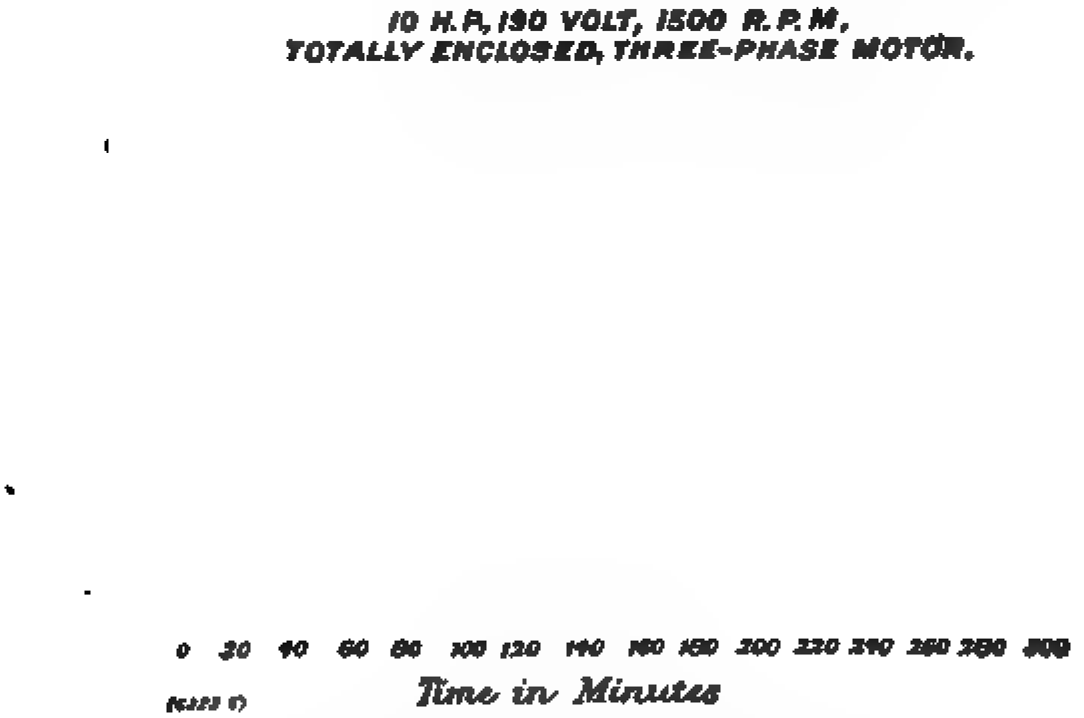


FIG. 454.—Cooling Tests on Totally-Enclosed Motor.

results, and affords a good occasion for considering the theory of heating and cooling as applied to induction motors.

Two factors chiefly determine the increase of temperature within a certain time for a given loss—first, the units of energy radiated per unit of time and per unit of increase in temperature, and secondly, the energy which the machine stores up, for every degree Centigrade increase in temperature.

Let S be the watt seconds radiated in one second by the machine for 1° C. difference in temperature, then the energy radiated in the time T for a difference $t°$ of temperature is obviously

$$S\, t°\, T \text{ watt seconds.}$$

The value of S may now be estimated from the final temperature obtained for a certain load.

§ 5. **Heat Radiation in Semi-Enclosed and Totally-Enclosed Motors Compared.**—Fig. 450, page 409, shows that at a load of 5 H.P. the temperature rise is 11·5° C.—*i.e.* the ultimate temperature exceeds the temperature of the surrounding air by 11·5°. The losses pertaining to a load of 5 H.P. are found from Fig. 449 to be 350 watts.

The final temperature is reached when the rate of radiation of energy is equal to the rate of generation of heat in the motor in virtue of the losses.

The energy radiated per second is $S \times 11·5$ watt seconds.

The losses per second are 350 watt seconds.

Therefore $350 = S \times 11·5$ and $S = \dfrac{350}{11·5} = 30·3$.

Similar experiments were made for loads of 7·5 H.P., 10 H.P., 12·5 H.P., and 15 H.P., and the corresponding results for the semi-enclosed motor are stated below.

The temperature rise and the losses were—

For	5	H.P.	11·5° C.	and	350	watt	seconds	per second.
„	7·5	„	16·5	„	540	„		„
„	10	„	20·5	„	800	„		„
„	12·5	„	32·5	„	1200	„		„
„	15	„	52	„	1860	„		„

The corresponding values for S together with similarly deduced values for the totally-enclosed motor are stated below.

TABLE LX.—COMPARISON OF HEAT RADIATED BY SEMI ENCLOSED AND TOTALLY-ENCLOSED MOTORS.

WATT SECONDS RADIATED PER SECOND PER DEGREE CENTIGRADE.		
Load H.P.	**Semi-Enclosed Motor. S**	**Totally-Enclosed Motor. S**
5	30	16
7·5	33	17
10	39	14
12·5	37	14
15	36	13
Average	35	14·8

It will be noted that the totally-enclosed motor radiates on the average less than half as much heat as the semi-enclosed motor, under the same conditions.

Besides the radiation per degree Centigrade per second, we must also consider the ability of the motor to store up energy in the form of heat.

Let Q be the energy in watt seconds which, if converted into heat in the motor, will increase its temperature by 1° C. It is evident that Q must be very nearly the same in the semi-enclosed as in the totally-enclosed motor, since the amount and distribution of material is nearly identical in the two cases. By calculating Q from the two sets of data we have an excellent opportunity for checking the reliability of the tests. During the first hour the semi-enclosed motor loaded with 5 H.P. increased in temperature by 7° C. The losses during that time were 3·5 watt hours. The radiation during the same time was comparatively small, as the average difference of temperature was only 3·5° C., the radiation therefore having been 122 watt hours $= S$ (semi-enclosed) $\times 3{\cdot}5$ watt hours $= 35 \times 3{\cdot}5 = 122$ watt hours.

The energy stored up in the motor in virtue of its heat capacity was therefore $350 - 122 = 228$ watt hours.

Hence the motor requires 228 watt hours $= 820{,}800$ watt seconds to increase its temperature by 7° C., or 117,000 watt seconds to increase it by 1° C. These and the corresponding values for other loads are arranged in tabular form, for the semi-enclosed motor, in Table LXI., and for the totally-enclosed motor in Table LXII.

TABLE LXI.—HEATING TESTS ON SEMI-ENCLOSED MOTOR ($S=35$).

Load in H.-P.	Rise in Temperature in ° C. at end of first hour.	Losses in Motor during 1 hour. Watt hours.	Radiation in Watt hours. (Taken as proportional to half the temperature increase.)	Energy in Watt sec. stored up in Motor at end of one hour.	Energy in Watt sec. stored up per degree Rise $= Q$.
5	7	350	122	822,000	117,000
7·5	10	540	175	1,310,000	131,000
10	14	800	245	2,000,000	142,000
12·5	20	1200	350	3,060,000	153,000
15	29·5	1860	515	4,850,000	164,000
17·5	44	2690	770	6,900,000	157,000

As may be seen from Table LXII., Q is very constant, while Table LXI. shows a gradual increase of Q with the load. A slight

discrepancy was to be expected, as the errors caused by the radiation from the interior must of course be large in the semi-enclosed type, but small in the totally-enclosed type, and as there may occur in an actual test variations in those factors affecting the radiation. But by far the greatest part of the discrepancy is due to the unequal distribution of the internal heat, which will be subsequently considered.

TABLE LXII.—HEATING TESTS TOTALLY-ENCLOSED MOTOR ($S=14{\cdot}5$).

Load in H.-P.	Rise in Temperature in °C. at end of first hour.	Losses in Motor during 1 hour. Watt hours.	Radiation in Watt hours. (Taken as proportional to half the temperature increase.)	Energy in Watt sec. stored up in Motor at end of one hour.	Energy in Watt sec. stored up per degree Rise $=Q$.
5	10	350	72	1,[illegible]00	100,000
7·5	[illegible]	540	101	1,[illegible]00	112,000
10	20	800	[illegible]	2,[illegible]00	118,000
11	23·5	940	170	2,[illegible]00	118,000
12	29	1100	210	3,[illegible]00	110,000
15	52·5	1860	280	5,[illegible]00	101,000

Let us take for the present, as an average value, $Q=120{,}000$ watt seconds.

The whole weight of the motor is 200 kilogrammes, therefore the heat capacity of the motor per kilogramme is $Q=600$ watt seconds.

This would correspond to a specific heat of the motor of 0·144.

As iron or steel constitutes by far the greatest component in the total weight, the specific heat should be somewhat less than 0·11 ; *i.e.* 30 per cent. lower than the experiments show.

There is a very natural explanation of this deviation, namely, that the temperatures observed upon the external surface were considerably lower than the inner temperatures. Indeed, this may also explain the fact that at the higher loads of the semi-enclosed motor, Q was so much higher; for the difference in temperature between inside and outside must be greater

(1) the better the ventilation, and
(2) the higher the losses.

§ 6. The Temperature at the Exterior Compare the Mean Temperature of the Whole Motor.—If w this explanation and attribute to it the difference in the ca values of Q, we obtain the following table, which is of

not because of the absolute values it contains, but because of its comparative values, especially between the semi-enclosed and the totally-enclosed types.

TABLE LXIII.—DIFFERENCE BETWEEN MEAN TEMPERATURE OF THE WHOLE MOTOR AND THE TEMPERATURE MEASURED AT THE EXTERIOR SURFACE OF THE STATOR PUNCHINGS AFTER ONE HOUR'S RUN.

Load in H.-P.	Semi-Enclosed Motor.	Totally-Enclosed Motor.
5	28%	20%
7·5	43%	"
10	55%	"
12·5	68%	"
15	80%	"
17 5	74%	"

§ 7. **The Rate of Cooling.**—Very interesting results may be obtained by the curves in figs. 453 and 454, showing respectively the law of cooling at standstill and when running at no load.

Let us first compare the cooling of the semi-enclosed motor with that of the totally-enclosed motor.

The semi-enclosed motor at standstill cooled down 23° C. in one hour, from 68° C. to 45° C. We may find two points of the corresponding curve for the totally-enclosed motor having the same mean value, and with abscissae differing by sixty minutes. These are 65° C. and 47·5° C., with a difference of 17·5° C.

As the mean temperature was in both cases the same, these different values of cooling (23 and 17·5) can only be explained by attributing to the semi-enclosed motor a higher value for S at standstill: therefore the ratio

$$\frac{S \text{ for semi-enclosed motor at standstill}}{S \text{ for totally-enclosed motor at standstill}} = \frac{23}{17{\cdot}6} = 1{\cdot}3.$$

Whereas we found that when running, S for the semi-enclosed motor is more than double the value of S for the totally-enclosed motor $\left(\frac{35}{14{\cdot}8} = 2{\cdot}4\right)$, this ratio has at standstill decreased to 1·3.

This ratio was the result of the first hour's cooling, viz., 1·3.

For the second hour's cooling this ratio was 1·45.

For the third hour's cooling it was 1·22.

The average value of the above being 1·32.

But we may also readily find the actual value for S, by taking Q in both cases equal to 110,000.

As the difference between the temperature in the interior and

that outside is now smaller, a smaller value of Q than that obtained from the load curves must be taken.

Taking the 23° C. decrease in temperature during the first hour (68° to 45°), the entire energy radiated during that time must have been:

$$23\ Q = \frac{68+45}{2} \times S \times 3600$$

$$S = \frac{23 \times 110{,}000}{56{\cdot}5 \times 3600} = 12{\cdot}5.$$

From the value of S at other parts of the same curve we obtain 13·2 and 11·2.

The average value of S may therefore be taken as 12·3 for the semi-enclosed motor, and $\frac{12{\cdot}3}{1{\cdot}32} = 9{\cdot}3$ for the totally-enclosed motor.

Therefore at standstill the totally-enclosed motor radiates 36 per cent. less heat under given conditions than when running. This is a very interesting result, as one might have thought that the totally-enclosed motor would radiate heat at the same rate, whether running or at a standstill, with the same difference of temperature.

We will tabulate our results for S in Table LXIV., below.

S is, of course, proportional to the total radiating surface, but to precisely define the total radiating surface is a most difficult matter.

It therefore seems preferable to take as radiating surface a value which is very nearly proportional to the true radiating surface, but which can be calculated in a very simple way.

Let λ = the length of rotor core between end flanges.

τ = the polar pitch at the air gap $= \frac{\text{circumference}}{\text{number of poles}}$.

d = diameter of rotor.

L = length over end connections.

$= \lambda + {\cdot}7\tau$ (approximate value for all sizes).

Then we may take

The radiating surface of rotor $= \pi\,\mathrm{d}\,\mathrm{L} = \pi\,\mathrm{d}\,(\lambda + {\cdot}7\tau)$.

The above motor had d = 2·2 dcm.; $\lambda = 1{\cdot}15$ dcm.; $\tau = 1{\cdot}7$ dcm.

In that case $\pi\ \mathrm{d}\ (\lambda + {\cdot}7\ \tau) = \pi \times 2{\cdot}2\ (1{\cdot}15 + {\cdot}7 \times 1{\cdot}7) = 16{\cdot}3$ sq. dcm.

Let S_a = the energy radiated per second, per degree centigrade, per square decimetre of surface.

Then $S_a = \frac{S}{\pi \mathrm{dL}}$, and we can determine this value for the two types of motor in the above analysis.

TABLE LXIV.—COMPARISON OF HEAT RADIATED BY SEMI-ENCLOSED AND TOTALLY-ENCLOSED MOTORS AT THE LIMITS OF SPEED.

		Watt-Seconds Radiated per Degree C. per Second.			
		Semi-Enclosed Motor.		Totally-Enclosed Motor.	
	Peripheral Speed—m.p.s.	*S.*	Per sq. dcm. *Sa.*	*S.*	Per sq. dcm. *Sa.*
Normal Speed .	17·2	35	2·15	14·7	·89
Standstill . .	0	12·3	·76	9·3	·57

§ 8. **Application of Results to Determination of Motor Rating for Intermittent Work.**—To show the application of these general considerations to a special case, we may set ourselves the following problem :—

What would be the rating of the above semi-enclosed motor, if after every five minutes' work there is ten minutes' rest, the temperature never to exceed 50° C. ?

The motor will have its maximum temperature after the five minutes' work, that is, it will cool down from 50° C. for ten minutes. During that time it radiates $(12{\cdot}3 \times 600 \times 50)$ watt sec. $= 369{,}000$ watt sec.

If Q is taken as 110,000, the temperature will fall $\frac{369{,}000}{110{,}000} = 3{\cdot}3°$ C.

The end temperature is therefore 46·7° C. We must now correct this decrease in the temperature, considering that the mean value of the temperature during that time was not 50° C. as assumed, but

$$\frac{50 + 46{\cdot}7}{2} = 48{\cdot}3° \text{ C.}$$

The decrease of temperature is therefore

$$3{\cdot}3 \frac{48{\cdot}3}{50} = 3{\cdot}2° \text{ C.}$$

and the temperature at the end of the ten minutes is 46·8° C.

The motor is loaded again with x horse-power, corresponding to a loss of y watts. The radiation during the next five minutes is $35 \times 300 \times 48{\cdot}3$ watt secs. $= 507{,}000$ watt secs. And as the motor can store up

$$110{,}000 \times 3{\cdot}2 = 352{,}000 \text{ watt secs.}$$

FIG. 455.—100 H.P. Motor, by Allmanna Svenska E. A. (see page 417).

Fig. 463.—185 H.P. 8000 Volt Motor, by Alioth Co. (see page 417).

Fig. 476.—Zani's 5 H.P. Motor, by Dick, Kerr & Co. (see page 427).

before it exceeds the maximum temperature, the losses during those five minutes may amount to

$$507{,}000 + 352{,}000 = 859{,}000 \text{ watt secs.}$$

The average watts lost therefore

$$= \frac{859{,}000}{5 \times 60} = 2863 \text{ watts,}$$

which corresponds to a load of 17·8 horse-power.

This calculation may be readily applied to any other case. Q can be estimated from the weight of the motor, and S from the radiating surface πdL and the special conditions, prevailing as to ventilation and speed of motor.

Of course, if the curves for cooling and heating have been found experimentally, this calculation may be made considerably shorter, as has been done by Ziehl in his article.

§ 9. **Data of a Westeras Motor and two Alioth Motors.**—In the following tabulated specification is set forth the data of three large induction motors. The design in column A was kindly supplied to the writer by Mr Ernst Danielson, and related to a 12-pole, 50-cycle, 500 volt, 500 r.p.m., 100 horse-power three-phase motor, built by the Allmanna Svenska Elektriska Aktiebolaget of Westeras, Sweden. A photograph of the motor is given in Fig. 455, Plate 23, and drawings in Figs. 456 to 458. Curves of test results are reproduced in Fig. 459.

The Alioth Company of Basel, Switzerland, has kindly given permission for the publication of the design of one of their three-phase motors. This is given in column B. The motor is a 14-pole, 50-cycle motor of 185 horse-power rated capacity at 8000 volts, and the synchronous no-load speed is 430 r.p.m. Drawings of the motor are given in Figs. 460 to 462, Plate 24, and a photograph in Fig. 463, Plate 23.

The design of a 30-pole, 25-cycle, 500 horse-power Alioth motor for 5000 volts, and for a no-load speed of 100 r.p.m., is given in column C. Details of slot construction are given in Fig. 464, and test results in Fig. 465, page 420. This is a remarkably interesting motor in virtue of its very large rated output and its very low speed.

In the calculation of these three motors, σ was determined by a slight modification of Behrend's formula,

$$\sigma = \mathrm{C}\frac{\Delta}{\tau}.$$

In this form in which it has heretofore been used, C has been

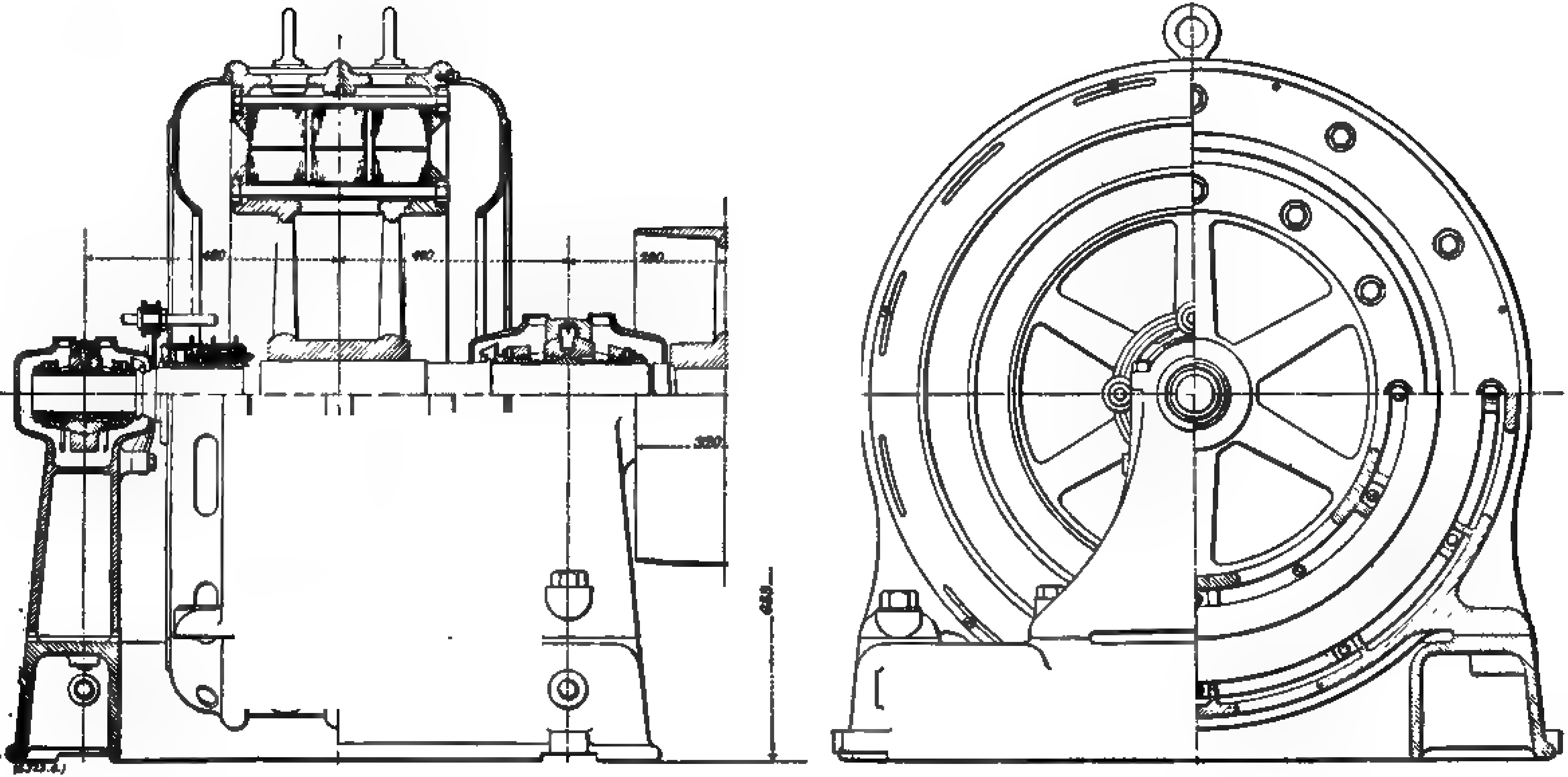


FIG. 456.—100 H.P. Motor, 12 Pole, 50 Cycle, 500 Volts, 500 r.p.m., Three-Phase, by Allmanna Svenska Elektriska Aktiebolaget.

determined from the curves of Fig. 431, page 387. This generally gives sufficient accuracy for ordinary purposes. A higher degree of

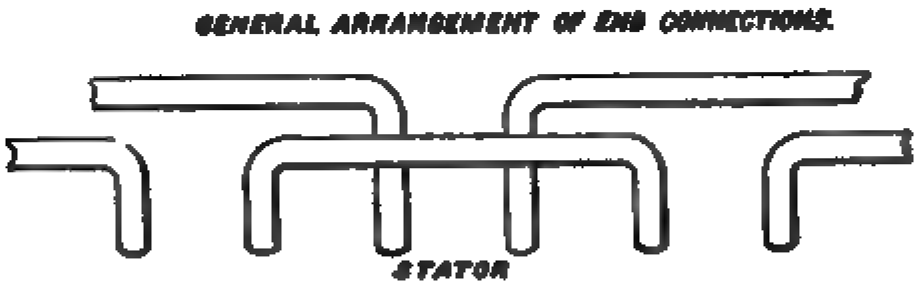


FIG. 457.—Slot and Gap Dimensions.

FIG. 458.—Stator and Rotor End Connections.

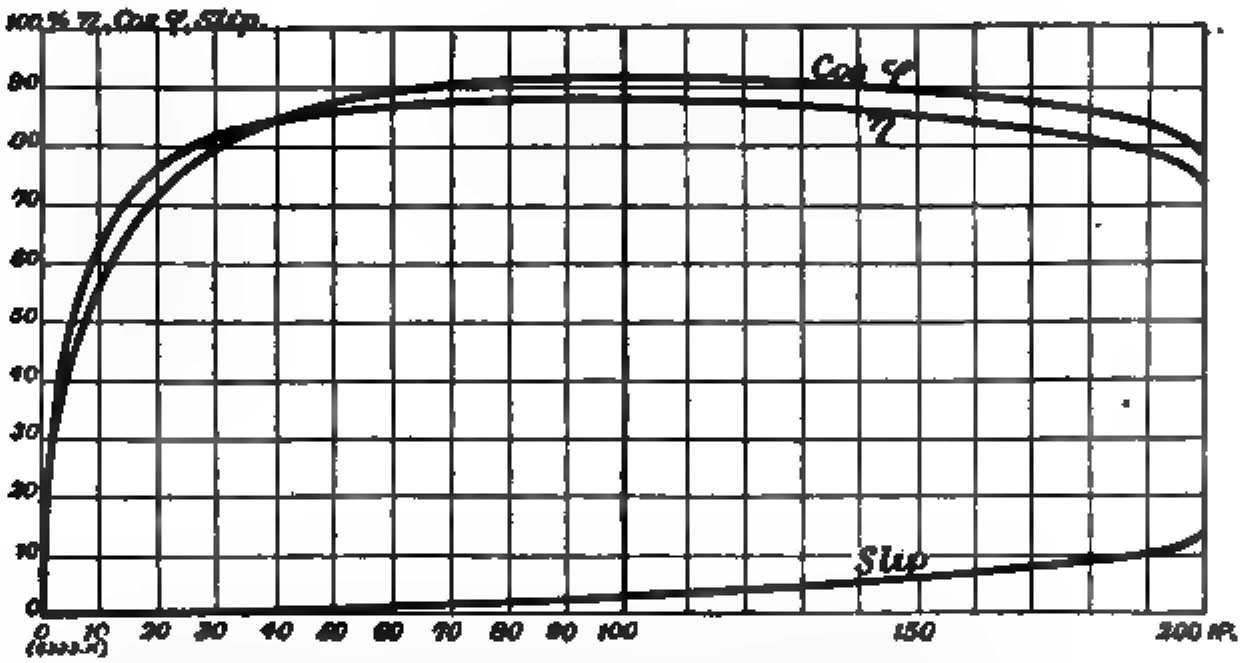


FIG. 459.—Efficiency, Power Factor and Slip, 100 H.P. Three-Phase, 480 r.p.m., 50 Cycle Allmanna Svenska Elektriska Aktiebolaget Motor.

accuracy may, however, be obtained by altering the formula to read

$$\sigma = C\,C'\,\frac{\Delta}{\tau}.$$

In this last form, C is determined, as before, from the curves

of Fig. 431, and C′ is determined from the curve of Fig. 466, and is a function of $\Delta \times H$, where Δ = radial depth of the air gap and H = average of the number of stator and rotor slots per pole.

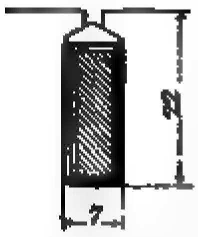


Fig. 464.—Section through Windings in Slots.

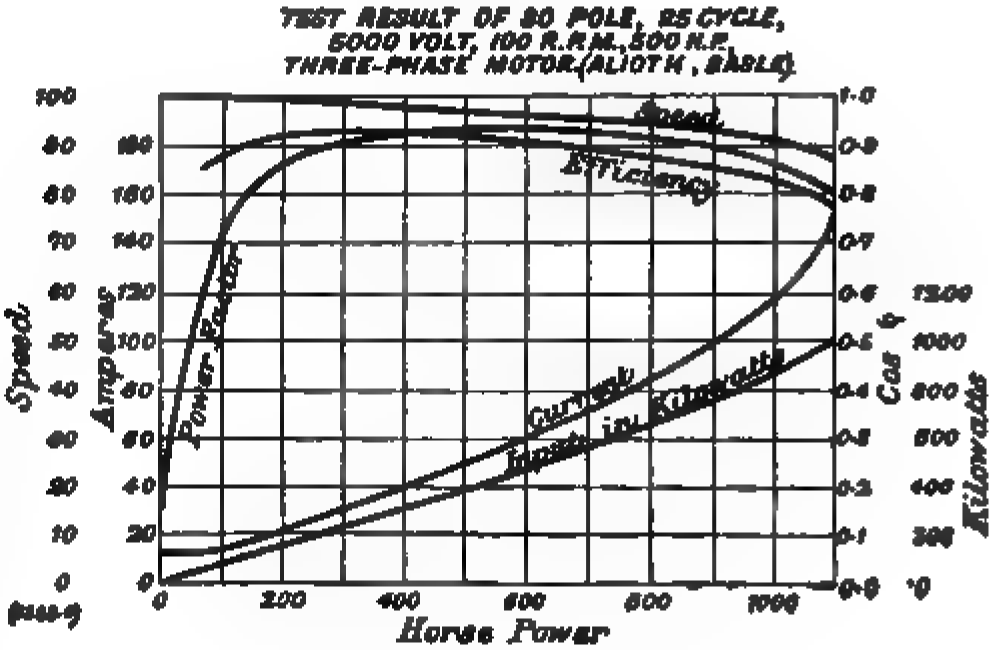


Fig. 465.—Test Results 500 H.P. Alioth Motor.

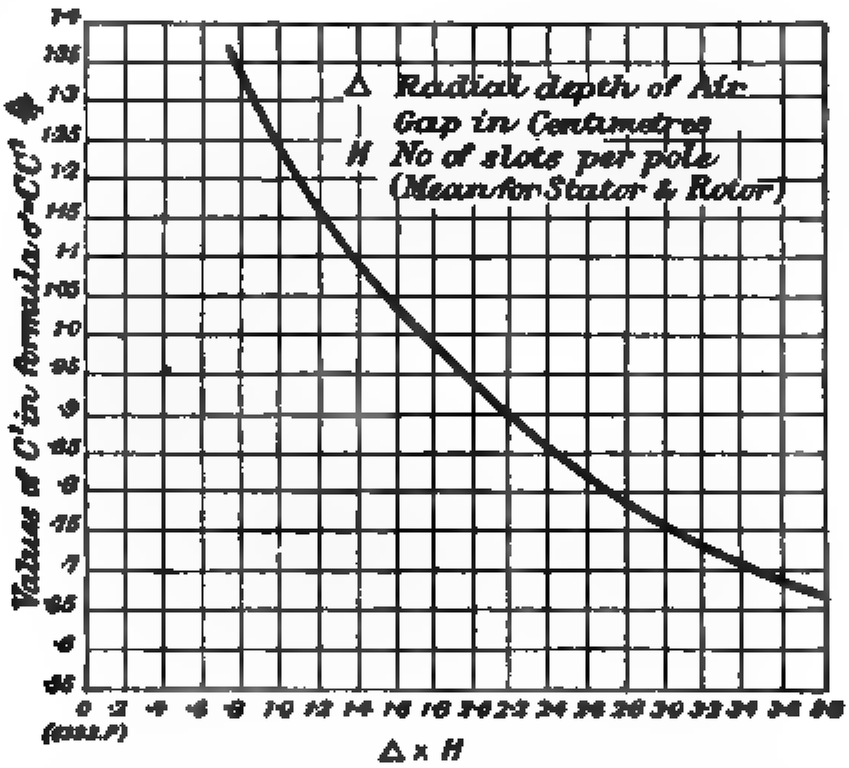
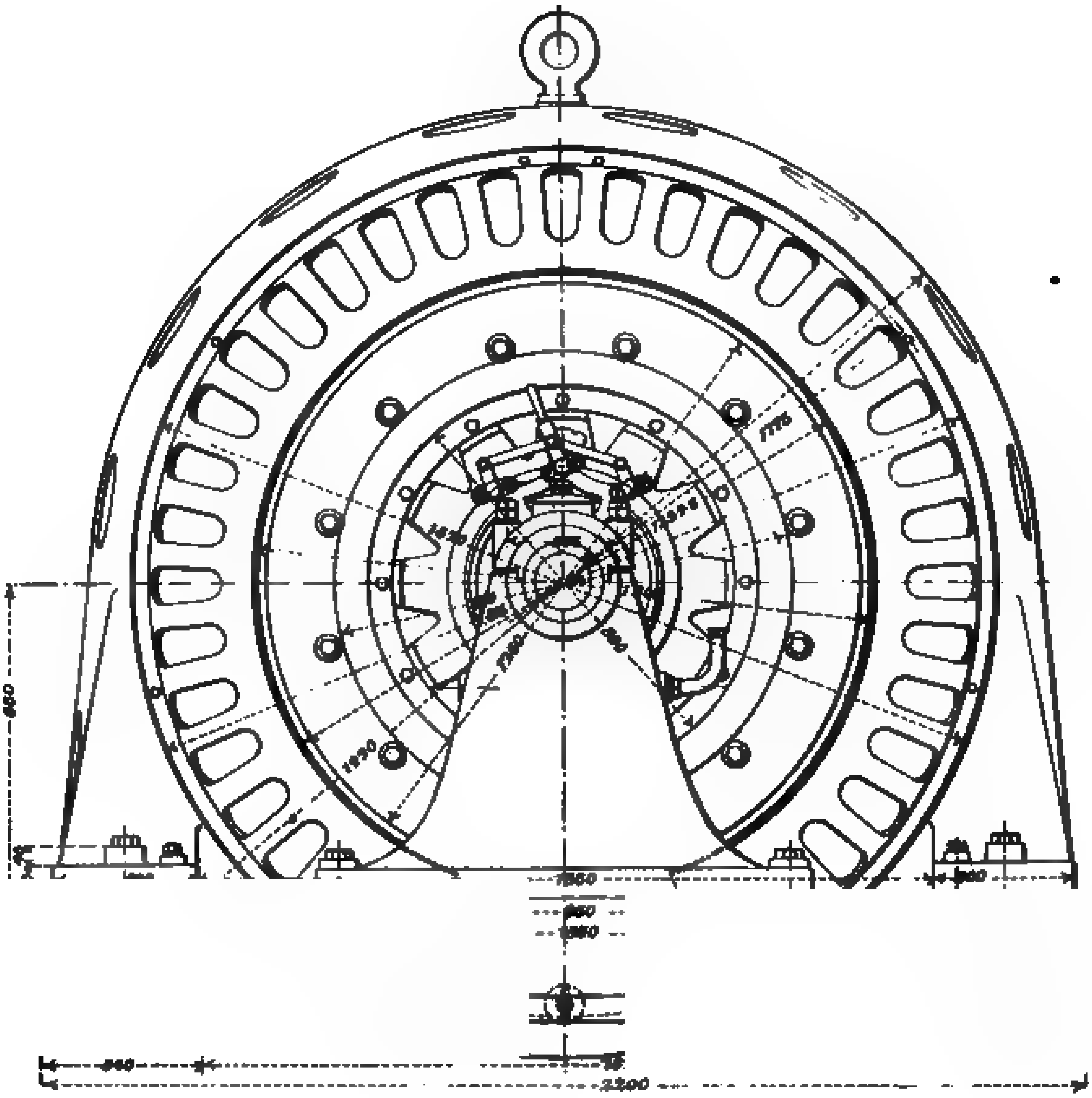


Fig. 466.—Relation between Δ H and C.

By means of this constant, C′, the so-called "zig-zag" dispersion along the heads of the teeth, is taken into account. The "zig-zag" dispersion may be very considerable when the product $\Delta \times H$ is small.

FIGS. 460, 461, and 462.—Section, Side Elevation, and Plan of



85 H.P. Alioth Co. Motor, 8000 Volts, 50 Cycles (see page 417).

	Allmanna Svenska Elektriska Aktiebolaget. A	Alioth. B	Alioth. C
Rated horse-power	100	185	500
Number of poles	12	14	30
Periodicity in cycles per sec.	50	50	25
Synchronous speed in R.P.M.	500	430	100
Voltage between terminals	500	8000	5000
Connection of stator winding	Y	Y	Y
Voltage per phase	288	4620	2880
Stator lamination—			
External diameter of stator punchings ...	103	162	322
Internal diameter of stator punchings ...	88	130	300
Gross length between flanges, λ_g	36	40	75
Number of ventilating ducts	2	none	none
Width of each duct	1·5	...	...
Net length of laminations between flanges, λ_n	29·7	36	67·5
Polar pitch at air-gap, τ	23	29·2	31·4
Number of stator slots	180	168	450
Number of stator slots per pole	15	12	15
Number of stator slots per pole per phase	5	4	5
Depth of stator slot	2·5	5·2	5·7
Width of stator slot	1·05	1·3	1·3
Width of stator slot opening	·3	·2	·3
Stator tooth pitch (at air-gap)	1·53	2·43	2·09
" " (at ends of the slots) ...	1·62	2·63	2·17
Minimum width of stator tooth	·48	1·13	·79
Maximum width of stator tooth	·57	1·33	·87
Weight of stator punchings	410	1750	3570
Rotor lamination—			
Radial depth of air-gap, Δ	·15	·125	·175
External diameter of rotor punchings ...	87·7	129·75	299·65
Internal diameter of rotor punchings ...	75·7	101·0	284·0
Gross length between flanges, λ_g ...	36	40	75
Number of ventilating ducts	2	none	none
Width of each duct	1·5	...	...
Net length of lamination between flanges, λ_n	29·7	36	67·5
Number of rotor slots	216	294	720
Number of rotor slots per pole	18	21	24
Number of rotor slots per pole per phase	6	7	8
Depth of rotor slot	2·15	2·3	2·2
Width of rotor slot	·8	·75	·7
Width of rotor slot opening	·3	·2	·2
Rotor tooth pitch (at air-gap)	1·28	1·39	1·31
Rotor tooth pitch (at bottom of slots) ...	1·21	1·34	1·29
Minimum width of rotor tooth	·41	·59	·59
Maximum width of rotor tooth	·48	·64	·61
Weight of rotor punchings	270	1340	3400

	Allmanna Svenska Elektriska Aktiebolaget. A	Alioth. B	Alioth. C
Stator winding—			
Number of stator conductors per slot ...	3 each with two components	27	8
Number of stator slots	180	168	450
Number of stator conductors	540	4536	3600
Number of stator turns	270	2268	1800
Number of stator turns per phase ...	90	756	600
Number of stator turns per pole per phase	7·5	54	20
Dimensions of conductors (bare) ...	2 × ·25 × ·65	·25 diam.	·5 diam.
„ „ (insulated)	2 × ·3 × ·7	·3 „	·56 „
Cross section of conductor	·324	·049	·197
Mean length of one turn	150	205	288
Total length of stator turns per phase ...	13,500	155,000	173,000
Resistance per phase at 60° C.	·084	6·3	1·75
Weight of stator copper in kilogrammes ...	117	203	910
Rotor winding—			
Number of rotor conductors per slot ...	2	1	1
Number of rotor slots	216	294	720
Number of rotor conductors	432	294	720
Number of rotor turns	216	147	360
Number of phases in rotor winding ...	3	3	3
Number of rotor turns per phase	72	49	120
Number of rotor turns per pole per phase	6	3·5	4
Dimensions of conductor (bare)	·45 × ·8	1·65 × ·5	1·6 × ·5
„ „ (insulated) ...	·5 × ·85	1·7 × ·55	1·65 × ·55
Cross-section of single conductor	·36	·825	·8
Mean length of one turn	144	185	240
Length of effective rotor turns per phase...	10,400	9060	28,800
Resistance per phase at 60° C.	·058	·022	·072
Weight of rotor copper	100	200	615
Power factor and current—			
$\frac{\text{Net length of lamination}}{\text{Pole pitch}} = \frac{\lambda_n}{\tau}$	1·28	1·23	2·15
Type of slot (average of stator and rotor slot)	two-thirds closed	nearly closed	nearly closed
C in Behrend's formula (from page 388) ...	10·6	12	10·5
$\Delta \times H =$	2·5	2·06	3·4
$C' =$	·84	·93	·71
Leakage factor σ	·058	·048	·042
Maximum power factor $= \frac{1}{1+2\sigma}$	·895	·912	·922
Watts output at full load	73,600	136,000	368,000
Full load efficiency	·90	·93	·910
Watts input at full load	81,900	146,000	405,000

	Allmanna Svenska Elektriska Aktiebolaget.	Alioth.	Alioth.
	A	B	C
Power factor and current—continued.			
Volt ampere input at full load	91,500	160,000	435,000
" " per phase	30,500	53,300	145,000
Volts per phase	288	4620	2880
Full load ampere per stator winding ...	106	11·5	50·5
Type of connection	Y	Y	Y
Full load line amperes	106	11·5	50·5
No load current and short circuit—			
Periodicity in cycles per second, N ...	50	50	25
Number of stator turns per phase, T ...	90	756	600
Approximate internal voltage per phase, E	280	4550	2820
Megalines flux per pole, $M \times 10^{-6}$ from $E = 4{\cdot}2$ T.N.M. 10^{-8}	1·48	2·86	4·5
Per cent. exposed iron at stator surface ...	80	92	85·5
" " " rotor " ...	80	92	90·5
Mean for stator and rotor, k " ...	80	92	88
Exposed cross section of iron at air-gap per pole $= \lambda_n \tau k$	544	965	1860
Spreading coefficient	1·15	1·15	1·15
Corrected air-gap cross section per pole ...	625	1110	2140
Average gap density	2370	2580	2100
Maximum gap density	4030	4380	3570
Ampere turns for gap per pole	480	436	500
Total ampere turns per pole	540	490	560
Ampere turns per pole per phase	270	245	280
Maximum current at no load	36	4·55	14
R.M.S. current at no load	25·5	3·2	9·8
R.M.S. in per cent. of full load current ...	23·5%	27·3%	19%
$\sigma =$	·058	·048	·042
Current corresponding to the diameter of the circle, $\frac{\text{no load current}}{\sigma}$	440	66·7	233
Ideal short circuit current	465	70	243
Secondary current at full load (from circle diagram for ratio 1 : 1 of transformation)	97·5	10·5	47·5
Number of stator conductors	540	4536	3600
Number of rotor conductors	432	294	720
Ratio of transformation	1·25	15·4	5
Actual secondary current	121	162	237
Losses—			
(I) I^2R loss of stator :			
Current in stator winding	106	11·5	50·5
Resistance of stator winding per phase ...	·084	6·3	1·75
I^2R loss of stator per phase	950	835	4450
Total I^2R loss of stator	2850	2505	13,350

	Allmanna Svenska Elektriska Aktiebolaget. A	Alioth. B	Alioth. C
Losses—continued.			
(II) I^2R loss of rotor :			
Current in rotor winding	121	162	237
Resistance of rotor winding per phase ...	·058	·0219	·072
I^2R loss of rotor per phase	850	575	4050
Total I^2R loss of rotor	2850	1625	12,150
Slip of rotor	3·4%	1·2%	3·1%
(III) Iron loss of stator :			
Minimum cross-section of stator teeth per pole	214	490	800
Average density at these points	6900	5840	5620
Maximum density at these points	11,700	9900	9600
Depth of stator iron above tooth	5·0	10·8	5·3
Cross section of stator iron	297	776	715
Density of stator iron	5000	3700	6300
Periodicity in cycles per second, N ...	50	50	25
Density in stator iron in kilolines, D ...	5	3·7	6 3
$\frac{DN}{100}$	2·5	1·85	1·58
Watts stator core loss per kilogramme (from fig. 384, page 331)	3·9	2·8	2·4
Weight of stator punchings, kilogrammes	410	1750	3570
Core loss in stator iron	1600	4900	8600
(IV) Iron loss of rotor :			
Cross section of rotor teeth at narrowest point	219	448	960
Average density of rotor teeth at narrowest point	6800	6400	4700
Maximum density of rotor teeth at narrowest point	11,550	10,900	8000
Depth of rotor iron below slots	3·85	12·1	5·6
Cross section of rotor iron	228	870	755
Density in rotor iron	6500	3300	5950
Slip	3·4%	1·2%	3·1%
Weight of rotor punchings	270	1340	3400
Iron loss in rotor	50	35	250
(V) Friction loss in bearings and through windings	1400	1500	2300
Total of losses—			
Variable losses (I and II)	5400	4130	25,500
Constant losses (III, IV and V)	3050	6535	11,150
Total losses	8450	10,665	36,650
Output in watts	73,600	136,000	368,000
Input in watts	82,050	146,665	404,650
Full load efficiency	89·9	92·8	91

	Allmanna Svenska Elektriska Aktiebolaget. A	Alioth. B	Alioth. C
Weights—			
Weight of stator copper, kilogrammes ...	117	203	910
„ rotor „ „ ...	100	200	615
Total weight of copper	217	403	1525
Weight of stator laminations	410	1750	3570
Weight of rotor laminations	270	1340	3400
Total weight of laminations	680	3090	6970
Total weight of active material	897	3493	8495
Weight of active material per horse-power output, kilogrammes	9	19	17

It appears that motor B is very conservatively rated. This explains also that the size of the motor is comparatively large for its output, and that the ratio of $\frac{\text{variable}}{\text{constant}}$ losses is so small. In fact, it is stated this motor has been frequently used for outputs 20 per cent. higher than its rated output.

It must be understood that in the calculations in the preceding tables, no reference is made to the actual performance of the motors as tested. This omission was intentional, and was made in order to allow all the calculations to be made on a uniform basis. But in justice to the manufacturer, and as at the same time showing the accuracy which is possible by the methods of calculation as set forth in the tabulated form, actual test results on the motors will now be given.

Motor A.

The no load current as observed was 23 amperes, at a cos. $\phi =$ 0·19. This is slightly lower than the calculated value 25·5 amperes, and this discrepancy may be due to small variations in the air gap. If we assume the reading of the small power factor 0·19 to have been correct, we obtain losses at no load $= 3 \times 288 \times 23 \times 0{\cdot}19 =$ 3700.

The losses from bearing and windage friction are given as 1·9 horse-power = 1400 watts. This has been given in the tables. The stator iron losses must therefore be equal to 3700 − friction losses $- I^2R$ losses at no load $= 3700 - 1400 - 140 = 2160$ watts.

The calculated value was 1600 watts.

The short circuit current is given as 438 amperes at a power factor of 0·383.

The calculated ideal short circuit value was 465, which agrees very well with the test result.

If, again, we take the power factor 0·383 to be correct, we find losses at standstill amounting to $3 \times 288 \times 438 \times 0{\cdot}383 = 145$ Kws.

The calculated value would be—

I^2R loss in stator	48,000
I^2R loss in rotor	47,000
Iron loss in stator	1,600
Iron loss in rotor	1,300
	97,900 watts.

This discrepancy may have its cause partly in the copper becoming very hot during this short circuit test, and thereby

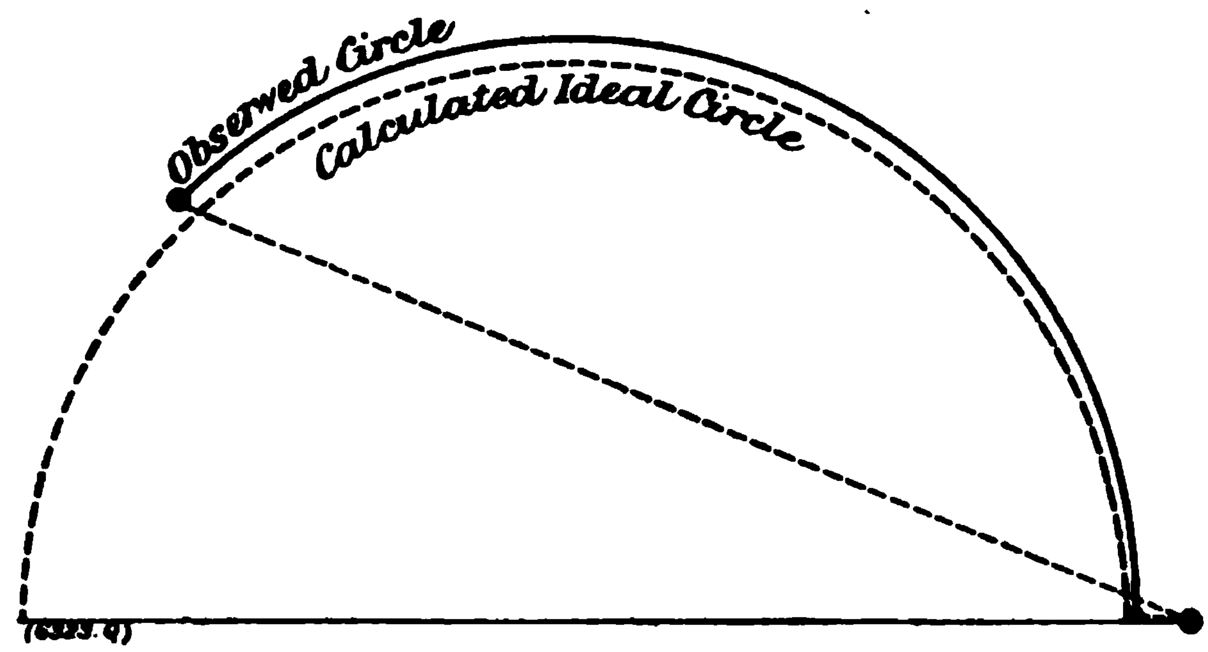


FIG. 467.—Circle Diagram of 12 Pole, 50 Cycle, 500 Volt, 500 r.p.m., 100 H.P. Three-Phase Motor (Allmanna Svenska).

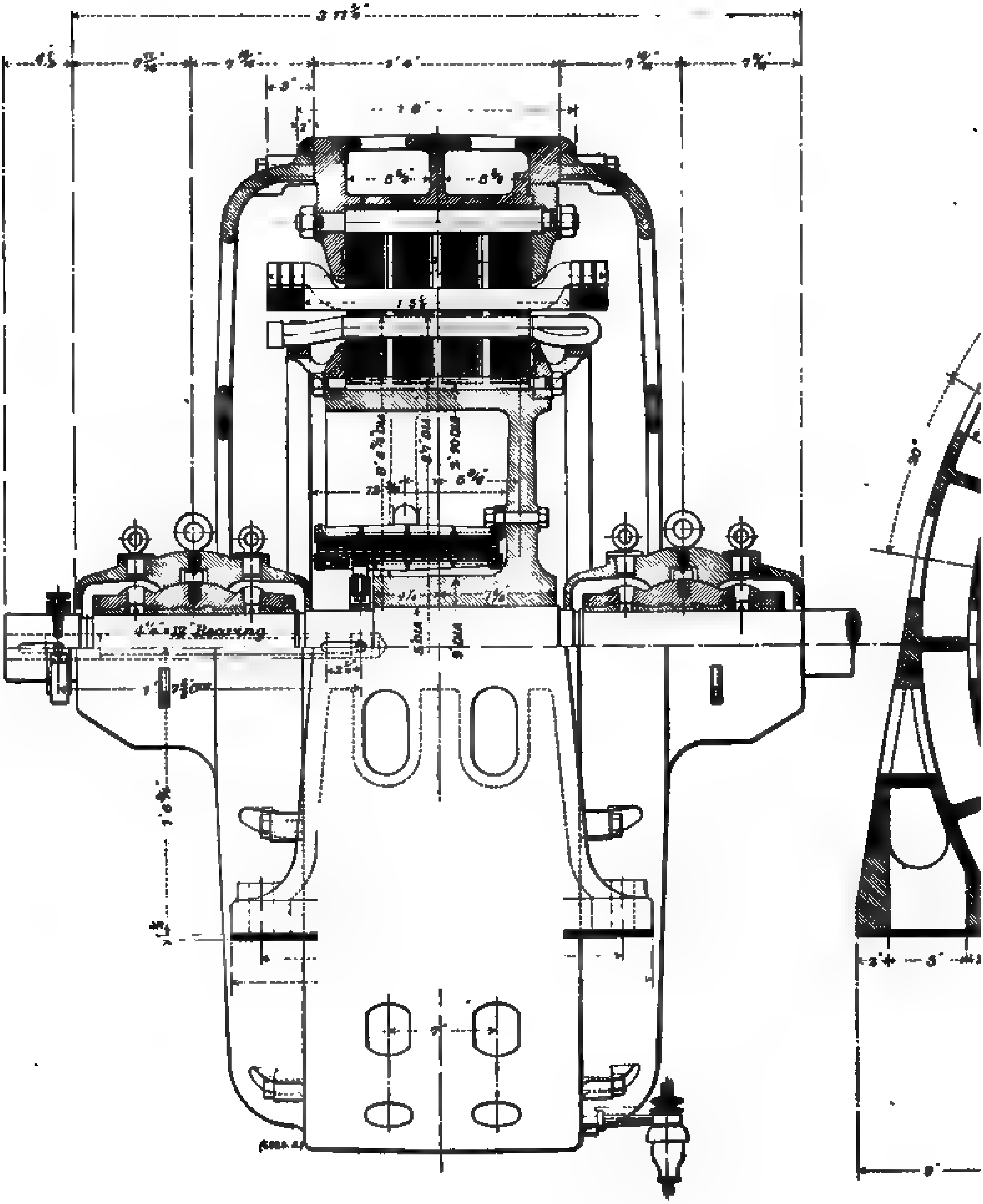
increasing in resistance, and partly in the unavoidable inaccuracy in the readings of such small power factors.

The ideal calculated circle diagram and the observed circle are shown in Fig. 467.

MOTOR B.

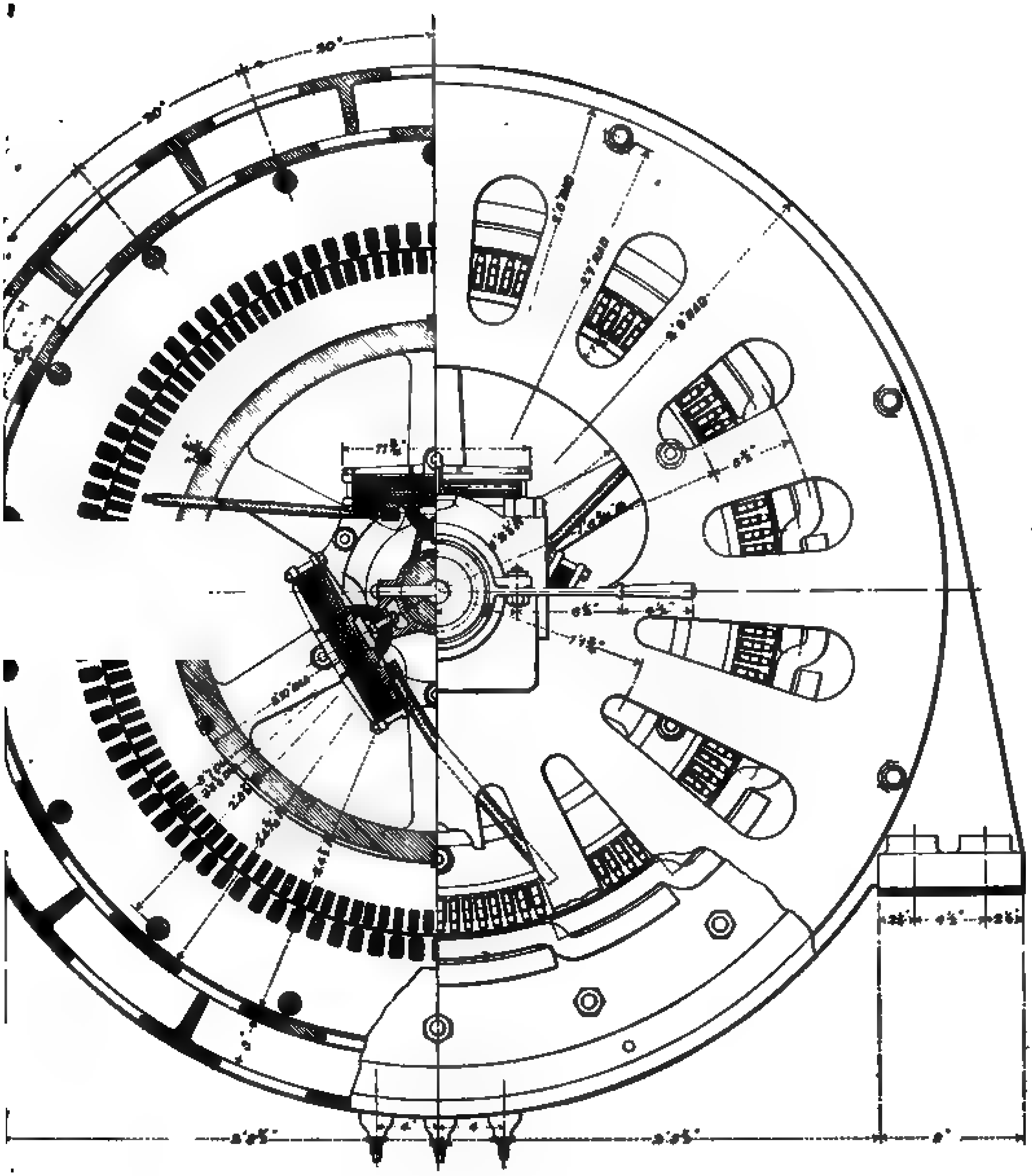
Very elaborate tests are available for the combined performance of motor B, directly coupled to a 140 Kw. direct current generator. But as it would lead too far to analyse these tests, the necessary results are alone quoted.

Fig. 468 shows the circle diagram as calculated in the tabulated form. The small circles and the crosses refer to analogous tests made on two motors built to exactly the same models and designs. It is seen that the results of one of these tests agree closely with the calculated diagram. The deviation in the other case is no greater than might readily be due to variations in the radial depth of the air gap.



Figs. 469 and 470.—Zani's 220 H.P., 5000 Volt I

[*Plate* 25.



iuction Motor, by Dick, Kerr & Co. (see page 427).

MOTOR C.

The observed no load current was 9·3 amperes, and the calculated value 9·8 amperes. The losses at no load amounted to 10,400 watts, which compare very well with the calculated value of $8600+2300+490=11,390$ watts.

The short circuit current at 976 volts, 25 cycles, was found to be 50 amperes.

The corresponding calculated value

$$243\,\frac{976}{5000}=47{\cdot}5 \text{ amperes.}$$

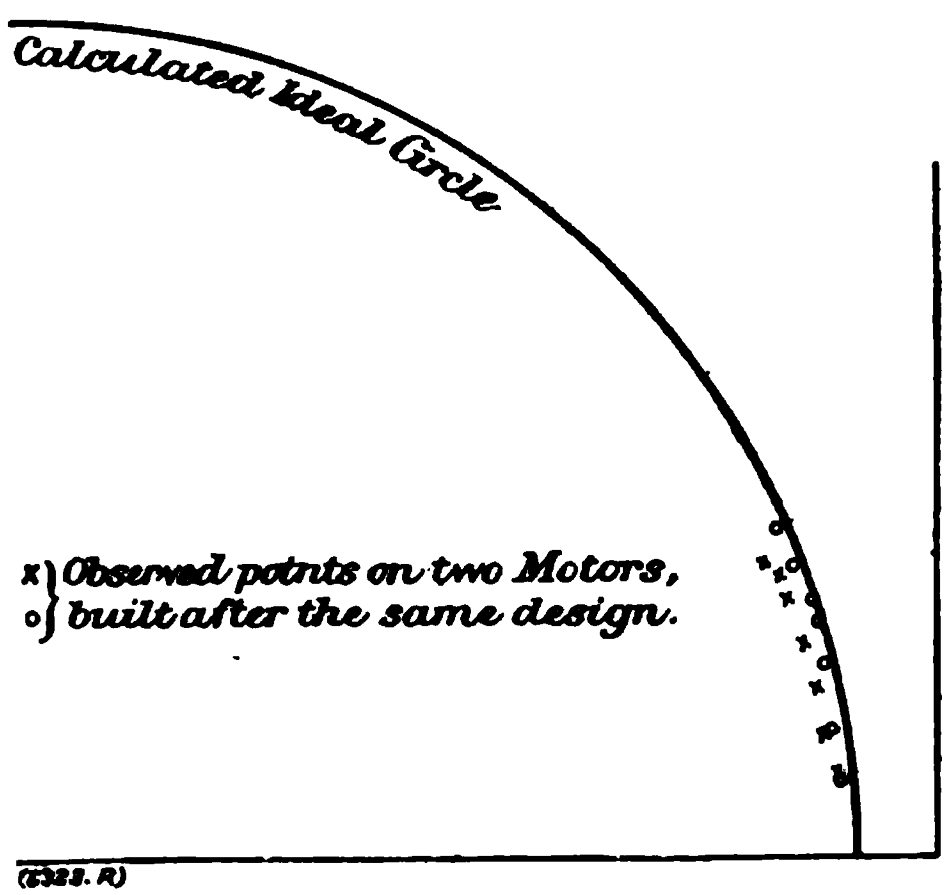


FIG. 468.—Circle Diagram of 14 Pole, 50 Cycle, 8000 Volt, 430 r.p.m., 185 H.P. Three-Phase Motor (Alioth, Basel).

§ 10. **Data of Messrs Dick, Kerr & Co.'s Motors.**—Mr A. P. Zani has, with the permission of Messrs Dick, Kerr & Company, very kindly furnished the writer with full particulars of two three-phase induction motors which he has designed for that firm. The designs are set forth in columns A and B of the following specification. Both motors are for a periodicity of 50 cycles per second. The motor described in column A has 6 poles, and is of a rated capacity of 5 horse-power at 400 volts. The no-load speed is 1000 r.p.m. A 12-pole, 220 horse-power, 5000 volt design is given in column B. The no-load speed is 500 r.p.m. Drawings and photographs of these two motors are given in Figs. 469 to 475, Plate 25, and in Fig. 476, Plate 23, is given a photograph of the 5 horse-power motor. Calculated curves of performance are given in Figs. 477 to 480, pages 431 and 432.

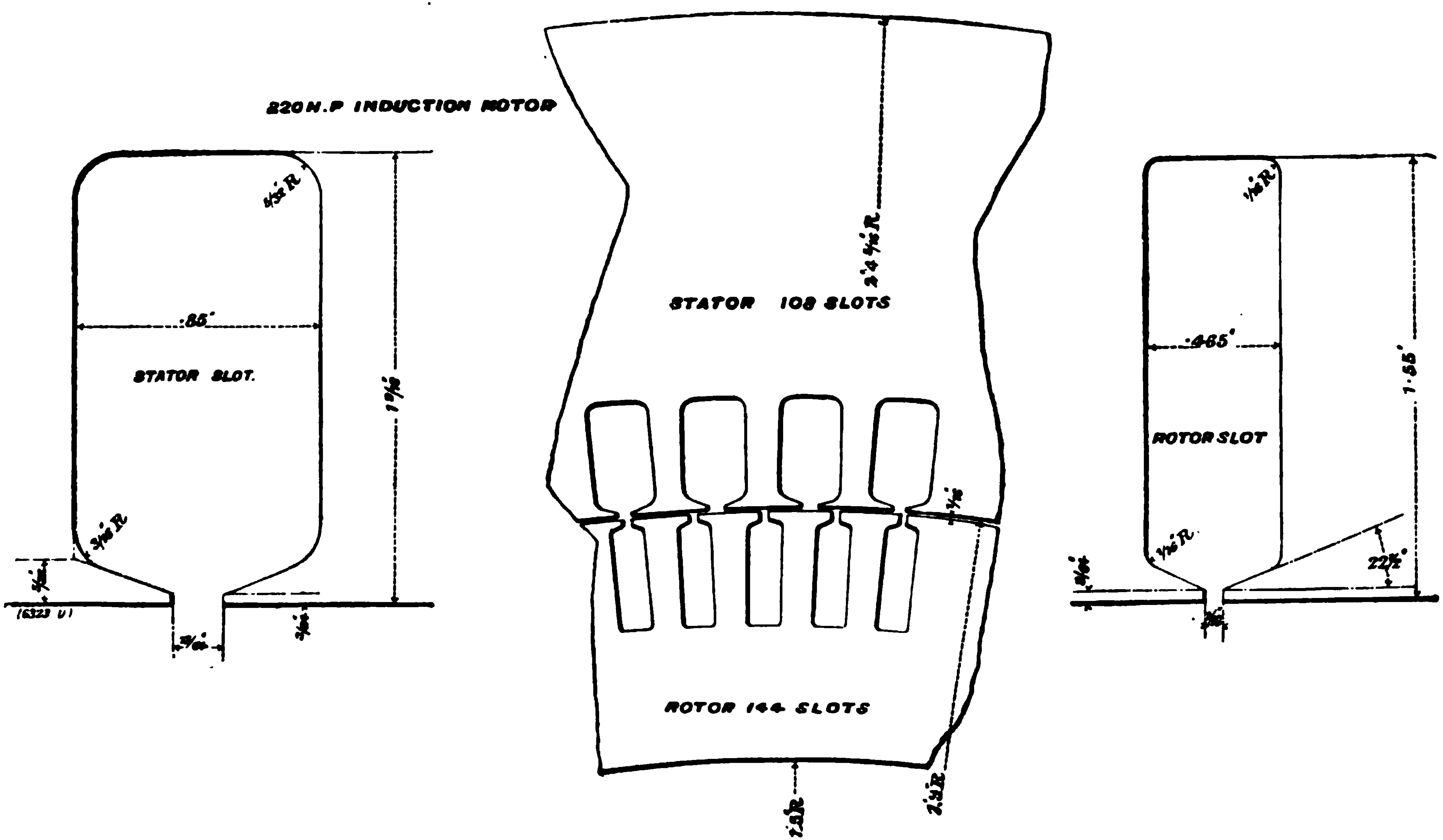
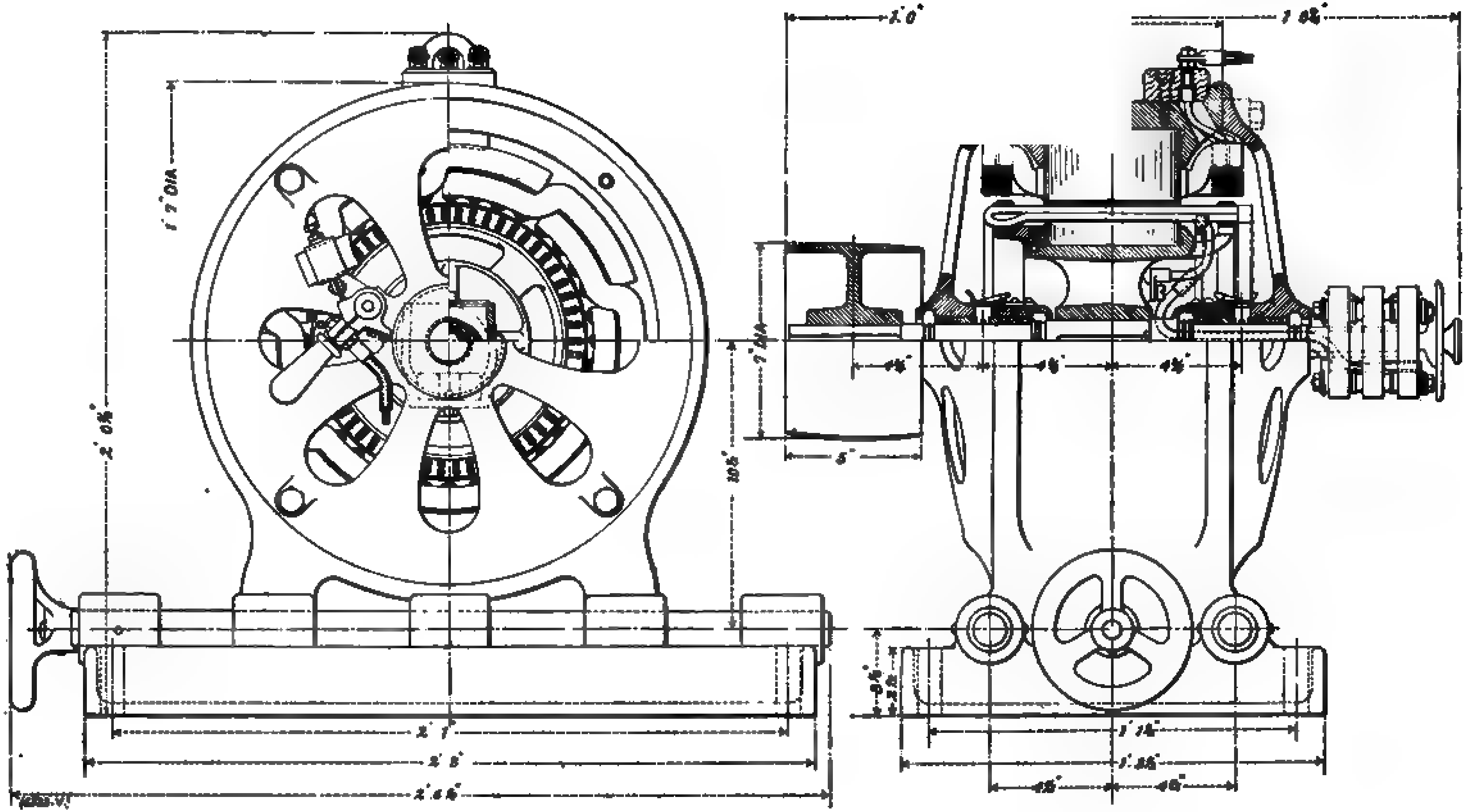


FIG. 472.—Slot and Core Dimensions, Dick, Kerr & Co.'s 200 H.P. Induction Motor.

FIG. 471.—Section through Zani Motor, 220 H.I

[*Plate* 26.

5000 Volts, by Dick, Kerr & Co. (see page 427).



FIGS. 473 and 474.—Sectional Elevations of Zani's 5 H.P. Motor, 1000 r.p.m. 50 Cycles, Three-Phase, Dick, Kerr & Co.

	Zani.	Zani.
Rated horse-power	5	220
Number of poles	6	12
Periodicity in cycles per sec.	50	50
Synchronous speed in R.P.M.	1000	500
Voltage between terminals	400	5000
Connection of stator winding	Δ	Y
Voltage per phase	400	2880

Stator lamination—

External diameter of stator punchings... ...	39·4	145
Internal diameter of stator punchings	25·4	109

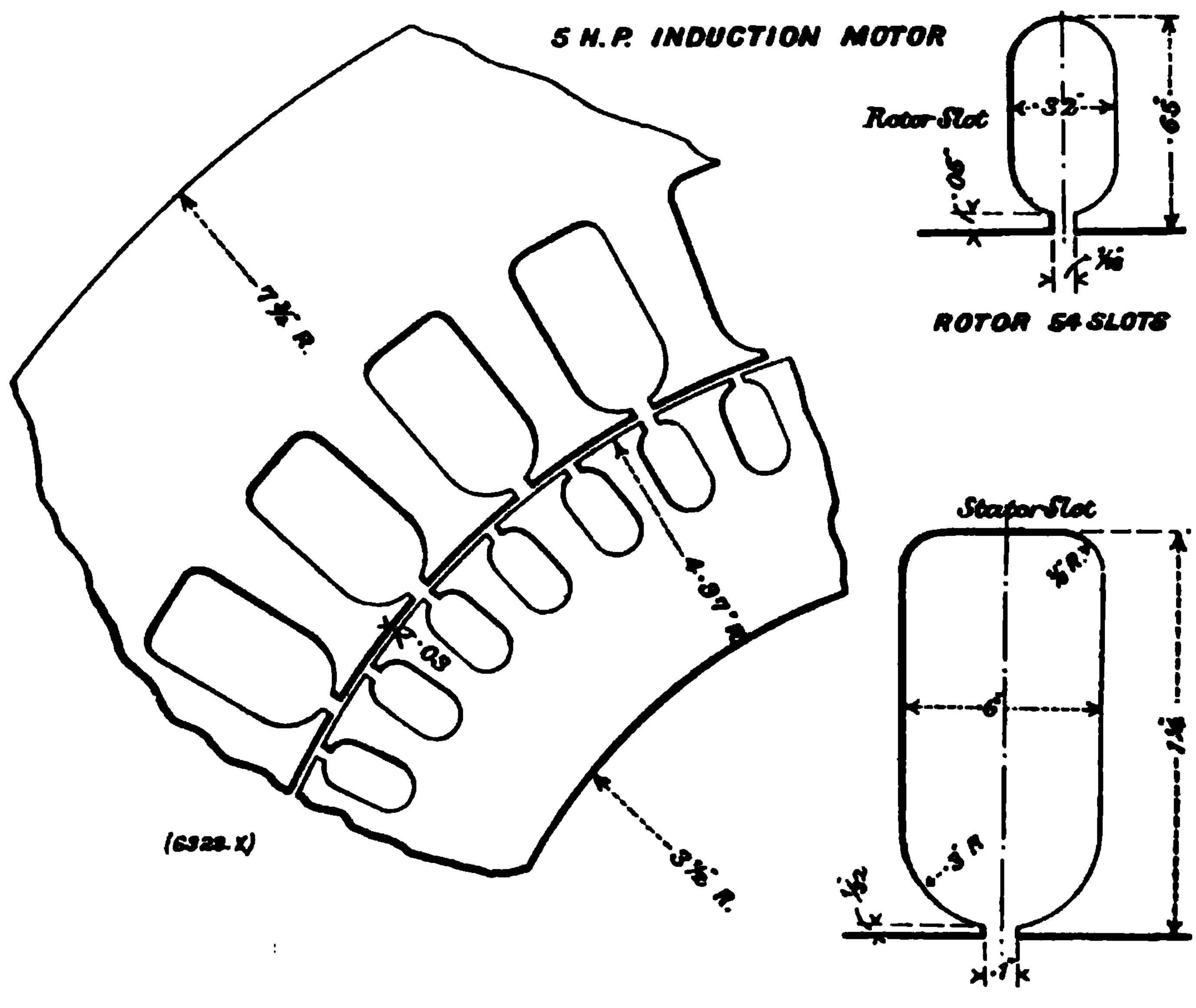


FIG. 475.—Stator and Rotor Slots, 5 H.P. Motor, by Dick, Kerr & Co.

Gross length between flanges, λ_g	12·7	30·5
Number of ventilating ducts	...	3
Width of each duct	...	1·3
Net length of laminations between flanges, λ_n	11·4	23·9
Polar pitch at air gap, τ	13·3	28·5
Number of stator slots	36	108
Number of stator slots per pole	6	9
Number of stator slots per phase	2	3
Depth of stator slot	3·18	3·97
Width of stator slot	1·52	21·6
Width of stator slot opening	·25	·4
Stator tooth pitch (at air gap)	2·21	3·16
Stator tooth pitch (at ends of the slots) ...	2·77	3·4

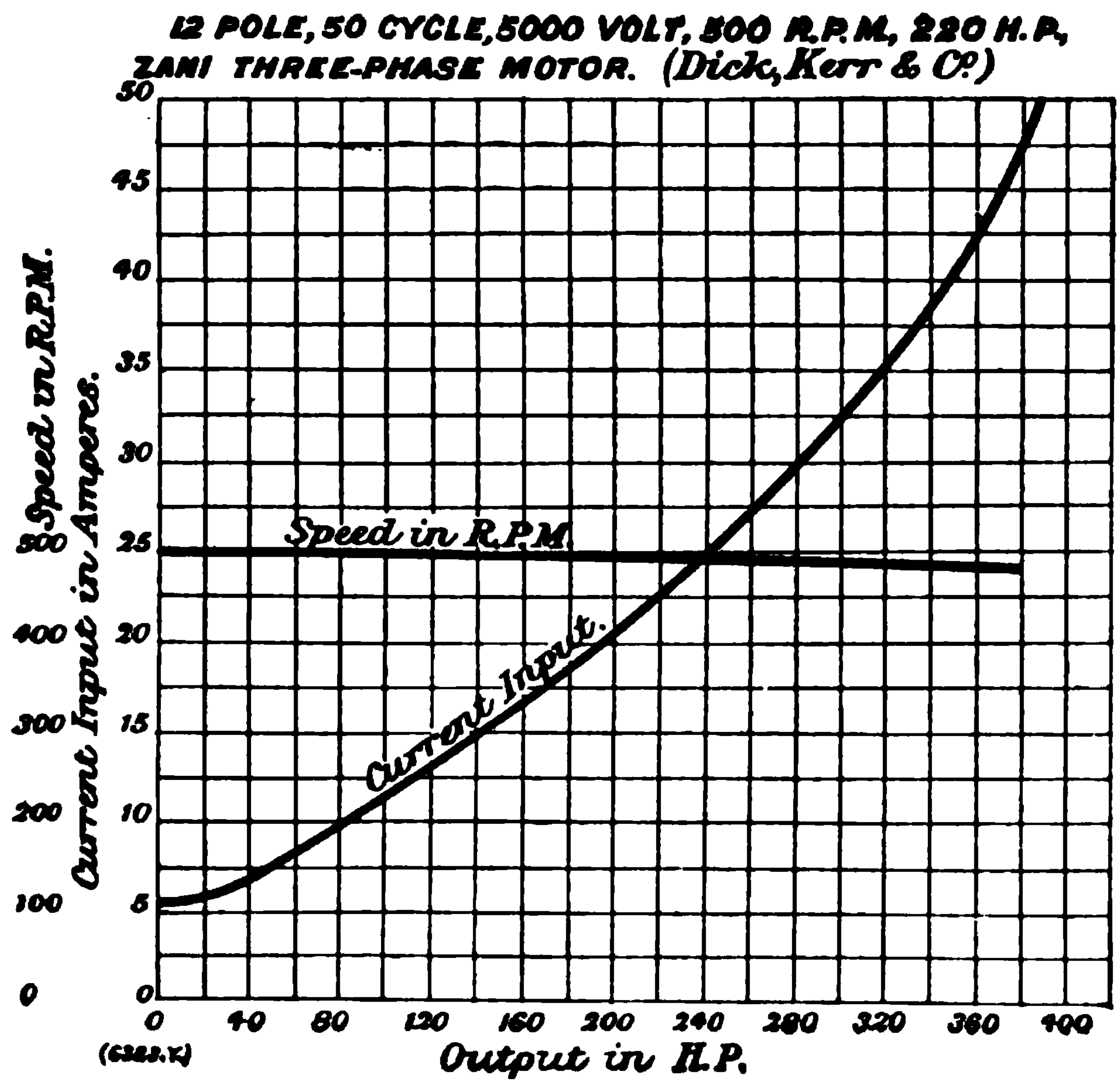


FIG. 477.—Speed, Current, and Horse-Power of 220 H.P. Three-Phase Motor.

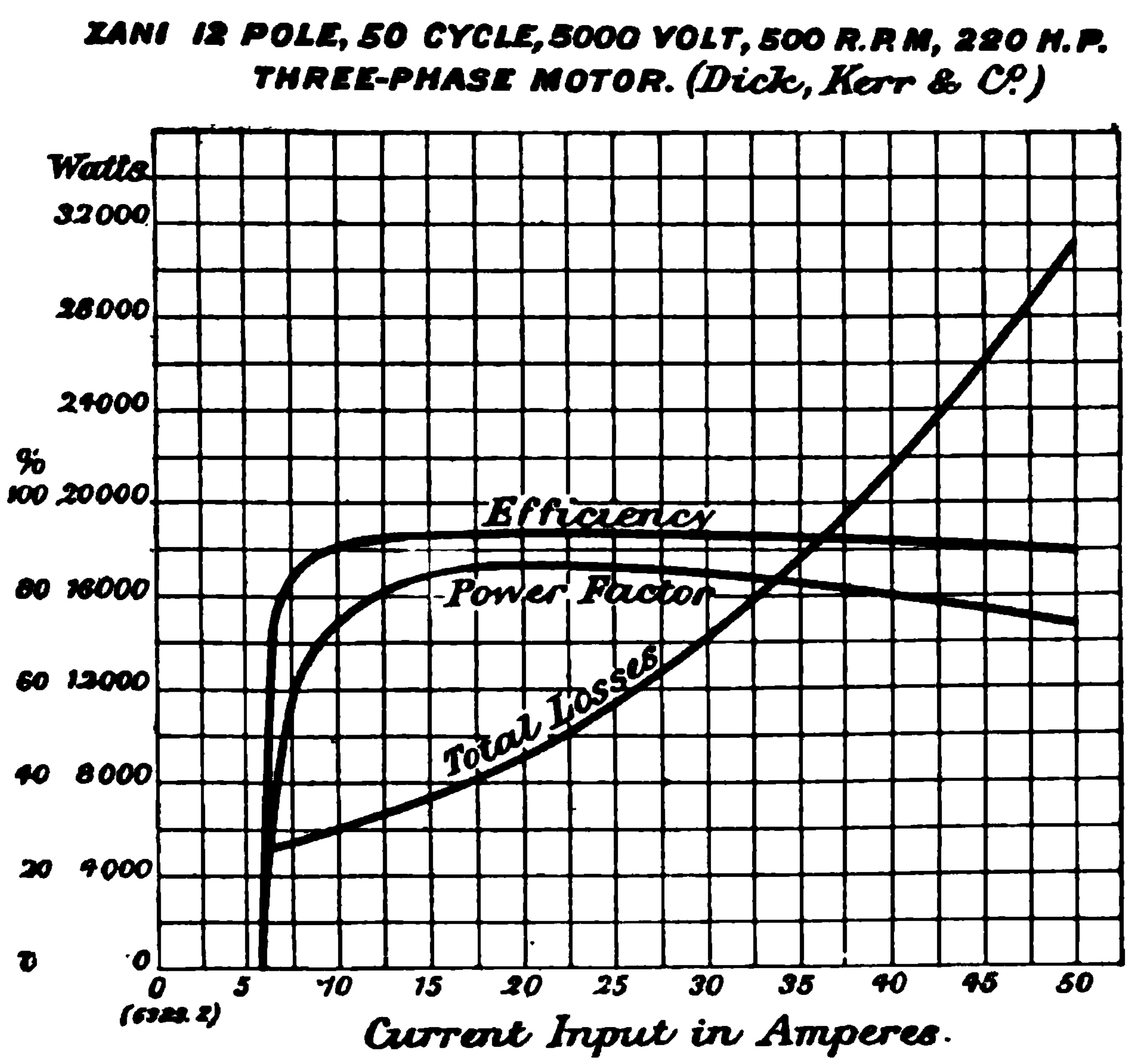


FIG. 478.—Characteristics of 220 H.P. Three-Phase Motor.

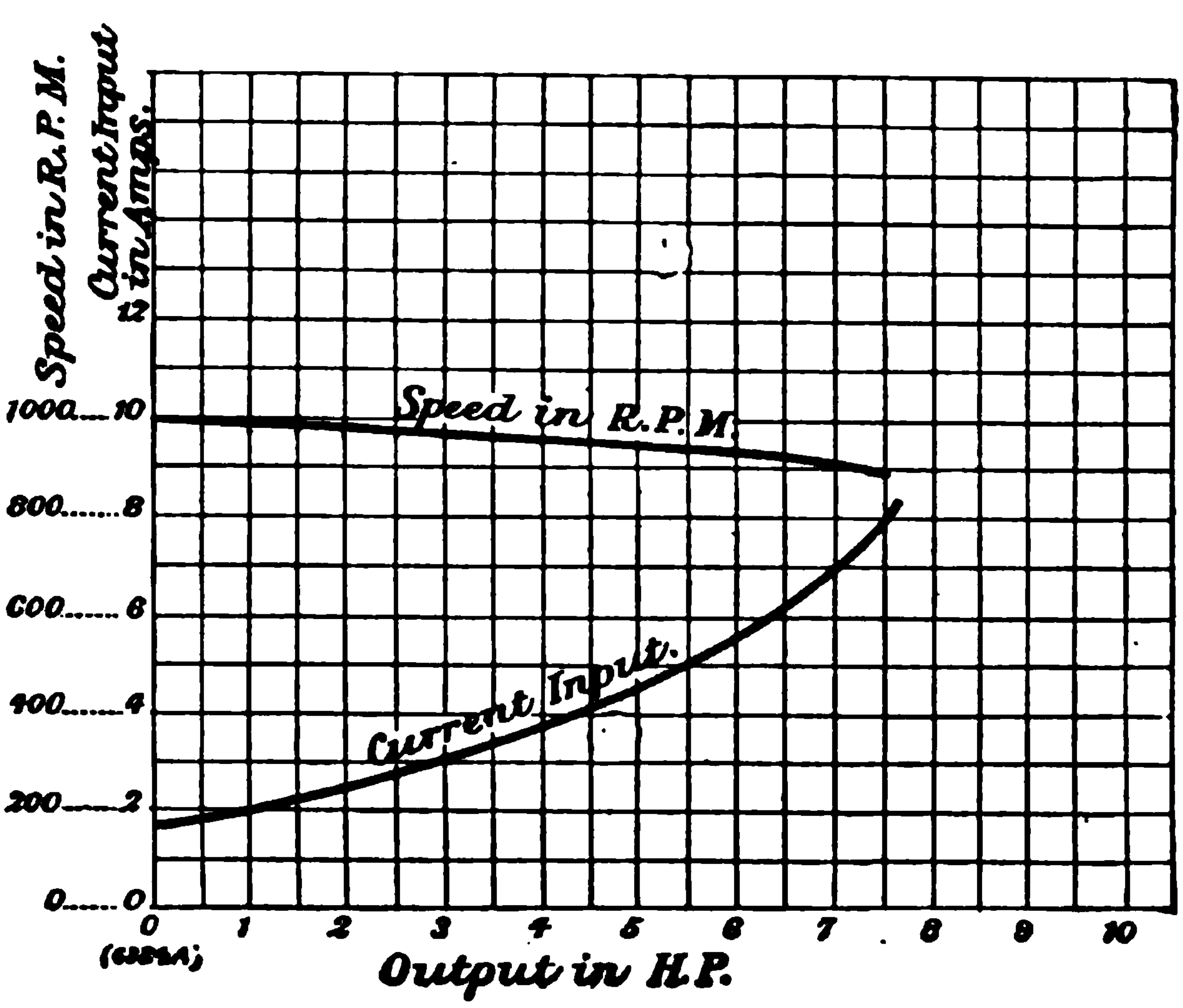


FIG. 479.—Speed, Input, and Output Curves 5 H.P. Three-Phase, 6 Pole, 50 Cycle, 400 Volt, 1000 r.p.m., Dick, Kerr & Co. Motor.

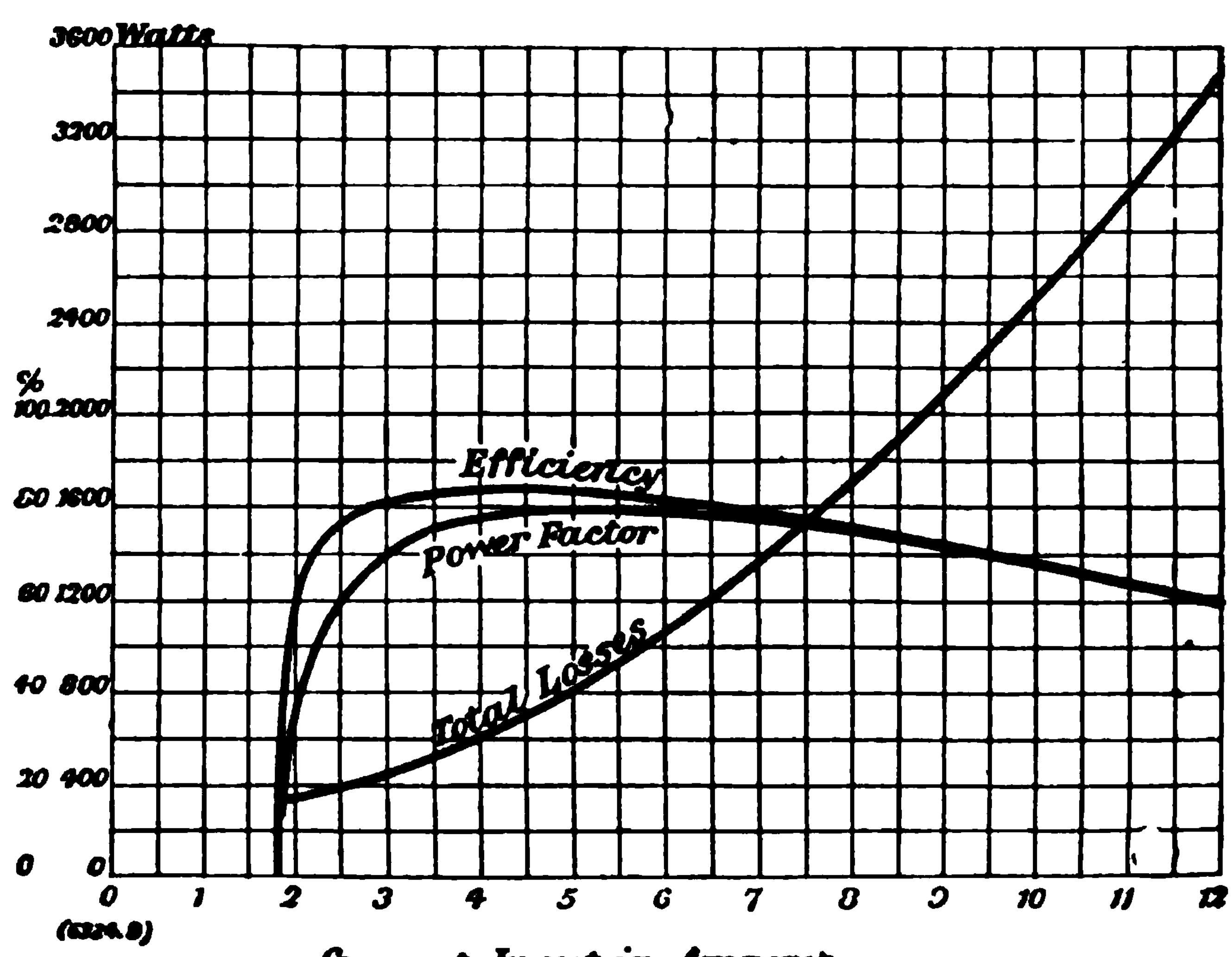


FIG. 480.—Characteristics of 5 H.P. Motor, Dick, Kerr & Co.

	Zani.	Zani.
***Stator lamination*—continued**		
Minimum width of stator tooth	0·69	1·0
Maximum width of stator tooth	1·25	1·24
Weight of stator punchings	49	1220
***Rotor lamination*—**		
Radial depth of air-gap (Δ)	·076	·16
External diameter of rotor punchings	25·25	108·68
Internal diameter of rotor punchings	17·8	86·5
Gross length between flanges, λ_g	12·7	30·5
Number of ventilating ducts	...	3
Width of each duct	...	1·3
Net length of lamination between flanges, λ_n	11·4	23·9
Number of rotor slots	54	144
Number of rotor slots per pole	9	12
Number of rotor slots per pole per phase ...	3	4
Depth of rotor slot	1·65	3·94
Width of rotor slot	·81	1·18
Width of rotor slot opening	·16	·16
Rotor tooth pitch (at air-gap)	1·47	2·37
Rotor tooth pitch (at bottom of the slots) ...	1·28	2·19
Minimum width of rotor tooth	0·47	1·01
Maximum width of rotor tooth	0·66	1·19
Weight of rotor punchings	17	515
***Stator winding*—**		
Number of stator conductors per slot	66	33 each consisting of two components
Number of stator slots	36	108
Number of stator conductors	2376	3564
Number of stator turns	1188	1782
Number of stator turns per phase	396	594
Number of stator turns per pole per phase ...	66	49·5
Dimensions of conductors (bare)	·145	Two 2·34 diam.
Cross section of conductor	·0165	·086
Mean length of one turn	75	159
Total length of stator turns per phase	29,700	94,000
Resistance per phase at 60° C.	3·6	2·2
Weight of total stator copper	13	216
***Rotor windings*—**		
Number of rotor conductors per slot	2	2
Number of rotor slots	54	144
Number of rotor conductors	108	288
Number of rotor turns	54	144
Number of phases in rotor winding	3	3
Number of rotor turns per phase	18	48

		Zani.	Zani.
Rotor windings—continued.			
Number of rotor turns per pole per phase	...	3	4
Dimensions of single conductors (bare)	...	...	1·6 × ·9
Dimensions of insulated conductor	... ...	...	1·65 × ·95
Cross section of conductor	... ...	·25	1·43
Mean length of one turn	... ...	70	150
Length of rotor turns per phase	... ...	1260	7200
Resistance per phase at 60° C.	... ...	·01	·01
Weight of rotor copper	... ...	8·4	273
Power factor and current—			
$\frac{\text{Net length of laminations}}{\text{Pole Pitch}} = \frac{\lambda_n}{\tau}$	... ...	·86	·84
Type of slot (average of stator and rotor)	...	nearly closed	nearly closed
C in Behrend's formula (from page 388)	...	13·5	13·5
$\Delta \times H =$	... ...	·57	1·68
$C' =$ (from page 420)	...	1·6	1·0
Leakage factor $\left(\sigma = CC'\frac{\Delta}{\tau}\right)$	... ...	·125	·076
Maximum power factor $= \frac{1}{1 \times 2\sigma}$	... ...	·80	·65
Watts output at full load	... ...	3730	164,000
Full load efficiency	... ...	83·5	93·5
Watts input at full load	... ...	4460	175,000
Volt amperes input at full load	... ...	5570	203,000
Volt amperes input per phase	... ...	1860	67,700
Volts per phase	... ...	400	2880
Full load ampere per stator winding	... ...	4·65	23·5
Type of connection	... ...	Δ	Y
Full load line amperes	... ...	8·0	23·5
No load current and short circuit current—			
Periodicity, cycles per second, N	... ...	50	50
Number of stator turns per phase, T	... ...	396	594
Approximate internal voltage per phase, E	...	380	2800
Megalines flux per pole from $E = 4{\cdot}2\,TNM\,10^{-8}$		·456	2·24
Per cent. exposed iron at stator surface	...	89	87
Per cent. exposed iron at rotor surface	...	·89	·88
Mean for stator and rotor (k)	... ...	·89	87·5
Exposed cross section of iron at air-gap per pole $= \lambda_n\ \tau\ k$	... ...	135	595
Spreading coefficient	... ...	1·15	1·15
Corrected air-gap cross section per pole	...	155	685
Average gap density	... ...	2940	3280
Maximum gap density	... ...	5000	5600
Ampere turns for gap per pole	... ...	304	720
Total ampere turns per pole	... ...	340	810
Ampere turns per pole per phase	... ...	170	405

	Zani.	Zani.
No load current and short circuit current—continued.		
Maximum current at no load	2·58	8·2
R.M.S. current at no load	1·82	5·8
R.M.S. current at no load in per cent. of full load current	41	24 6
$\sigma =$	·125	·076
Current corresponding to the diameter of the circle $= \frac{\text{no load current}}{\sigma}$	14·5	76·5
Ideal short circuit current	16·3	82·3
Secondary current at full load (from circle diagram for ratio 1 : 1 of transformation) ...	3·9	21·5
Number of stator conductors	2376	3564
Number of rotor conductors	108	288
Ratio of transformation	21·9	12·3
Actual secondary current	85·5	265
Losses—		
(I) I^2R loss of stator:		
Current in stator winding	4·65	23·5
Resistance of stator winding per phase ...	3·6	2·2
I^2R loss of stator per phase	78	1210
Total I^2R loss of stator	234	3630
(II) I^2R loss of rotor:		
Current in rotor winding	85·5	265
Resistance of rotor winding per phase ...	·01	·01
I^2R loss of rotor per phase	73	700
Total I^2R loss of rotor	220	2100
Slip of rotor	5·5%	1·3%
(III) Iron loss of stator:		
Minimum cross section of stator teeth per pole	47	215
Average density at these points	9700	10,400
Maximum density at these points	16,500	17,700
Depth of stator iron above teeth	3·82	14
Cross section of stator iron	87	670
Density of stator iron	5250	3350
Periodicity in cycles per second, N	50	50
Density of stator iron in kilolines, D	5·25	3·35
$\frac{DN}{100}$	2·62	1·67
Watts stator core loss per kilogramme	4	2·5
Weight of stator punchings, kilogrammes ...	49	1220
Core loss in stator iron	200	3050
(IV) Iron loss of rotor:		
Cross section of rotor teeth at narrowest point	48	288
Average density of rotor teeth at narrowest point	9500	7800
Maximum density of rotor teeth	16,200	13,200
Depth of rotor iron below slot	2·07	7
Cross section of rotor iron	47·5	335

	Zani.	Zani.
Losses—continued.		
Density in rotor iron	9600	6700
Slip	5·5%	1·3%
Weight of rotor punchings	17	515
Iron loss in rotor	10	40
Friction loss in bearings and through windage	100	2000
Efficiency—		
Variable losses (I and II)	454	5730
Constant losses (III, IV and V)	310	5090
Total losses	764	10,820
Output in watts	3730	164,000
Input in watts	4494	174,820
Full load efficiency	83·3%	93·8%[1]
Weight—		
Weight of stator copper (kilogrammes) ...	13	216
Weight of rotor copper " ...	8·4	273
Total weight of copper	21·4	489
Weight of stator laminations	49	1220
Weight of rotor laminations	17	515
Total weight of laminations	66	1735
Total weight of active material	87·4	2224
Weight of active material per horse-power output (kilogramme)	17·3	10

In a recent contribution[2] to the Proceedings of the Institution of Electrical Engineers, Dr Behn-Eschenburg proposed a very interesting formula for the estimation of σ.

It reads as follow:

$$\sigma = \frac{3}{H^2} + \frac{\Delta}{X\,H\,\tau} + \frac{6\Delta}{\tau},$$

in which

N = average number of slots per pole for stator and rotor.

X = Average width of slot opening, and σ, τ and λ have the same significance as in all preceding chapters (see Index).

This formula is more especially valuable because it shows the subdivision of the total dispersion in three components, namely:

1st. The dispersion due to the end connections.

2nd. The dispersion associated with the active length of the winding, *i.e.* the embedded portion.

3rd. The "zig-zag" dispersion over the heads of the teeth.

[1] The efficiency assumed at first was therefore not quite correct, but the small differences (0·3 per cent.) do not justify repeating the calculation.

[2] "On the magnetic dispersion in induction motors, and its influence on the design of these machines." By Dr Hans Behn-Eschenburg, of the Oerlikon Machine Works.

It is very important for a designer to obtain a clear appreciation of the relative magnitudes of these three components and the influence upon them of the general dimensions and the type of design.

The writer has tested Dr Behn-Eschenburg's formula, and finds it to give fairly good results. By means of the large amount of reliable data contained in Dr Behn-Eschenburg's paper, the writer has also tested his own method based on Behrend's formula when thrown into the form

$$\sigma = C\,C'\frac{\Delta}{\tau},$$

where C and C′ are obtained respectively from the curves of Figs. 431 and 466. This method leads to still closer results. Table LXV., page 452, sets forth the results on the motors for which Dr Behn-Eschenburg's paper contained data, together with the result on a number of other motors for which the writer has reliable experimental data. The only cases excluded are those for which the value of $\Delta \times H$ is less than 0·75, as good commercial motors ought not to have so low a value for this product, for the curve for C′ rises sharply from this point (page 420). From Fig. 431, page 387, we see the futility of designing with high values for the pole pitch τ, since this leads to high magnetic dispersion in the end connections. Fig. 466, page 420, shows that it is futile to employ an excessively small radial depth (Δ) of the air gap, unless a large number of slots per pole are used, for this will lead to a high "zig-zag" dispersion over the heads of the stator and rotor teeth.

Thus the general idea that a large polar pitch (τ) and a short air-gap (Δ) contribute very greatly to a high maximum power factor, is shown to be far from generally true. τ may safely be chosen large only when the ratio $\frac{\lambda}{\tau}$ is not thereby too greatly decreased. A small radial depth of air-gap will only be of advantage in cases where the number of slots per pole is so great as to give a reasonably high value for $\Delta \times H$. With the increasing use of low periodicities it will be much more practicable to obtain the advantages associated with high values of $\Delta \times H$, and these will be the lines on which designers will henceforth proceed in designing for high power factors.

For squirrel cage motors a third factor C″ is introduced, and Behrend's formula assumes the form $\sigma = C\,C'\,C''\,\frac{\Delta}{\tau}$.

A good average value for C″ will be found to be 0·75.

CHAPTER XVII

COMMUTATORS IN ALTERNATING CURRENT MACHINERY

§ 1. **Development.**—Various types of series-wound, single-phase commutator motors are now in process of development, and it may be expected that by the end of another six months they will have come into limited commercial use. At present considerable secrecy is necessarily observed with regard to the details of their construction, and no approach to standardisation has yet been arrived at by any of the firms engaged in their development. Such motors are at present being advocated chiefly for railway work. For stationary work, the high frequencies at present customary in power distribution undertakings will be an obstacle to their general introduction, although in small capacities they are practicable at moderately high periodicities.

§ 2. **Bibliography of Alternating Current Commutating Machinery.**—The following bibliography relates not only to single-phase commutator motors, but to the subject of the employment of commutators in alternating current machinery in general, the two subjects being practically inseparable.

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APPENDIX

ENGLISH AND METRIC EQUIVALENTS.

1 inch	= 2·54 centimetres.	1 cm.	= ·3937 inch.
1 foot	= ·3048 metre.	1 metre	= 3·2808 feet.
1 mile	= 1·6093 kilometres.	1 km.	= ·6214 mile.
1 sq. in.	= 6·4515 sq. cm.	1 sq. cm.	= ·1550 sq. in.
1 sq. ft.	= ·0929 sq. m.	1 sq. m.	= 10·7641 sq. ft.
1 cu. in.	= 16·387 cu. cm.	1 cu. cm.	= ·0610 cu. in.
1 cu. ft.	= ·0283 cu. m.	1 cu. m.	= 35·3156 cu. ft.
1° F.	= ·5° C.	1° C.	= 1·8° F.
1 English H.P.	= ·746 Kw.	1 Kw.	= 1·3404 English H.P.
1 Metric H.P.	= ·736 Kw.	1 Kw.	= 1·359 Metric H.P.
1 lb.	= ·4536 kilogramme.	1 Kgm.	= 2·205 lbs.
1000 amperes per sq. in.	= 155 amps. per sq. cm.	100 amps. per sq. cm.	= 645 amps. per sq. in.

TABLE OF THE PROPERTIES OF COPPER WIRES FOR THE PRINCIPAL GAUGES IN USE IN DIFFERENT COUNTRIES.

DIAMETERS AND AREAS.

Gauge Name.	Gauge No.	Diameter.								Cross Section.		Gauge Name.	Gauge No.
		Bare.		S.C.C.		D.C.C.		T.C.C.					
		Millimetres.	Inches.	Millimetres	Ins.	Millimetres	Ins.	Millimetres	Ins.	Sq. Cms.	Sq. Ins.		
S.W.G.	7/0	12·70	·500	...	...	...	...	13·20	·520	1·265	·196	S.W.G.	7/0
...	...	12	·473	...	...	...	...	12·5	·492	1·135	·176	...	...
S.W.G.	6/0	11·80	·464	...	...	...	...	12·30	·484	1·090	·169	S.W.G.	6/0
B. & S.	0000	11·64	·460	...	...	...	...	12·20	·480	1·070	·166	B. & S.	0000
B.W.G.	0000	11·50	·454	...	...	...	...	12·00	·474	1·045	·162	B.W.G.	0000
S.W.G.	5/0	11	·432	...	...	...	...	11·40	·452	·950	·147	S.W.G.	5/0
B.W.G.	000	10·80	·425	...	...	...	...	11·30	·445	·915	·142	B.W.G.	000
B. & S.	000	10·40	·410	...	...	...	...	11·00	·430	·850	·132	B. & S.	000
S.W.G.	4/0	10·15	·400	...	...	...	..	10·65	·420	·814	·126	S.W.G.	4/0
...	...	10	·394	...	...	...	...	10·45	·410	·787	·122	...	...
B.W.G.	00	9·65	·380	...	...	...	...	10·20	·400	·730	·113	B.W.G.	00
S.W.G.	000	9·44	·372	...	...	...	...	9·90	·392	·705	·109	S.W.G.	000
B. & S.	00	9·25	·365	...	...	...	...	9·75	·385	·680	·105	B. & S.	00
...	...	9	·354	...	...	...	...	9·45	·372	·638	·0985	...	...
S.W.G.	00	8·84	·348	...	...	...	...	9·30	·366	·614	·0951	S.W.G.	00
B.W.G.	0	8·64	·340	...	...	...	..	9·10	·358	·586	·0908	B.W.G.	0
B. & S.	0	8·30	·325	...	...	...	...	8·70	·343	·534	·0829	B. & S.	0
S.W.G.	0	8·23	·324	...	...	...	...	8·67	·342	·531	·0824	S.W.G.	0
...	...	8	·315	...	...	...	...	8·25	·333	·503	·0780	...	...
S.W.G.	1	7·62	·300	...	...	...	...	8·05	·318	·456	·0707	S.W.G.	1
B.W.G.	1	7·62	·300	...	...	...	...	8·05	·318	·456	·0707	B.W.G.	1
B. & S.	1	7·35	·289	...	...	7·70	·303	7·80	·307	·424	·0657	B. & S.	1
B.W.G.	2	7·20	·284	...	...	7·55	·298	7·65	·302	·408	·0633	B.W.G.	2
S.W.G.	2	7	·276	...	...	7·35	·290	7·45	·294	·385	·0598	S.W.G.	2
B.W.G.	3	6·60	·259	...	...	6·95	·273	7·00	·277	·340	·0527	B.W.G.	3
B. & S.	2	6·55	·258	...	..	6·90	·272	7·00	·276	·336	·0521	B. & S.	2
S.W.G.	3	6·40	·252	...	...	6·75	·266	6·85	·270	·320	·0499	S.W.G.	3
B.W.G.	4	6·05	·238	...	...	6·40	·252	6·50	·256	·288	·0445	B.W.G.	4
...	...	6	·236	...	...	6·35	·250	6·45	·254	·283	·0439	...	...
S.W.G.	4	5·90	·232	...	...	6·25	·246	6·30	·250	·274	·0423	S.W.G.	4
B. & S.	3	5·80	·229	...	...	6·15	·243	6·25	·247	·265	·0418	B. & S.	3
B.W.G.	5	5·60	·220	...	...	5·95	·234	6·00	·238	·245	·0380	B.W.G.	5
S.W.G.	5	5·40	·212	...	...	5·65	·224	5·80	·230	·228	·0353	S.W.G.	5
B. & S.	4	5·20	·204	...	...	5·50	·216	5·55	·220	·2115	·0328	B. & S.	4
B.W.G.	6	5·15	·203	...	...	5·45	·215	5·50	·219	·209	·0324	B.W.G.	6
...	...	5	·197	...	...	5·31	·209	5·35	·213	·197	·0305	...	...
S.W.G.	6	4·90	·192	...	...	5·15	·204	5·25	·208	·187	·0290	S.W.G.	6
B. & S.	5	4·63	·182	...	..	4·90	·194	5·00	·198	·1675	·0260	B. & S.	5
B.W.G.	7	4·58	·180	...	...	4·85	·192	4·95	·196	·164	·0254	B.W.G	7
S.W.G.	7	4·47	·176	...	...	4·75	·188	4·85	·192	·157	·0248	S.W.G.	7
B.W.G.	8	4·20	·165	...	...	4·50	·177	4·60	·181	·138	·0214	B.W.G.	8
B. & S.	6	4·10	·162	...	...	4·40	·174	4·50	·178	·133	·0206	B. & S.	6
S.W.G.	8	4·07	·160	...	...	4·35	·172	4·45	·176	·1295	·0201	S.W.G.	8
...	...	4	·157	...	...	4·32	·170	4·40	·174	·126	·0196	...	...

DIAMETERS AND AREAS—*continued.*

Gauge Name.	Gauge No.	Diameter. Bare. Millimetres.	Bare. Inches.	S.C.C. Millimetres	S.C.C. Ins.	D.C.C. Millimetres	D.C.C. Ins.	T.C.C. Millimetres	T.C.C. Ins.	Cross Section. Sq. Cms.	Cross Section. Sq. Ins.	Gauge Name.	Gauge No.
B.W.G.	9	3·75	·148	...	...	4·05	·160	4·15	·164	·111	·0172	B.W.G.	9
B. & S.	7	3·65	·144	...	...	3·95	·156	4·05	·160	·105	·0163	B. & S.	7
S.W.G.	9	3·65	·144	..	...	3·95	·156	4·05	·160	·105	·0163	S.W.G.	9
B.W.G.	10	3·40	·134	...	...	3·70	·146	3·80	·150	·0910	·0141	B.W.G.	10
B. & S.	8	3·25	·128	...	...	3·55	·140	3·65	·144	·0833	·0129	B. & S.	8
S.W.G.	10	3·25	·128	...	...	3·55	·140	3·65	·144	·0833	·0129	S.W.G.	10
B.W.G.	11	3·05	·120	...	...	3·35	·132	3·45	·136	·0730	·0113	B.W.G.	11
...	...	3	·118	...	...	3·28	·129	3·40	·135	·0710	·0110	...	...
S.W.G.	11	2·95	·116	...	...	3·25	·128	3·35	·132	·0685	·0106	S.W.G.	11
B. & S.	9	2·90	·114	...	...	3·20	·126	3·25	·130	·0663	·0103	B. & S.	9
B.W.G.	12	2·77	·109	...	...	3·00	·119	3·15	·124	·0601	·00933	B.W.G.	12
S.W.G.	12	2·65	·104	...	...	2·90	·114	3·00	·119	·0546	·00849	S.W.G.	12
B. & S.	10	2·60	·102	2·75	·108	2·85	·112	2·95	·116	·0513	·00815	B. & S.	10
B.W.G.	13	2·41	·0950	2·55	·101	2·65	·105	2·75	·109	·0452	·00709	B.W.G.	13
S.W.G.	13	2·34	·0920	2·50	·098	2·60	·102	2·70	·106	·0429	·00665	S.W.G.	13
B. & S.	11	2·30	·0907	2·45	·097	2·55	·101	2·65	·105	·0418	·00647	B. & S.	11
B.W.G.	14	2·11	·0836	2·25	·089	2·35	·093	2·45	·097	·0349	·00541	B.W.G.	14
B. & S.	12	2·05	·0808	2·20	·087	2·30	·091	2·40	·095	·0330	·00513	B. & S.	12
S.W.G.	14	2·03	·0800	2·20	·086	2·30	·091	2·38	·094	·0324	·00503	S.W.G.	14
...	...	2	·0788	2·15	·085	2·26	·089	2·35	·092	·0314	·00490	...	...
B.W.G.	15	1·83	·0720	2·00	·078	2·10	·082	2·20	·086	·0264	·00407	B.W.G.	15
S.W.G.	15	1·83	·0720	2·00	·078	2·10	·082	2·20	·086	·0264	·00407	S.W.G.	15
B. & S.	13	1·83	·0720	2·00	·078	2·10	·082	2·20	·086	·0264	·00407	B. & S.	13
B.W.G.	16	1·65	·0650	1·80	·071	1·90	·075	2·00	·079	·0214	·00332	B.W.G.	16
B. & S.	14	1·63	·0641	1·80	·071	1·90	·075	1·98	·078	·0208	·00323	B. & S.	14
S.W.G.	16	1·63	·0641	1·80	·070	1·90	·075	1·98	·078	·0208	·00323	S.W.G.	16
B.W.G.	17	1·47	·0580	1·60	·063	1·70	·068	1·80	·072	·0170	·00264	B.W.G.	17
B. & S.	15	1·45	·0571	1·60	·063	1·70	·068	1·78	·071	·0165	·00256	B. & S.	15
S.W.G.	17	1·42	·0560	1·55	·061	1·65	·066	1·75	·070	·01585	·00246	S.W.G.	17
B. & S.	16	1·29	·0508	1·40	·055	1·50	·059	1·60	·063	·0131	·00203	B. & S.	16
B.W.G.	18	1·245	·0490	1·36	·054	1·45	·057	1·55	·061	·0122	·00189	B.W.G.	18
S.W.G.	18	1·22	·0480	1·346	·053	1·42	·056	1·52	·060	·0117	·00181	S.W.G.	18
B. & S.	17	1·15	·0453	1·243	·049	1·346	·053	1·48	·057	·0104	·00161	B. & S.	17
B.W.G.	19	1·065	·0420	1·192	·047	1·270	·050	1·37	·054	·00898	·00139	B.W.G.	19
B. & S.	18	1·023	·0403	1·118	·044	1·220	·048	1·32	·052	·00826	·00128	B. & S.	18
S.W.G.	19	1·017	·0400	1·142	·045	1·220	·048	1·32	·052	·00814	·00126	S.W.G.	19
...	...	1·0	·0393	1·122	·044	1·19	·047	1·30	·051	·00794	·00123	...	...
S.W.G.	20	·915	·0360	1·017	·040	1·118	·044	1·22	·048	·00659	·00102	S.W.G.	20
B. & S.	19	·910	·0358	·990	·039	1·118	·044	1·19	·047	·00659	·00102	B. & S.	19
...	...	·9	·0354	·990	·039	1·09	·043	1·18	·047	·00635	·000985	...	...
B.W.G.	20	·890	·0350	·990	·039	1·090	·043	1·17	·046	·00622	·000961	B.W.G.	20
S.W.G.	21	·8125	·0320	·915	·036	1·015	·040	1·11	·044	·00520	·000805	S.W.G.	21
B.W.G.	21	·8125	·0320	·915	·036	1·015	·040	1·11	·044	·00520	·000805	B.W.G.	21
B. & S.	20	·8125	·0320	·915	·036	1·015	·040	1·11	·044	·00520	·000805	B. & S.	20
...	...	·8	·0315	·890	·035	·990	·039	1·09	·043	·00502	·000780	...	...
B. & S.	21	·7240	·0285	·825	·032	·915	·036	1·02	·040	·00411	·000639	B. & S.	21
B.W.G.	22	·7110	·0280	·812	·032	·915	·036	1·01	·040	·00398	·000618	B.W.G.	22
S.W.G.	22	·7110	·0280	·812	·032	·915	·036	1·01	·040	·00398	·000618	S.W.G.	22
...	...	·7	·0276	·801	·032	·890	·035	1·00	·039	·00387	·000600	...	...
B. & S.	22	·6425	·0253	·736	·029	·838	·033	·940	·037	·00325	·000502	B. & S.	22
B.W.G.	23	·6350	·0250	·736	·029	·838	·033	·930	·037	·00317	·000491	B.W.G.	23
S.W.G.	23	·6100	·0240	·711	·028	·812	·032	·914	·036	·00293	·000453	S.W.G.	23
...	...	·6	·0236	·701	·028	·812	·032	·900	·035	·00283	·000438	...	...
B. & S.	23	·5749	·0226	·686	·027	·786	·031	·890	·035	·00259	·000402	B. & S.	23
B.W.G.	24	·5590	·0220	·660	·026	·762	·030	·862	·034	·00245	·000380	B.W.G.	24
S.W.G.	24	·5590	·0220	·660	·026	·762	·030	·862	·034	·00245	·000380	S.W.G.	24

DIAMETERS AND AREAS—*continued.*

Gauge Name.	Gauge No.	Diameter. Bare. Milli-metres.	Bare. Inches.	S.C.C. Milli-metres	S.C.C. Ins.	D.C.C. Milli-metres	D.C.C. Ins.	T.C.C. Milli-metres	T.C.C. Ins.	Cross Section. Sq. Cms.	Cross Section. Sq. Ins.	Gauge Name.	Gauge No.
B. & S.	24	·5100	·0201	·610	·024	·710	·028	·812	·032	·00204	·000317	B. & S.	24
B.W.G.	25	·5080	·0200	·610	·024	·710	·028	·812	·032	·00203	·000315	B.W.G.	25
S.W.G.	25	·5080	·0200	·610	·024	·710	·028	·812	·032	·00203	·000315	S.W.G.	25
...	...	·5	·0197	·602	·024	·710	·028	·790	·031	·00196	·000305	...	...
B.W.G.	26	·4570	·0180	·559	·022	·660	·026	·762	·030	·001645	·000255	B.W.G.	26
S.W.G.	26	·4570	·0180	·559	·022	·660	·026	·762	·030	·001645	·000255	S.W.G.	26
B. & S.	25	·4550	·0179	·559	·022	·660	·026	·762	·030	·001625	·000252	B. & S.	25
S.W.G.	27	·4160	·0164	·508	·020	·610	·024	..	...	·00136	·000211	S.W.G.	27
B.W.G.	27	·4060	·0160	·508	·020	·610	·024	...	...	·00130	·000201	B.W.G.	27
B. & S.	26	·4040	·0159	·508	·020	·610	·024	...	...	·00128	·000198	B. & S.	26
...	...	·4	·0157	·504	·020	·610	·024	...	...	·00125	·000194	...	...
S.W.G.	28	·3760	·0148	·483	·019	·585	·023	...	...	·00111	·000172	S.W.G.	28
B. & S.	27	·3605	·0142	·457	·018	·559	·022	...	...	·00102	·000158	B. & S.	27
B.W.G.	28	·3560	·0140	·457	·018	·559	·022	...	...	·000994	·000154	B.W.G.	28
S.W.G.	29	·3460	·0136	·457	·018	·559	·022	...	...	·000932	·000145	S.W.G.	29
B.W.G.	29	·3300	·0130	·432	·017	·533	·021	...	...	·000852	·000132	B.W.G.	29
B. & S.	28	·3200	·0126	·432	·017	·533	·021	...	...	·000806	·000125	B. & S.	28
S.W.G.	30	·3150	·0124	·432	·017	·533	·021	...	...	·000780	·000121	S.W.G.	30
B.W.G.	30	·3050	·0120	·417	·016	·508	·020	...	...	·000730	·000113	B.W.G.	30
B. & S.	29	·3	·0118	·381	·015	·508	·020	...	...	·000705	·000109	B. & S.	29
S.W.G.	31	·2950	·0116	·407	·016	·508	·020	...	...	·000685	·000106	S.W.G.	31
S.W.G.	32	·2740	·0109	·381	·015	·482	·019	...	...	·000590	·0000918	S.W.G.	32
B. & S.	30	·2540	·0100	·356	·014	·457	·018	...	...	·000508	·0000787	B. & S.	30
B.W.G.	31	·2540	·0100	·356	·014	·457	·018	...	...	·000508	·0000787	B.W.G.	31
S.W.G.	33	·2540	·0100	·356	·014	·457	·018	...	...	·000508	·0000787	S.W.G.	33
S.W.G.	34	·2340	·00920	·330	·013	...	...	...	...	·000429	·0000665	S.W.G.	34
B.W.G.	32	·2280	·00900	·318	·0125	...	...	...	...	·000410	·0000636	B.W.G.	32
B. & S.	31	·2270	·00893	·318	·0125	...	...	...	...	·000404	·0000626	B. & S.	31
S.W.G.	35	·2138	·00840	·305	·012	...	...	...	...	·000357	·0000554	S.W.G.	35
B.W.G.	33	·2030	·00800	·292	·0115	...	...	...	...	·000324	·0000503	B.W.G.	33
B. & S.	32	·2020	·00795	·292	·0115	..	...	...	...	·000320	·0000496	B. & S.	32
...	...	·2	·00788	·290	·0114	...	...	...	...	·000314	·0000490	...	...
S.W.G.	36	·1930	·00760	·280	·0110	...	...	...	..	·000292	·0000454	S.W.G.	36
B. & S.	33	·1800	·00708	·266	·0105	...	...	...	...	·000254	·0000394	B. & S.	33
B.W.G.	34	·1780	·00700	·254	·0100	...	...	...	...	·000248	·0000385	B.W.G.	34
S.W.G.	37	·1730	·00680	·254	·0100	...	...	...	...	·000234	·0000363	S.W.G.	37
B. & S.	34	·1600	·00631	·249	·0098	...	...	...	...	·000201	·0000312	B. & S.	34
S.W.G.	38	·1525	·00600	·241	·0095	...	...	...	...	·000182	·0000283	S.W.G.	38
B. & S.	35	·1430	·00562	·218	·0086	...	...	...	...	·00160	·0000248	B. & S.	35
S.W.G.	39	·1320	·00520	·216	·0085	·280	·011	...	...	·000137	·0000212	S.W.G.	39
B. & S.	36	·1270	·00500	·203	·0080	·280	·011	...	...	·000126	·0000196	B. & S.	36
B.W.G.	35	·1270	·00500	·203	·0080	·280	·011	...	...	·000126	·0000196	B.W.G.	35
S.W.G.	40	·1220	·00480	·203	·0080	...	...	..	...	·000117	·0000181	S.W.G.	40
B. & S.	37	·1130	·00445	·190	·0075	...	...	...	...	·000101	·0000156	B. & S.	37
S.W.G.	41	·1120	·00440	·190	·0075	...	...	...	...	·000098	·0000152	S.W.G.	41
B.W.G.	36	·1016	·00400	·177	·0070	...	...	...	...	·000081	·0000126	B.W.G.	36
S.W.G.	42	·1016	·00400	·177	·0070	...	...	...	...	·000081	·0000126	S.W.G.	42
B. & S.	38	·1010	·00397	...	...	...	...	...	...	·000079	·0000123	B. & S.	38
...	...	·1	·00393	...	...	...	...	...	...	·000079	·0000123	...	...
S.W.G.	43	·0915	·00360	...	...	...	...	...	...	·0000657	·0000102	S.W.G.	43
B. & S.	39	·0895	·00353	...	...	...	...	...	...	·0000631	·00000979	B. & S.	39
S.W.G.	44	·0813	·00320	...	...	...	...	...	...	·0000518	·00000804	S.W.G.	44
B. & S.	40	·0800	·00315	...	...	...	...	...	...	·0000500	·00000776	B. & S.	40
S.W.G.	45	·0711	·00280	...	...	...	...	...	...	·0000398	·00000616	S.W.G.	45
S.W.G.	46	·0610	·00240	...	...	...	...	...	...	·0000292	·00000452	S.W.G.	46
S.W.G.	47	·0509	·00200	...	...	...	...	...	...	·0000200	·00000314	S.W.G.	47
S.W.G.	48	·0406	·00160	...	...	...	...	...	...	·0000130	·00000201	S.W.G.	48
S.W.G.	49	·0305	·00120	...	...	...	...	...	...	·0000073	·00000113	S.W.G.	49
S.W.G.	50	·0254	·00100	...	...	...	...	...	...	·0000050	·000000785	S.W.G.	50

RESISTANCES.

Gauge Name.	Gauge No.	Ohms. 0° Cent. Km.	0° Cent. 1000′	20° Cent. Km.	20° Cent. 1000′	40° Cent. Km.	40° Cent. 1000′	60° Cent. Km.	60° Cent. 1000′	80° Cent. Km.	80° Cent. 1000′	100° Cent. Km.	100° Cent. 1000′	Gauge Name.	Gauge No.
S.W.G.	7/0	·1260	·0383	·1360	·0414	·1460	·0446	·1580	·0480	·1690	·0515	·1780	·0545	S.W.G.	7/0
12 mm.	...	·1415	·0431	·153	·0466	·165	·0503	·177	·0539	·19	·058	·202	·0614	12 mm.	...
S.W.G.	6/0	·1460	·0445	·158	·0480	·1700	·0518	·1820	·0556	·1960	·0597	·2050	·0631	S.W.G.	6/0
B. & S.	0000	·1480	·0452	·1600	·0489	·1720	·0526	·1860	·0565	·1980	·0606	·2100	·0643	B. & S.	0000
B.W.G.	0000	·1520	·0464	·1650	·0502	·1780	·0542	·1920	·0583	·2050	·0622	·2150	·0660	B.W.G.	0000
S.W.G.	5/0	·1660	·0512	·1820	·0553	·1960	·0597	·2100	·0641	·226	·0687	·2400	·0728	S.W.G.	5/0
B.W.G.	000	·1730	·0529	·1880	·0573	·2045	·0619	·2180	·0665	·2330	·0710	·2450	·0751	B.W.G.	000
B. & S.	000	·1870	·0570	·2050	·0617	·2175	·0663	·2340	·0718	·2500	·0764	·2650	·0812	B. & S.	000
S.W.G.	4/0	·1960	·0597	·2100	·0645	·2275	·0696	·2450	·0747	·2650	·0802	·2800	·0850	S.W.G.	4/0
10 mm.	...	·2035	·062	·22	·0671	·237	·0722	·254	·0775	·273	·0832	·29	·0885	10 mm.	...
B.W.G.	00	·2170	·0663	·2350	·0717	·255	·0775	·2720	·0833	·2900	·0888	·3100	·0943	B.W.G.	00
S.W.G.	000	·2260	·0690	·2450	·0746	·265	·0805	·2840	·0865	·3050	·0929	·3200	·0983	S.W.G.	000
B. & S.	00	·2350	·0720	·2550	·0778	·275	·0837	·2950	·0900	·3150	·0963	·3350	·102	B. & S.	00
9 mm.	...	·252	·0767	·271	·0827	·292	·089	·314	·0959	·337	·103	·358	·109	9 mm.	...
S.W.G.	00	·2590	·0789	·2800	·0852	·300	·0920	·3240	·0988	·3500	·106	·3600	·112	S.W.G.	00
B.W.G.	0	·2700	·0820	·2950	·0896	·320	·0970	·3400	·104	·3600	·111	·3850	·118	B.W.G.	0
B. & S.	0	·2990	·0909	·3200	·0981	·350	·106	·3700	·113	·4000	·122	·4250	·129	B. & S.	0
S.W.G.	0	·2990	·0910	·3200	·0984	·350	·106	·3750	·114	·4000	·122	·4250	·130	S.W.G.	0
8 mm.	...	·319	·0973	·344	·1048	·372	·1133	·399	·1215	·429	·131	·455	·1385	8 mm.	...
S.W.G.	1	·350	·107	·3750	·115	·405	·124	·4350	·133	·4675	·1425	·4975	·1515	S.W.G.	1
B.W.G.	1	·350	·107	·3750	·115	·405	·124	·4350	·133	·4675	·1425	·4975	·1515	B.W.G.	1
B. & S.	1	·3770	·115	·4050	·124	·440	·134	·4700	·144	·5000	·153	·5300	·162	B. & S.	1
B.W.G	2	·3900	·119	·4200	·128	·450	·138	·4850	·148	·5200	·159	·5550	·169	B.W.G.	2
S.W.G.	2	·4150	·126	·4450	·136	·480	·147	·5150	·157	·5550	·169	·5900	·179	S.W.G.	2
B.W.G.	3	·470	·143	·5050	·154	·545	·166	·5900	·179	·6300	·191	·6700	·204	B.W.G.	3
B. & S.	2	·473	·144	·5100	·156	·550	·168	·5900	·180	·6350	·193	·6700	·204	B. & S.	2
S.W.G.	3	·500	·151	·5350	·163	·580	·176	·6200	·189	·6600	·202	·7000	·214	S.W.G.	3
B.W.G.	4	·555	·169	·6000	·183	·650	·197	·7000	·212	·7400	·226	·7900	·240	B.W.G.	4
6 mm.	...	·564	·172	·61	·186	·657	·201	·705	·215	·757	·231	·803	·245	6 mm.	...
S.W.G.	4	·585	·178	·6300	·192	·680	·207	·7300	·222	·7800	·238	·8300	·252	S.W.G.	4
B. & S.	3	·600	·182	·650	·197	·695	·212	·7500	·228	·8000	·244	·8500	·258	B. & S.	3
B.W.G.	5	·650	·198	·7000	·214	·760	·231	·8100	·248	·8700	·265	·9300	·282	B.W.G.	5
S.W.G.	5	·700	·213	·7550	·230	·810	·245	·8700	·266	·9400	·286	·9900	·302	S.W.G.	5
B. & S.	4	·755	·230	·810	·248	·880	·268	·945	·288	1·010	·307	1·070	·325	B. & S.	4
B.W.G.	6	·760	·232	·820	·251	·890	·271	·960	·291	1·020	·311	1·080	·330	B.W.G.	6
5 mm.	...	·810	·247	·875	·267	·940	·287	1·010	·308	1·090	·334	1·150	·351	5 mm.	...
S.W.G.	6	·850	·260	·920	·280	·990	·302	1·060	·324	1·140	·348	1·200	·368	S.W.G.	6
B. & S.	5	·950	·290	1·03	·313	1·10	·337	1·19	·362	1·28	·387	1·34	·410	B. & S.	5
B.W.G.	7	·970	·296	1·05	·320	1·14	·346	1·22	·371	1·30	·396	1·38	·421	B.W.G.	7
S.W.G.	7	1·02	·310	1·10	·334	1·18	·360	1·27	·387	1·36	·415	1·44	·440	S.W.G.	7
B.W.G.	8	1·16	·352	1·25	·380	1·34	·410	1·45	·441	1·54	·471	1·64	·500	B.W.G.	8
B. & S.	6	1·20	·365	1·30	·394	1·40	·425	1·50	·456	1·64	·488	1·69	·516	B. & S.	6
S.W.G.	8	1·23	·374	1·32	·404	1·42	·435	1·54	·468	1·65	·502	1·74	·532	S.W.G.	8
4 mm.	...	1·27	·389	1·37	·419	1·48	·451	1·59	·485	1·71	·522	1·81	·551	4 mm.	...
B.W.G.	9	1·44	·437	1·56	·473	1·68	·511	1·80	·549	1·92	·585	2·05	·621	B.W.G.	9
B. & S.	7	1·51	·460	1·64	·497	1·76	·535	1·89	·576	2·03	·617	2·15	·655	B. & S.	7
S.W.G.	9	1·51	·460	1·64	·497	1·76	·535	1·89	·576	2·03	·617	2·15	·655	S.W.G.	9
B.W.G.	10	1·76	·534	1·90	·577	2·05	·624	2·20	·670	2·35	·714	2·50	·760	B.W.G.	10
B. & S.	8	1·91	·582	2·05	·628	2·22	·677	2·39	·728	2·56	·780	2·70	·828	B. & S.	8

RESISTANCES—*continued.*

Gauge Name.	Gauge No.	Ohms. 0° Cent. Km.	0° Cent. 1000′	20° Cent. Km.	20° Cent. 1000′	40° Cent. Km.	40° Cent. 1000′	60° Cent. Km.	60° Cent. 1000′	80° Cent. Km.	80° Cent. 1000′	100° Cent. Km.	100° Cent. 1000′	Gauge Name.	Gauge No.
S.W.G.	10	1·91	·582	2·05	·628	2·22	·677	2·39	·728	2·56	·780	2·70	·828	S.W.G.	10
B.W.G.	11	2·20	·665	2·35	·719	2·55	·776	2·75	·834	2·93	·890	3·10	·945	B.W.G.	11
3 mm.	...	2·26	·688	2·44	·745	2·63	·801	2·81	·860	3·03	·925	3·21	·980	3 mm.	...
S.W.G.	11	2·30	·710	2·50	·766	2·70	·827	2·90	·888	3·15	·954	3·30	1·01	S.W.G.	11
B. & S.	9	2·40	·732	2·60	·791	2·80	·852	3·00	·915	3·20	·979	3·40	1·04	B. & S.	9
B.W.G.	12	2·65	·806	2·85	·872	3·10	·942	3·30	1·01	3·55	1·08	3·75	1·15	B.W.G.	12
S.W.G.	12	2·90	·885	3·15	·956	3·40	1·03	3·65	1·11	3·90	1·19	4·15	1·26	S.W.G.	12
B. & S.	10	3·00	·924	3·25	·997	3·55	1·08	3·80	1·16	4·10	1·24	4·35	1·32	B. & S.	10
B.W.G.	13	3·50	1·07	3·75	1·15	4·05	1·24	4·35	1·33	4·65	1·42	5·00	1·53	B.W.G.	13
S.W.G.	13	3·70	1·13	4·00	1·22	4·30	1·31	4·65	1·41	4·95	1·51	5·25	1·60	S.W.G.	13
B. & S.	11	3·85	1·17	4·15	1·26	4·45	1·36	4·80	1·47	5·10	1·56	5·40	1·65	B. & S.	11
B.W.G.	14	4·55	1·39	4·90	1·50	5·30	1·62	5·70	1·74	6·10	1·86	6·50	1·98	B.W.G.	14
B. & S.	12	4·80	1·47	5·20	1·59	5·60	1·71	6·05	1·84	6·40	1·96	6·80	2·06	B. & S.	12
S.W.G.	14	4·90	1·50	5·30	1·62	5·72	1·75	6·15	1·87	6·55	2·00	7·00	2·13	S.W.G.	14
2 mm.	...	5·09	1·55	5·48	1·66	5·88	1·80	6·32	1·93	6·80	2·08	7·22	2·21	2 mm.	...
B.W.G.	15	6·10	1·85	6·55	2·00	7·10	2·16	7·60	2·32	8·10	2·47	8·60	2·63	B.W.G.	15
S.W.G.	15	6·10	1·85	6·55	2·00	7·10	2·16	7·60	2·32	8·10	2·47	8·60	2·63	S.W.G.	15
B. & S.	13	6·10	1·85	6·55	2·00	7·10	2·16	7·60	2·32	8·10	2·47	8·60	2·63	B. & S.	13
B.W.G.	16	7·50	2·27	8·05	2·45	8·60	2·64	9·30	2·84	10·00	3·03	10·60	3·23	B.W.G.	16
B. & S.	14	7·70	2·34	8·30	2·52	8·90	2·72	9·60	2·92	10·20	3·13	10·80	3·32	B. & S.	14
S.W.G.	16	7·70	2·34	8·30	2·52	8·90	2·72	9·60	2·92	10·20	3·13	10·80	3·32	S.W.G.	16
B.W.G.	17	9·35	2·85	10·10	3·08	10·90	3·32	11·70	3·57	12·50	3·81	13·30	4·06	B.W.G.	17
B. & S.	15	9·70	2·94	10·50	3·18	11·20	3·42	12·10	3·68	13·00	3·94	13·70	4·18	B. & S.	15
S.W.G.	17	10·00	3·06	10·50	3·30	11·70	3·56	12·50	3·82	13·40	4·10	14·20	4·35	S.W.G.	17
B. & S.	16	12·20	3·72	13·20	4·01	14·2	4·33	15·20	4·65	16·20	4·96	17·20	5·25	B. & S.	16
B.W.G.	18	13·00	3·99	14·00	4·31	15·30	4·66	16·40	5·00	17·60	5·34	18·60	5·68	B.W.G.	18
S.W.G.	18	13·60	4·15	14·75	4·49	15·90	4·84	17·05	5·20	18·30	5·58	19·40	5·91	S.W.G.	18
B. & S.	17	15·40	4·70	16·60	5·06	17·90	5·45	19·30	5·87	20·6	6·26	21·7	6·62	B. & S.	17
B.W.G.	19	17·80	5·43	19·3	5·87	20·8	6·34	22·8	6·80	23·8	7·27	25·4	7·72	B.W.G.	19
B. & S.	18	19·4	5·90	20·9	6·37	22·6	6·87	24·2	7·38	25·9	7·89	27·4	8·35	B. & S.	18
S.W.G.	19	19·6	5·97	21·2	6·45	22·9	6·96	24·5	7·47	26·3	8·01	27·9	8·5	S.W.G.	19
1·0 mm.	...	20·4	6·20	22·0	6·71	23·7	7·22	25·4	7·75	27·3	8·32	29·0	8·85	1·0 mm.	...
S.W.G.	20	24·2	7·37	26·2	7·96	28·20	8·6	30·28	9·23	32·5	9·9	34·48	10·5	S.W.G.	20
B. & S.	19	24·5	7·45	26·4	8·04	28·44	8·66	30·5	9·3	32·6	9·95	34·8	10·6	B. & S.	19
·9 mm.	...	25	7·6	27	8·3	29·3	8·95	31·4	9·6	33·8	10·3	35·7	10·9	·9 mm.	...
B.W.G.	20	25·6	7·8	27·75	8·45	29·9	9·13	32·2	9·8	34·48	10·5	36·4	11·1	B.W.G.	20
S.W.G.	21	30·7	9·35	33·1	10·1	35·8	10·9	38·4	11·7	41·2	12·5	43·60	13·3	S.W.G.	21
B.W.G.	21	30·7	9·35	33·1	10·1	35·8	10·9	38·4	11·7	41·2	12·5	44·3	13·5	B.W.G.	21
B. & S.	20	30·8	9·39	33·1	10·1	35·8	10·9	38·4	11·7	41·2	12·6	44·0	13·4	B. & S.	20
·8 mm.	...	31·6	9·7	34·3	10·5	37·2	11·4	39·8	12·1	42·8	13·2	45·0	13·8	·8 mm.	...
B. & S.	21	38·7	11·8	42·0	12·8	45·2	13·7	48·5	14·8	51·8	15·8	55·1	16·8	B. & S.	21
B.W.G.	22	40·05	12·2	43·4	13·2	46·7	14·2	50·2	15·3	53·8	16·4	57·1	17·4	B.W.G.	22
S.W.G.	22	40·05	12·2	43·4	13·2	46·7	14·2	50·2	15·3	53·8	16·4	56·8	17·3	S.W.G.	22
·7 mm.	...	41·2	12·6	44·5	13·6	48·2	14·7	51·7	15·7	55·5	17·0	58·6	17·8	·7 mm.	...
B. & S.	22	49·0	15·1	53·2	16·3	57·5	17·6	62·0	18·9	65·8	20·0	69·5	21·2	B. & S.	22
B.W.G.	23	50·3	15·4	54·5	16·6	58·8	17·9	63·0	19·2	67·5	20·5	71·8	21·9	B.W.G.	23
S.W.G.	23	54·3	16·6	58·75	17·9	63·5	19·3	67·9	20·7	72·8	22·2	77·0	23·5	S.W.G.	23
·6 mm.	...	56·2	17·2	61·0	18·3	66·0	20·1	70·8	21·6	75·8	23·2	80·0	24·5	·6 mm.	...
B. & S.	23	61·6	18·8	66·5	20·3	72·0	21·9	77·4	23·6	82·6	25·2	87·5	26·7	B. & S.	23
B.W.G.	24	65·0	19·8	70·5	21·4	75·8	23·1	81·3	24·8	87·1	26·5	92·5	28·2	B.W.G.	24
S.W.G.	24	65·0	19·8	70·5	21·4	75·8	23·1	81·3	24·8	87·1	26·6	92·5	28·2	S.W.G.	24
B. & S.	24	78·0	23·7	84·5	25·6	91·0	27·6	97·0	29·6	104·0	31·7	110·0	33·6	B. & S.	24
B.W.G.	25	78·5	24·0	84·6	25·8	92·0	28·0	98·4	30·0	105·2	32·0	112·0	34·0	B.W.G.	25

RESISTANCES—*continued.*

Gauge Name.	Gauge No.	Ohms.												Gauge Name.	Gauge No.
		0° Cent.		20° Cent.		40° Cent.		60° Cent.		80° Cent.		100° Cent.			
		Km.	1000′	Km.	1000′	Km.	1000′	Km.	1000′	Km.	1000′	Km.	1000′		
S.W.G.	25	78·5	24·0	84·6	25·8	92·0	28·0	98·4	30·0	105·0	32·0	112·0	34·0	S.W.G.	25
·5 mm.	...	81·3	24·8	88·0	26·9	95	29	102	31·1	109·0	33·2	116·0	35·5	·5 mm.	...
B.W.G.	26	96·8	29·6	105·0	32·0	113	34·5	121	37·0	130·0	39·6	138·0	42·0	B.W.G.	26
S.W.G.	26	96·8	29·6	105·0	32·0	113	34·5	121	37·0	130·0	39·6	138·0	42·0	S.W.G.	26
B. & S.	25	98·0	29·9	106·0	32·3	114	34·8	123·0	37·4	131·0	40·0	139·0	42·0	B. & S.	25
S.W.G.	27	117	35·6	126·0	38·4	136	41·5	146	44·5	157	47·7	155	50·5	S.W.G.	27
B.W.G.	27	123·1	37·4	132·8	40·5	143·2	43·7	154	47·0	164·4	50·1	174·2	53·2	B.W.G.	27
B. & S.	26	124·3	37·8	134·0	40·8	144·3	44·0	155	47·2	165·0	50·5	175·5	53·5	B. & S.	26
·4 mm.	...	128	39	137·0	42·2	149	45·5	160	48·8	172·0	52·2	182·0	55·4	·4 mm.	...
S.W.G.	28	145·0	43·6	154·2	47·1	167	50·9	179	54·6	192·0	58·6	203·5	62·0	S.W.G.	28
B. & S.	27	156·0	47·5	168·5	51·4	182	55·3	195	59·4	209	63·6	221·0	67·5	B. & S.	27
B.W.G.	28	161·0	48·8	173·0	52·8	187	57·0	201	61·3	214·0	65·4	228	69·4	B.W.G.	28
S.W.G.	29	169·9	51·7	185	55·1	198	60·3	212·3	64·8	223·0	69·5	241·0	73·5	S.W.G.	29
B.W.G.	29	187·0	56·6	204	61·3	217	66·2	233	71·1	248·5	75·9	264·0	80·5	B.W.G.	29
B. & S.	28	197·0	60·0	214	64·7	230	69·7	246	75·0	263·0	80·2	279	85·0	B. & S.	28
S.W.G.	30	204·0	62·1	220	67·1	233	72·5	254·5	77·8	274·0	83·5	290·0	88·5	S.W.G.	30
B.W.G.	30	218·0	66·4	236	71·9	255	77·6	273	83·4	294·0	89·0	310·0	94·5	B.W.G.	30
B. & S.	29	227	69	245	75	264	81·0	285	87	305	93	320	98	B. & S.	29
S.W.G.	31	233	71·0	251	76·6	271	82·7	291	88·8	313	95·3	330	101	S.W.G.	31
S.W.G.	32	270	82·0	292	88·5	314	95·5	338	103	361·0	110	384	117	S.W.G.	32
B. & S.	30	314	95·5	338	103	368·0	112·0	390	119	420	128	446	136	B. & S.	30
B.W.G.	31	314	95·5	338	103	368	112	390	119	420	128	446	136	B.W.G.	31
S.W.G.	33	314	95·5	338	103	368	112	390	119	420	128	446	136	S.W.G.	33
S.W.G.	34	372	113	402	122	432	131	462	141	496	151	525	160	S.W.G.	34
B.W.G.	32	388	118	422	128	453	138	486	148	518	158	552	168	B.W.G.	32
B. & S.	31	395	120	427	130	459	140	496	151	528	161	562	171	B. & S.	31
S.W.G.	35	444	135	481	146	517	157	554	169	598	181	633	192	S.W.G.	35
B W.G.	33	493	150	531	162	574	175	617	188	665	200	702	214	B.W.G.	33
B. & S.	32	500	152	539	164	580	177	624	190	668	208	709	216	B. & S.	32
·2 mm.	...	510	156	548	167	594	182	64	195	680	207	720	220	·2 mm.	...
S.W.G.	36	545	166	587	179	638	193	685	207	738	222	780	235	S.W.G.	36
B. & S.	33	627	191	679	207	732	223	787	240	840	256	893	272	B. & S.	33
B.W.G.	34	640	195	692	211	748	228	804	245	860	262	913	278	B.W.G.	34
S.W.G.	37	680	207	734	223	787	240	850	258	910	277	965	294	S.W.G.	37
B. & S.	34	795	242	855	261	925	282	990	302	1060	323	1120	342	B. & S.	34
S.W.G.	38	874	266	940	287	1018	310	1089	332	1168	356	1240	378	S.W.G.	38
B. & S.	35	998	304	1076	328	1160	354	1245	380	1335	407	1414	431	B. & S.	35
S.W.G.	39	1165	355	1257	383	1354	413	1456	444	1562	476	1650	504	S.W.G.	39
B. & S.	36	1257	383	1360	414	1465	445	1572	480	1690	513	1788	545	B. & S.	36
B.W.G.	35	1257	383	1360	414	1465	446	1572	480	1690	513	1788	545	B.W.G.	35
S.W.G.	40	1360	415	1470	448	1587	483	1700	519	1830	557	1935	590	S.W.G.	40
B. & S.	37	1590	484	1713	522	1850	564	1983	605	2122	647	2250	686	B. & S.	37
S.W.G.	41	1620	494	1750	533	1880	575	2025	618	2178	663	2300	701	S.W.G.	41
B.W.G.	36	1955	595	2121	645	2282	696	2459	749	2628	801	2785	846	B.W.G.	36
S.W.G.	42	1630	497	2121	645	2280	695	2460	747	2628	801	2785	850	S.W.G.	42
B. & S.	38	2005	610	2160	659	2350	710	2510	762	2680	815	2840	865	B. & S.	38
·1 mm.	...	204	620	2200	671	2370	722	2540	775	2730	832	2900	885	·1 mm.	...
S.W.G.	43	2420	738	2590	796	2820	860	3023	922	3245	990	3430	1050	S.W.G.	43
B. & S.	39	2530	770	2728	830	2940	895	3164	965	3380	1030	3579	1090	B. & S.	39
S.W.G.	44	3070	935	3325	1010	3579	1090	3840	1170	4120	1250	4360	1330	S.W.G.	44
B. & S.	40	3180	970	3449	1050	3710	1130	3970	1210	4256	1300	4520	1380	B. & S.	40
S.W.G.	45	4000	1220	4330	1320	4660	1420	5020	1530	5380	1640	5710	1740	S.W.G.	45
S.W.G.	46	5450	1660	5900	1790	6380	1930	6890	2070	7320	2200	7700	2360	S.W.G.	46
S.W.G.	47	7900	2390	8530	2580	9190	2780	9800	2984	10550	3200	11150	3400	S.W.G.	47
S.W.G.	48	12250	3740	13280	4040	14270	4350	15380	4680	16450	5020	17400	5310	S.W.G.	48
S.W.G.	49	21800	6640	23590	7170	25400	7740	27390	8310	29400	8920	31000	9450	S.W.G.	49
S.W.G.	50	31300	9530	33950	10300	36600	11100	39400	11900	42200	12800	44300	13500	S.W.G.	50

RELATIONS BETWEEN WEIGHTS, LENGTHS, AND RESISTANCES.

auge ame.	Gauge No.	Metres per Ohm at 20° C.	Feet per Ohm at 20° C.	Kilograms per Ohm at 20° C.	Lbs. per Ohm at 20° C.	Metres per Kilogram.	Feet per lb.	Kilo-grams per Kilo-metre (Bare).	Lbs. per 1000′ (Bare).	Gauge Name.	Gauge No.
W.G.	7/0	7380	24200	8300	18300	·89	1·32	1126	756	S.W.G.	7/0
mm.	...	6550	21400	6600	14600	·995	1·48	1010	680	12 mm.	...
W.G.	6/0	6370	20900	6170	13600	1·04	1·54	970	651	S.W.G.	6/0
& S.	0000	6215	20400	5940	13100	1·06	1·56	955	641	B. & S.	0000
W.G.	0000	6065	19900	5620	12400	1·08	1·60	930	624	B.W.G.	0000
W.G.	5/0	5500	18100	4620	10200	1·19	1·77	840	564	S.W.G.	5/0
W.G.	000	5320	17500	4330	9540	1·23	1·83	815	547	B.W.G.	000
& S.	000	4950	16200	3740	8230	1·32	1·97	758	508	B. & S.	000
W.G.	4/0	4720	15500	3400	7500	1·39	2·07	720	484	S.W.G.	4/0
mm.	...	4550	14900	3180	7030	1·425	2·12	701	471	10 mm.	...
W.G.	00	4260	14000	2760	6100	1·54	2·29	650	437	B.W.G.	00
W.G.	000	4100	13400	2540	5600	1·61	2·39	622	419	S.W.G.	000
& S.	00	3950	12900	2350	5180	1·67	2·48	600	403	B. & S.	00
mm.	...	3690	12100	2090	4625	1·76	2·61	567	382	9 mm.	...
W.G.	00	3600	11800	1950	4300	1·84	2·73	545	366	S.W.G.	00
W.G.	0	3400	11200	1780	3910	1·92	2·86	522	350	B.W.G.	0
& S.	0	3100	10200	1480	3260	2·10	3·13	477	320	B. & S.	0
W.G.	0	3100	10200	1460	3220	2·12	3·15	475	318	S.W.G.	0
mm.	...	2910	9550	1300	2870	2·23	3·31	448	302	8 mm.	...
W.G.	1	2660	8730	1080	2380	2·47	3·67	405	272	S.W.G.	1
W.G.	1	2650	8690	1075	2370	2·47	3·67	405	272	B.W.G.	1
& S.	1	2450	8080	930	2050	2·65	3·95	376	253	B. & S.	1
W.G.	2	2370	7790	860	1900	2·75	4·10	364	244	B.W.G.	2
W.G.	2	2245	7370	770	1700	2·92	4·34	342	230	S.W.G.	2
W.G.	3	1970	6480	600	1320	3·30	4·93	302	203	B.W.G.	3
& S.	2	1960	6410	585	1290	3·35	4·98	300	201	B. & S.	2
W.G.	3	1870	6150	535	1180	3·50	5·20	285	192	S.W.G.	3
W.G.	4	1660	5470	425	933	3·92	5.83	256	172	B.W.G.	4
mm.	...	1640	5390	414	915	3·97	5·90	252	169	6 mm.	...
W.G.	4	1590	5210	385	849	4·12	6·14	243	163	S.W.G.	4
& S.	3	1540	5080	367	810	4·20	6·28	237	159	B. & S.	3
W.G.	5	1420	4680	310	685	4·60	6·83	219	147	B.W.G.	5
W.G.	5	1320	4350	269	592	4·95	7·35	202	136	S.W.G.	5
& S.	4	1230	4030	230	509	5·31	7·91	188	126	B. & S.	4
W.G.	6	1210	3980	225	497	5·40	8·02	186	125	B.W.G.	6
mm.	...	1140	3750	200	440	5·69	8·48	175·5	118	5 mm.	...
W.G.	6	1090	3580	180	398	6·00	8·97	167	112	S.W.G.	6
& S.	5	975	3200	146	320	6·70	9·98	149	100	B. & S.	5
W.G.	7	950	3130	140	307	6·85	10·2	146	98·1	B.W.G.	7
W.G.	7	915	3000	127	280	7·20	10·7	140	93·7	S.W.G.	7
W.G.	8	800	2630	98	217	8·10	12·1	123	82·4	B.W.G.	8
& S.	6	775	2540	91	202	8·45	12·6	118	79·5	B. & S.	6
W.G.	8	760	2480	87	192	8·70	12·9	116	77·4	S.W.G.	8
mm.	...	730	2390	81·9	180	8·9	13·2	112	75·7	4 mm.	...
W.G.	9	645	2120	68	140	10·15	15·1	99	66·3	B.W.G.	9
& S.	7	610	2010	57·5	127	10·70	15·9	94	63·0	B. & S.	7
W.G.	9	615	2020	57·5	127	10·70	15·9	93·5	62·7	S.W.G.	9
W.G.	10	525	1730	42·6	94·3	12·40	18·4	81·2	54·4	B.W.G.	10
& S.	8	490	1600	36·0	79·7	13·40	20·0	74·5	50·0	B. & S.	8
W.G.	10	485	1590	35·5	78·5	13·60	20 2	74·00	49·6	S.W.G.	10

RELATIONS BETWEEN WEIGHTS, LENGTHS, AND RESISTANCES—*continued.*

Gauge Name.	Gauge No.	Metres per Ohm at 20° C.	Feet per Ohm at 20° C.	Kilograms per Ohm at 20° C.	Lbs. per Ohm at 20° C.	Metres per Kilogram.	Feet per lb.	Kilo-grams per Kilo-metre (Bare).	Lbs. per 1000′ (Bare).	Gauge Name.	Gauge No.
B.W.G.	11	425	1390	27·5	60·6	15·40	22·9	65·00	43·6	B.W.G.	11
3 mm.	...	410	1342	25·9	57·0	15·8	23·5	63·2	42·5	3 mm.	...
S.W.G.	11	395	1300	24	53·0	16·50	24·6	60·80	40·7	S.W.G.	11
B. & S.	9	385	1270	22·7	50·1	16·90	25·2	59·00	39·6	B. & S.	9
B.W.G.	12	350	1150	18·8	41·3	18·70	27·8	53·8	36·0	B.W.G.	12
S.W.G.	12	320	1050	15·5	34·2	20·50	30·6	48·7	32·7	S.W.G.	12
B. & S.	10	305	1000	14·2	31·5	21·40	31·8	46·9	31·4	B. & S.	10
B.W.G.	13	265	872	10·8	23·8	24·50	36·6	40·5	27·3	B.W.G.	13
S.W.G.	13	250	820	9·5	21·0	26·2	39·1	38·00	25·6	S.W.G.	13
B. & S.	11	240	795	9·0	19·8	27·00	40·1	37·00	24·9	B. & S.	11
B.W.G.	14	208	665	6·3	13·9	32·20	48·0	31·20	20·9	B.W.G.	14
B. & S.	12	192	631	5·68	12·5	34·30	50·6	29·50	19·8	B. & S.	12
S.W.G.	14	189	621	5·50	12·1	34·60	51·6	29·00	19·4	S.W.G.	14
2 mm.	...	182·5	599	5·11	11·2	35·5	53·0	28·2	18·85	2 mm.	...
B.W.G.	15	152	501	3·55	7·86	42·50	63·7	23.40	15·7	B.W.G.	15
S.W.G.	15	153	503	3·59	7·90	42·70	63·8	23·40	15·7	S.W.G.	15
B. & S.	13	152	500	3·55	7·84	42·70	63·8	23·40	15·7	B. & S.	13
B.W.G.	16	124	408	2·86	5·22	52·50	78·2	19·10	12·8	B.W.G.	16
B. & S.	14	120	397	2·23	4·93	54·00	80·4	18·50	12·4	B. & S.	14
S.W.G.	16	120	397	2·23	4·91	54·20	80·7	18·50	12·4	S.W.G.	16
B.W.G.	17	99	325	1·50	3·31	66·00	98·2	15·20	10·2	B.W.G.	17
B. & S.	15	96	315	1·40	3·10	68·00	101	14·70	9·86	B. & S.	15
S.W.G.	17	92	304	1·30	2·88	70·50	105	14·15	9·49	S.W.G.	17
B. & S.	16	78	249	·885	1·95	86·00	128	11·80	7·82	B. & S.	16
B.W.G.	18	71	232	·767	1·69	92·70	138	10·30	7·27	B.W.G.	18
S.W.G.	18	68·2	224	·707	1·56	96·10	143	10·39	6·97	S.W.G.	18
B. & S.	17	60·4	198	·558	1·23	108	161	9·23	6·20	B. & S.	17
B.W.G.	19	51·8	170	·413	·910	126	187	7·95	5·34	B.W.G.	19
B. & S.	18	47·8	157	·350	·772	136	208	7·33	4·92	B. & S.	18
S.W.G.	19	47·3	155	·340	·750	139	207	7·20	4·84	S.W.G.	19
1 mm.	...	45·5	149	·320	·705	142	212	7·0	4·73	1 mm.	...
S.W.G.	20	38·4	126	·222	·491	171	258	5·84	3·92	S.W.G.	20
B. & S.	19	37·8	124	·220	·485	173	257	5·81	3·90	B. & S.	19
·9 mm.	...	37·0	121	·210	·460	176	263	5·6	3·79	·9 mm.	...
B.W.G.	20	36·0	118	·199	·439	182	270	5·52	3·71	B.W.G.	[illegible]
S.W.G.	21	30·2	99·0	·139	·307	217	323	4·62	3·10	S.W.G.	[illegible]
B.W.G.	21	30·2	98·9	·139	·307	217	323	4·62	3·10	B.W.G.	[illegible]
B. & S.	20	30·2	98·7	·138	·305	217	323	4·62	3·10	B. & S.	[illegible]
·8 mm.	...	29·1	95·0	·130	·285	223	332	4·48	3·0	·8 mm.	...
B. & S.	21	23·7	77·5	·0870	·192	274	408	3·65	2·45	B. & S.	[illegible]
B.W.G.	22	23·0	75·5	·0816	·180	283	421	3·53	2·37	B.W.G.	[illegible]
S.W.G.	22	23·0	75·8	·0816	·180	284	422	3·53	2·37	S.W.G.	[illegible]
·7 mm.	...	22·5	73·5	·077	·169	290	434	3·42	2·3	·7 mm.	...
B. & S.	22	18·9	62·1	·0548	·121	346	514	2·90	1·95	B. & S.	[illegible]
B.W.G.	23	18·4	60·0	·0516	·114	356	529	2·82	1·89	B.W.G.	[illegible]
S.W.G.	23	17·0	55·5	·0442	·0975	386	570	2·59	1·74	S.W.G.	[illegible]
·6 mm.	...	16·4	53·5	·041	·090	398	593	2·5	1·68	·6 mm.	...
B. & S.	23	15·0	49·2	·0348	·0759	435	648	2·29	1·54	B. & S.	[illegible]
B.W.G.	24	14·2	46·8	·0308	·068	460	683	2·19	1·47	B.W.G.	[illegible]
S.W.G.	24	14·2	46·8	·0308	·0685	460	683	2·18	1·46	S.W.G.	[illegible]
B. & S.	24	11·8	38·6	·0216	·0477	550	818	1·82	1·22	B. & S.	[illegible]
B.W.G.	25	11·8	38·6	·0216	·0468	555	826	1·80	1·21	B.W.G.	[illegible]
S.W.G.	25	11·8	38·6	·0216	·0468	555	826	1·80	1·21	S.W.G.	[illegible]

RELATIONS BETWEEN WEIGHTS, LENGTHS, AND RESISTANCES—*continued.*

Gauge Name.	Gauge No.	Metres per Ohm at 20° C.	Feet per Ohm at 20° C.	Kilograms per Ohm at 20° C.	Lbs. per Ohm at 20° C.	Metres per Kilogram.	Feet per lb.	Kilo-grams per Kilo-metre (Bare).	Lbs. per 1000′ (Bare).	Gauge Name.	Gauge No.
·5 mm.	...	11·4	37·2	·0198	·0435	580	850	1·74	1·17	·5 mm.	...
B.W.G.	26	9·5	31·0	·0139	·0307	686	1020	1·46	·981	B.W.G.	26
S.W.G.	26	9·5	31·0	·0139	·0308	686	1020	1·46	·980	S.W.G.	26
B. & S.	25	9·45	30·8	·0136	·0300	693	1030	1·45	·97	B. & S.	25
S.W.G.	27	7·95	26·1	·00960	·0212	827	1230	1·21	·814	S.W.G.	27
B.W.G.	27	7·53	24·7	·00865	·0191	868	1290	1·15	·775	B.W.G.	27
B. & S.	26	7·46	24·5	·00857	·0189	875	1300	1·14	·769	B. & S.	26
·4 mm.	...	7·3	24	·00815	·0178	894	1350	1·12	·740	·4 mm.	...
S.W.G.	28	6·46	21·2	·00639	·0141	1015	1510	·986	·663	S.W.G.	28
B. & S.	27	5·94	19·5	·00539	·0119	1100	1640	·908	·610	B. & S.	27
B.W.G.	28	5·76	18·9	·00508	·0112	1135	1690	·882	·593	B.W.G.	28
S.W.G.	29	5·46	17·9	·00454	·0100	1203	1790	·834	·560	S.W.G.	29
B.W.G.	29	4·97	16·3	·00378	·00835	1319	1960	·762	·512	B.W.G.	29
B. & S.	28	4·69	15·4	·00337	·00747	1390	2070	·720	·484	B. & S.	28
S.W.G.	30	4·54	14·9	·00314	·00684	1444	2150	·692	·465	S.W.G.	30
B.W.G.	30	4·24	13·9	·00275	·00606	1540	2200	·650	·436	B.W.G.	30
B. & S.	29	4·1	13·4	·00258	·00570	1580	2360	·625	·41	B. & S.	29
S.W.G.	31	3·96	13·0	·00240	·00530	1653	2460	·606	·407	S.W.G.	31
S.W.G.	32	3·45	11·3	·00181	·00399	1900	2830	·525	·353	S.W.G.	32
B. & S.	30	2·96	9·71	·00134	·00295	2210	3290	·452	·304	B. & S.	30
B.W.G.	31	2·95	9·66	·00132	·00292	2218	3300	·445	·303	B.W.G.	31
S.W.G.	33	2·96	9·70	·00133	·00294	2224	3310	·451	·303	S.W.G.	33
S.W.G.	34	2·50	8·20	·000952	·00210	2628	3910	·381	·256	S.W.G.	34
B.W.G.	32	2·36	7·82	·000870	·00192	2741	4080	·365	·245	B.W.G.	32
B. & S.	31	2·34	7·70	·000844	·00186	2790	4150	·359	·241	B. & S.	31
S.W.G.	35	2·09	6·85	·000666	·00147	3145	4680	·318	·213	S.W.G.	35
B.W.G.	33	1·89	6·18	·000544	·00120	3470	5160	·289	·194	B.W.G.	33
B. & S.	32	1·86	6·11	·000530	·00117	3515	5230	·284	·191	B. & S.	32
·2 mm.	...	1·82	6·0	·000505	·00111	3600	5400	·277	·185	·2 mm.	...
S.W.G.	36	1·71	5·60	·000442	·000975	3840	5720	·261	·175	S.W.G.	36
B. & S.	33	1·48	4·84	·000333	·000735	4430	6590	·226	·152	B. & S.	33
B.W.G.	34	1·44	4·73	·000318	·000702	4525	6740	·220	·148	B.W.G.	34
S.W.G.	37	1·37	4·49	·000283	·000625	4820	7150	·208	·140	S.W.G.	37
B. & S.	34	1·17	3·84	·000210	·000462	5580	8310	·179	·120	B. & S.	34
S.W.G.	38	1·05	3·45	·000172	·000379	6170	9180	·162	·109	S.W.G.	38
B. & S.	35	9·30	3·05	·000132	·000291	7060	10500	·142	·0954	B. & S.	35
S.W.G.	39	·798	2·62	·0000970	·000214	8200	12200	·122	·0818	S.W.G.	39
B. & S.	36	·735	2·41	·0000830	·000183	8875	13200	·118	·0757	B. & S.	36
B.W.G.	35	·735	2·41	·0000830	·000183	8875	13200	·118	·0757	B.W.G.	35
S.W.G.	40	·683	2·24	·0000707	·000156	9610	14300	·104	·0697	S.W.G.	40
B. & S.	37	·585	1·92	·0000522	·000115	10220	16700	·0894	·0600	B. & S.	37
S.W.G.	41	·574	1·88	·0000499	·000110	11500	17100	·0874	·0586	S.W.G.	41
B.W.G.	36	·472	1·55	·0000339	·0000748	13900	20700	·0720	·0484	B.W.G.	36
S.W.G.	42	·472	1·55	·0000340	·0000750	13900	20700	·0720	·0484	S.W.G.	42
B. & S.	38	·463	1·52	·0000327	·0000721	14120	21000	·0709	·0476	B. & S.	38
·1 mm.	...	·455	1·49	·000032	·0000705	14200	21200	·07	·0473	·1 mm.	...
S.W.G.	43	·382	1·25	·0000223	·0000492	17130	25500	·0584	·0392	S.W.G.	43
B. & S.	39	·366	1·20	·0000206	·0000455	17800	26500	·0561	·0377	B. & S.	39
S.W.G.	44	·302	·990	·0000139	·0000306	21700	32300	·0462	·0310	S.W.G.	44
B. & S.	40	·291	·955	·0000130	·0000286	22440	33400	·0445	·0299	B. & S.	40
S.W.G.	45	·231	·758	·00000816	·0000180	28400	42200	·0353	·0237	S.W.G.	45
S.W.G.	46	·171	·560	·00000442	·00000975	38600	57400	·0259	·0174	S.W.G.	46
S.W.G.	47	·118	·388	·00000213	·00000469	55500	82600	·0180	·0121	S.W.G.	47
S.W.G.	48	·0756	·248	·000000875	·00000193	86750	126000	·0115	·00774	S.W.G.	48
S.W.G.	49	·0424	·139	·000000274	·000000605	154800	230000	·00650	·00436	S.W.G.	49
S.W.G.	50	·0296	·0970	·000000133	·000000293	218500	331000	·00451	·00303	S.W.G.	50

TABLE LXV.—VALUES OF σ IN MOTORS BY VARIOUS MANUFACTURERS.

Reference No.	Manufacturer.	Number of Poles.	Squirrel Cage S or Wound Rotor W.	C″ for Sq. Cage Rotor.	H_1 Number of Stator Slots.	H_2 Number of Rotor Slots.	H Average of Stator and Rotor Slots per pole.	X Average Slot Opening for Stator and Rotor.	λ_g Gross Length of Core.	τ Polar Pitch at Air Gap.	Δ.	$\frac{\lambda_g}{\tau}$.	$\Delta \times$ H.	In Behrend's Formula $\sigma = CC' \frac{\Delta}{\tau}$. C	C′	Estimated Value of σ.	Observed Value of σ.	Disagreement between Estimated and Observed Values in per cent. of Observed Values σ.
1	Oerlikon	4	W	...	72	96	21·0	0·10	10	22·8	0·065	0·44	1·37	16·0	1·11	0·051	0·06	15
2	,,	6	W	...	72	96	14·0	0·10	10	15·2	0·065	0·66	0·91	14·5	1·26	0·078	0·09	13
3	,,	4	W	...	72	96	21·0	0·10	14·5	22·8	0·065	0·64	1·37	14·7	1·11	0·047	0·045	4
4	,,	6	W	...	72	96	14·0	0·10	14·5	15·2	0·065	0·94	0·91	13·2	1·26	0·071	0·075	5
5	,,	6	W	...	72	120	16·0	0·10	19·0	25·7	0·080	0·74	1·28	14·1	1·13	0·050	0·05	0
6	,,	8	W	...	72	120	12·0	0·10	19·0	19·3	0·080	0·99	0·96	18·1	1·25	0·068	0·063	8
7	,,	6	W	...	72	120	16·0	0·10	28·0	25·7	0·080	1·09	1·28	12·6	1·13	0·044	0·042	5
8	,,	8	W	...	72	120	12·0	0·10	28·0	19·3	0·080	1·45	0·96	11·8	1·25	0·061	0·056	9
9	,,	12	W	...	144	180	13·5	0·10	17·0	23·6	0·100	0·72	1·35	14·1	1·11	0·066	0·067	2
10	,,	12	W	...	144	180	13·5	0·10	40·0	23·6	0·100	1·70	1·35	11·7	1·11	0·055	0·046	20
11	,,	8	W	...	96	144	15·0	0·10	24·0	22·8	0·090	1·05	1·35	12·6	1·11	0·055	0·054	2
12	,,	12	W	...	144	180	13·5	0·0	32·5	23·6	0·110	1·38	1·49	12·5	1·06	0·062	0·070	11
13	,,	12	W	...	144	180	13·5	0·35	32·5	23·6	0·110	1·38	1·49	10·3	1·06	0·051	0·06	15
14	,,	8	W	...	96	144	15·0	0	24·0	22·8	0·090	1·05	1·35	13·3	1·11	0·058	0·054	7
15	,,	8	W	...	96	144	15·0	0·70	24·0	22·8	0·110	1·05	1·65	10·6	1·01	0·052	0·062	16
16	,,	14	W	...	168	210	13·5	0·15	28·0	33·8	0·100	0·85	1·35	18·1	1·11	0·048	0·046	7
17	,,	14	W	...	168	210	13·5	0·15	28·0	33·8	0·140	0·85	1·88	13·1	0·96	0·052	0·055	5
18	,,	12	W	...	108	144	10·5	0	40·0	24·6	0·100	1·63	1·05	12·4	1·20	0·061	0·054	13
19	,,	8	W	...	120	160	17·5	0·10	30·0	22·8	0·070	1·31	1·22	12·0	1·14	0·042	0·042	0
20	,,	4	W	...	120	160	35·0	0·10	30·0	45·6	0·070	0·66	2·44	14·5	0·85	0·0189	0·022	14
21	,,	12	W	...	144	180	13·5	0·15	23·0	23·6	0·100	0·98	1·35	12·9	1·09	0·060	0·067	10
22	,,	6	W	...	144	180	27·0	0·15	23·0	47·2	0·100	0·49	2·7	15·3	0·86	0·0260	0·038	21
23	,,	8	W	...	72	120	12·7	0·10	19·0	19·3	0·070	0·99	0·84	12·9	1·30	0·061	0·064	5
24	,,	4	W	...	72	120	24·0	0·10	19·0	38·5	0·070	0·52	1·68	15·0	1·01	0·0276	0·034	19
25	,,	12	W	...	144	180	13·5	0·15	32·0	23·6	0·100	1·36	1·35	11·8	1·09	0·055	0·06	8

26	,,	8	W	...	144	180	20·3	0·15	32·0	35·4	0·100	0·91	2·03	12·9	0·92	0·0335	0·043	22
27	,,	6	W	...	54	72	10·5	0·20	24·0	19·9	0·080	1·21	0·84	11·8	1·29	0·061	0·054	13
28	,,	6	W	...	108	144	21·0	0·10	24·0	19·9	0·080	1·21	1·68	11·4	1·02	0·047	0·039	21
29	,,	16	W	...	144	192	10·5	0	22·0	29·6	0·150	0·75	1·57	14·6	1·05	0·078	0·075	4
30	,,	16	W	...	192	216	12·8	0·25	22·0	29·6	0·150	0·75	1·92	13·2	0·94	0·063	0·065	3
31	,,	8	S	0·75	96	144	15·0	0·10	24·0	22·8	0·090	1·05	1·35	12·6	1·09	0·041	0·037	11
32*	A. E. G.	6	W	...	90	140	19·2	0·25	16·6	28·0	0·100	0·72	1·92	12·9	0·94	0·053	0·042	26
33†	,,	10	W	...	150	252	20·0	0·35	25·0	26·8	0·150	0·94	3·00	12·0	0·75	0·051	0·065	22
34‡	S. & H.	8	W	...	120	72	12·0	0·20	25·0	20·6	0·125	1·22	1·50	12·0	1·06	0·077	0·089	14
35	Alioth	14	W	...	168	294	16·5	0·20	40·0	29·2	0·125	1·37	2·06	12·0	0·92	0·47	0·050	6
36	,,	30	W	...	450	720	19·5	0·25	75·0	31·4	0·175	2·39	3·40	10·6	0·70	0·41	0·036	14
37	,,	12	W	...	180	216	16·5	0·30	36·0	23·0	0·150	1·56	2·46	10·7	0·85	0·59	0·052	14
38	Almanna Svenska Elektriska Aktiebolaget.	4	S	0·75	72	105	22·1	0·25	20	29·9	0·19	0·67	4·2	14·6	0·65	0·045	0·034	32
39		6	W	...	72	105	14·8	0·25	20	19·9	0·10	1·01	1·48	12·5	1·07	0·067	0·066	2
40		8	W	...	72	105	11·1	0·25	20	15·0	0·11	1·34	1·22	11·5	1·14	0·096	0·081	19
41		10	W	...	90	105	9·8	0·25	20	11·9	0·11	1·68	1·07	11·2	1·20	0·124	0·099	25
42		4	W	...	60	84	18·0	0·25	21·5	39·4	0·15	0·55	2·7	14·8	0·80	0·045	0·051	12
43		6	W	...	108	126	19·5	0·25	21·5	26·3	0·16	0·82	3·13	13·1	0·74	0·059	0·058	2
44		8	W	...	120	141	16·3	0·25	21·5	19·7	0·13	1·09	2·12	12·2	0·91	0·073	0·065	12
45		10	W	...	120	141	13·1	0·25	21·5	15·8	0·14	1·37	1·83	11·6	0·97	0·100	0·078	28
46		16	W	...	192	288	15·0	0·3	42	20·8	0·145	2·03	2·18	11·0	0·91	0·070	0·062	13
47		28	W	...	420	378	14·3	0·3	22·5	19·2	0·16	1·17	2·28	11·5	0·88	0·084	0·073	15
48		72	W	...	864	648	10·5	0·4	40·4	13·7	0·23	2·94	2·42	10·7	0·85	0·153	0·129	19
49	Mavor & Coulson	4	Sq.	0·75	48	67	14·4	0·30	15·2	24	0·08	0·68	1·12	14	1·2	0·042	0·056	25
50	,,	4	Sq.	0·75	72	119	24	0·35	30·4	36	0·25	0·84	6	12	·63	0·04	0·048	17
51	Firm A	8	W	...	72	96	10·5	0·40	11·4	14·0	0·082	0·82	0·360	12·0	1·3	0·09	0·143	37
52	,,	8	W	...	72	96	10·5	0·40	15·8	14·0	0·082	1·13	0·860	10·6	1·3	0·080	0·112	20
53	,,	8	W	...	72	120	12·0	0·40	14·6	20·8	0·114	0·70	1·37	13·3	1·1	0·08	0·091	12
54	,,	4	S	0·75	72	67	17·4	0·40	19·7	41·6	0·114	0·475	1·98	14·9	0·94	0·029	0·0297	2
55	,,	6	W	...	108	126	19·5	0·40	22·3	39·6	0·158	0·565	3·12	14·0	·74	0·041	0·0365	12
56	Brown, Boveri & Co.	6	W	...	54	72	10·5	0·3	20·0	16·8	0·075	1·19	0·79	11·6	1·34	0·069	0·055	2
57	,,	8	W	...	72	96	10·5	0·3	28·0	19·7	0·100	1·42	1·05	11·0	1·21	0·068	0·060	13

* Item 32 is from Kapp's Elek. Kon., p. 174. † Item 33 is from Kapp's Elek. Kon., p. 161. ‡ Item 34 is from Kapp's Elec. Kon., p. 261.

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